

## STRESS ANALYSIS OF LINERS FOR PRESTRESSED CONCRETE PRESSURE VESSELS

J.V. PARKER,

*The Nuclear Power Group Ltd., Knutsford, Cheshire, United Kingdom*

### ABSTRACT

The paper presents the methods used by the Author since 1962 for the stress analysis of prestressed concrete pressure vessel (PCPV) liners. Whilst a finite element analysis of the PCPV can include the liner as a structural component, the behaviour of the liner near structural discontinuities in either the vessel or the liner, and also in the region of high strain gradients, can be analysed independently of the concrete provided that the load-deflexion characteristics of the liner to concrete shear connectors or anchorages are known.

The analysis takes into account the non-linear load-deflexion characteristics of the shear connectors and assumes that the liner material is ideally elastic-plastic. The idealized behaviour of liner panels in biaxial compression and the idealized behaviour of shear connectors are based upon laboratory experiments.

Practical applications of the methods are presented with particular reference to the PCPV liners for the Hinkley Point 'B' and Hunterston 'B' AGR Power Stations.

### 1 INTRODUCTION

The cavity liner to a prestressed concrete reactor pressure vessel (PCPV) usually contributes very little to the structural behaviour of the vessel. Its main function is, of course, to provide a pressure boundary and structural strength of the liner is not a primary requirement.

Normally a steel liner is attached to the concrete at discrete points by studs or their equivalent which are referred to as ties or shear connectors depending upon the nature of their duty. Ties are necessary in order to limit deflexions of the liner away from the concrete while shear connectors are necessary in order to maintain reasonable compatibility between the strain distribution over the lined surface of the concrete and the in-plane or membrane strains in the liner. The forces generated in the liner and shear connectors are due to deformation of the vessel and the restrained thermal expansion between the liner and the vessel and are thus strain controlled.

Because of the discrete nature of the attachment of the liner to the concrete it is desirable to analyse the behaviour of the liner and these connections (the ties and shear connectors) to the concrete as a problem divorced from the analysis of the complete vessel. Indeed, while it is a simple matter to include a representation of the liner in a finite element idealization of the PCPV, it is not really practical to include the ties and shear connectors.

In fact the inclusion of the liner in such an analysis is, in many cases, not essential due to its small strength or stiffness contribution to that of the vessel. Consequently, the starting point for the design and analysis of the cavity liner must be the distributions of cavity surface strains obtained from an analysis of the PCPV either including or ignoring the strength of the liner. This approach also has the advantage that the inclusion of non-linear and time-dependent effects such as plasticity, local crushing, cracking and creep can be taken into account more readily.

This paper discusses methods of PCPV liner analysis which have been used for vessels built by The Nuclear Power Group (TNP) Ltd. since 1962. The methods rely partially on basic experimental test work carried out by TNP Ltd. and by Whessoe Ltd. of Darlington, U.K.

## 2 FUNDAMENTALS OF THE UNCOUPLED APPROACH

If  $\epsilon_c$  is the compressive strain at a point on the inside surface of the concrete ignoring the presence of the liner and  $\alpha T$  is the thermal strain in the liner at this point (or adjacent to it), then, assuming that the liner is fully adhered to the concrete, forces will be present in the liner corresponding to some compressive strain  $\gamma$ . These liner forces are equilibrated by bearing forces between the shear connectors and the concrete and by shearing forces between the liner panels (and other attachments) and the concrete.

If we assume that these bearing and shearing forces do not deform the concrete, then

$$\begin{aligned}\gamma &= \epsilon_c + \alpha T \\ &= \epsilon, \text{ say} \quad \text{where } \epsilon = \epsilon_c + \alpha T\end{aligned}\quad (1)$$

$\epsilon$  is the strain which the liner has to be designed to accommodate. The true liner strain  $\gamma$ , however, depends upon the design of the liner anchorage and/or keying system.

Analysis of the liner is strictly a two-dimensional problem but its behaviour both generally and locally may be represented essentially by the one-dimensional model shown in Fig 1.

In this one-dimensional model the liner is idealized as an assemblage of panels  $i-1, i$  having a constant radius of curvature  $a_{i-1, i}$  in the  $xz$  plane and shear connectors and/or ties  $i$  making angles  $\theta_{i, i-1}$  and  $\theta_{i, i+1}$  with the directions normal to panels  $i-1, i$  and  $i, i+1$  respectively at  $i$ . The system is cylindrical in the  $y$  direction and consequently

$$\gamma_y = \epsilon_y \quad (2)$$

Furthermore we consider the liner to be in a membrane state only or, if local bending of panels does occur, the in-plane forces dictate the average behaviour.

- Let  $P$  = compressive force in liner panel in direction  $x$
- $p$  = bearing pressure between liner panel and concrete
- $q$  = shear per unit length of panel between liner panel and concrete

Then the equilibrium conditions for any point on panel  $i-1, i$  are (Fig 2)

$$p = \frac{P}{a_{i-1, i}} \quad (3)$$

$$q = \frac{dP}{ds} \quad (4)$$

integrating eq (4) over the length  $l_{i-1,i}$  of panel  $i-1,i$  gives

$$l_{i-1,i} q_{i-1,i} = P_{i,i-1} - P_{i-1,i} \quad (4a)$$

in which  $q_{i-1,i}$  is the average shear flow over panel  $i-1,i$ .

Also, if  $F_i$  = bearing force between shear connector  $i$  and the concrete  
and  $T_i$  = tension in shear connector or tie  $i$  then we have, referring to Fig 2,  
the equilibrium conditions

$$F_i = P_{i,i+1} \cos \theta_{i,i+1} - P_{i,i-1} \cos \theta_{i,i-1} \quad (5)$$

and

$$T_i = P_{i,i-1} \sin \theta_{i,i-1} - P_{i,i+1} \sin \theta_{i,i+1} \quad (6)$$

eq (6) being valid only for positive values of  $T_i$ .

If  $u_s$  and  $w_s$  are the displacements of any point on liner panel  $i-1,i$  in directions  $x$  and  $z$  respectively (Fig 3) and  $u_c$  and  $w_c$  are the corresponding displacements at an adjacent point on the surface of the concrete, then the liner and concrete strains in direction  $x$  are respectively

$$\epsilon_x = \frac{du_s}{ds} + \frac{w_s}{a_{i-1,i}} \quad (7a)$$

$$\text{and } \epsilon_x = \frac{du_c}{ds} + \frac{w_c}{a_{i-1,i}} \quad (7b)$$

Introducing the relative displacements

$$u = u_s - u_c$$

$$\text{and } w = w_s - w_c$$

We obtain the compatibility conditions

$$\frac{du}{ds} = \epsilon_x - \epsilon_x - \frac{w}{a_{i-1,i}} \quad (8)$$

which becomes, on integrating over the length of the panel

$$u_{i,i-1} - u_{i-1,i} = (\epsilon_{x_{i-1,i}} - \epsilon_{x_{i-1,i}} - \frac{w_{i-1,i}}{a_{i-1,i}}) l_{i-1,i} \quad (8a)$$

in which  $\epsilon_{x_{i-1,i}}$ ,  $\epsilon_{x_{i-1,i}}$  and  $w_{i-1,i}$  are average values.

Further, referring to Fig 3, the shear connector displacement is

$$r_i = u_{i,i-1} \cos \theta_{i,i-1} = u_{i,i+1} \cos \theta_{i,i+1} \quad (9)$$

hence the compatibility condition given by eq (8a) can be written as

$$\frac{r_i}{\cos \theta_{i,i-1}} - \frac{r_{i-1}}{\cos \theta_{i-1,i}} = (\gamma_{x_{i-1,i}} - \epsilon_{x_{i-1,i}} - \frac{w_{i-1,i}}{a_{i-1,i}}) l_{i-1,i} \quad (10)$$

Finally, load-displacement relationships may be formulated in the form

$$l_{i-1,i} \gamma_{i-1,i} = \psi_{i-1,i} (P_{i-1,i}, P_{i,i-1}, l_{i-1,i} \cdot q_{i-1,i}) \quad (11)$$

$$r_i = \phi_i (F_j, l_{k-1,k} \cdot q_{k-1,k}) \quad (12)$$

$$\text{and } w_i = \zeta_i (P, T_j, l_{k-1,k} \cdot q_{k-1,k}) \quad (13)$$

Eq (2) to (13) together with the appropriate boundary conditions and yield criteria completely define the problem. For practical problems however, these equations simplify dramatically and we now consider a number of useful special cases.

Since in general the shear connectors are normal to the plane of the liner panels,  $\theta = 0$ . It is also reasonable to take the shear between the liner panel and concrete surface as zero, i.e.  $q = 0$ . Panel  $i-1,i$  may then be referred to as panel  $i$  for convenience and we get, from eq (4a)

$$P_{i,i-1} = P_{i-1,i} = P_i$$

while from eq (6)

$$T_i = 0$$

Eq (3) and (5) then give

$$P_i = \frac{P_i}{a_i} \quad (14)$$

$$F_i = P_{i+1} - P_i \quad (15)$$

The compatibility condition defined by eq (10) becomes

$$r_i - r_{i-1} = (\gamma_{x_i} - \epsilon_{x_i} - \frac{w_i}{a_i}) l_i \quad (16)$$

If the load-displacement relationships are linearly elastic then eq (11), (12) and (13) take the form

$$P_i = C_i (\gamma_{x_i} + \nu_i \gamma_{y_i}) \quad (17)$$

$$F_i = K_i r_i \quad (18)$$

$$p_i = k_i w_i \quad (19)$$

Substituting  $\gamma_{x_i}$ ,  $r_i$  and  $w_i$  from eq (17), (18) and (19) in eq (16) and using eq (14) and (15) to eliminate  $p_i$  and  $F_i$  we get, making use of eq (2)

$$P_{i+1} - \left(1 + \frac{K_i}{K_{i-1}} - \frac{K_i l_i}{k_i a_i^2} + \mu_i^2\right) P_i + \frac{K_i}{K_{i-1}} P_{i-1} = -\mu_i^2 \bar{P}_i \quad (20)$$

in which

$$\mu_i^2 = \frac{K_i l_i}{C_i} \quad (21)$$

$$\bar{P}_i = C_i (\epsilon_x + \nu_i \gamma_{y_i}) \quad (22)$$

Now  $P_i$  will usually be known at the two extremities of the system under consideration. For example

$$P_i = P_0 \text{ at } i = 0$$

$$P_i = P_m \text{ at } i = m$$

These boundary conditions, together with eq (20) form  $m-1$  linear simultaneous equations in  $P$  from which  $P_i$  ( $i = 1$  to  $m-1$ ) can be found. Eq (14) and (15) can then be used to evaluate the average bearing pressures  $p_i$  and the shear connector forces  $F_i$ .

This linear problem is almost trivial for solution by computer and the inclusion of non-linearity is simple.

Liner panel characteristics are discussed in section 3 where it is argued that it is reasonable to idealize the load-displacement characteristic as the linear relationship given by eq (17) for values of  $P_i$  up to some value, referred to as the yield load  $P_{y_i}$ , which then remains constant for further increases in the strain quantity  $(\gamma_{x_i} + \nu_i \gamma_{y_i})$ . These idealized panel characteristics are shown in Fig 4. If we now consider three adjacent panels  $i-1, i$  and  $i+1$  to be in a state of yield and with identical yield load

$$\text{i.e. } P_{y_{i-1}} = P_{y_i} = P_{y_{i+1}} = P_y$$

Then from eq (14), (15), (16) and (19) we arrive at the interesting and important results

$$F_{i-1} = F_i = 0 \quad (23a)$$

$$r_{i-1} = r_i = 0 \quad (23b)$$

$$\gamma_{x_i} = \epsilon_{x_i} + \frac{P_y}{k_i a_i^2} \quad (23c)$$

Thus, the state of strain in panel  $i$  is dictated entirely by the behaviour of the concrete. Furthermore, it is clear that flow laws for yielded liner panels are generally not necessary.

Shear connector characteristics are discussed in Section 4 and the form of the load-displacement relationship which is illustrated in Fig 6 is taken as

$$F_i = \frac{a_{0i} + a_{1i} r_i + a_{2i} r_i^2 + a_{3i} r_i^3}{b_{0i} + b_{1i} r_i + b_{2i} r_i^2} \quad (24)$$

in which the  $a$ 's and  $b$ 's are constants.

We can obtain a solution to this non-linear problem by increasing the boundary values in small steps from zero to the prescribed final values thereby allowing  $r_i$  to vary slowly. Consequently we may linearize eq (24) for the  $s+1$  th step in which  $r_i$  varies from  $r_{i,s}$  to  $r_{i,s+1}$  by using the truncated Taylor series

$$\begin{aligned} F_{i,s+1} &= F_i(r_{i,s+1}) \\ &= F_i(r_{i,s}) + (r_{i,s+1} - r_{i,s}) F_i'(r_{i,s}) \\ &= F_{i,s} + (r_{i,s+1} - r_{i,s}) K_{i,s} \end{aligned}$$

in which  $K_{i,s} = \left( \frac{d F_i}{d r_i} \right)_s = F_i'(r_{i,s})$

i.e.  $r_{i,s+1} = \frac{F_{i,s+1}}{K_{i,s}} + r_{i,s} - \frac{F_{i,s}}{K_{i,s}}$  (25)

Using eq (15) we can rewrite eq (25) as

$$r_{i,s+1} = \frac{P_{i+1,s+1} - P_{i,s+1}}{K_{i,s}} + r_{i,s} - \frac{F_{i,s}}{K_{i,s}} \quad (26)$$

While substituting  $\gamma_{x_i}$  and  $w_i$  from eq (17) and (19) in eq (16) and using eq (14) to eliminate  $p_i$  we get, making use of eq (2)

$$\ddot{r}_{i,s+1} - r_{i-1,s+1} - \frac{1}{C_i} \left( 1 - \frac{C_i}{k_i a_i^2} \right) P_{i,s+1} = - \frac{1}{C_i} \bar{P}_{i,s+1} \quad (27)$$

from eq (26) and (27) we finally obtain

$$\begin{aligned} \frac{P_{i+1,s+1}}{K_{i,s}} - \left\{ \frac{1}{K_{i,s}} + \frac{1}{K_{i-1,s}} + \frac{1}{C_i} \left( 1 - \frac{C_i}{k_i a_i^2} \right) \right\} P_{i,s+1} + \frac{P_{i-1,s+1}}{K_{i-1,s}} \\ = - \frac{1}{C_i} \bar{P}_{i,s+1} - \left\{ \left( r_{i,s} - \frac{F_{i,s}}{K_{i,s}} \right) - \left( r_{i-1,s} - \frac{F_{i-1,s}}{K_{i-1,s}} \right) \right\} \end{aligned} \quad (28)$$

Clearly, for linear load-deflection characteristics as defined by eq (18)

$$r_{i,s} - \frac{F_{i,s}}{K_{i,s}} = 0$$

and eq (28) becomes identical with eq (20).

Let us now consider the special, but very common case in which the panels are identical and the shear connectors are identical and elastic. i.e.  $l_i = 1$ ,  $a_i = a$ ,  $C_i = C$ ,  $K_i = K$ ,  $k_i = k$ .

Then eq (20) becomes

$$P_{i+1} - (2 + \beta^2) P_i + P_{i-1} = -\mu^2 \bar{P}_i \quad (29)$$

in which

$$\begin{aligned} \beta^2 &= \mu^2 - \frac{Kl}{ka^2} \\ &= \mu^2 \left(1 - \frac{C}{ka^2}\right) \end{aligned} \quad (30)$$

The solution of the difference equation (eq (29)) consists of a complementary function given by

$$P_i = A_1 \eta^i + A_2 \eta^{-i} \quad (31)$$

in which  $A_1$  and  $A_2$  are constants and

$$\eta = \left(1 + \frac{\beta^2}{2}\right) + \sqrt{\left(1 + \frac{\beta^2}{2}\right)^2 - 1} \quad (32)$$

and a particular solution.

Taking, for example,

$$\bar{P}_i = \bar{P} = \text{constant},$$

the complete solution is

$$P_i = A_1 \eta^i + A_2 \eta^{-i} + \left(\frac{\mu}{\beta}\right)^2 \bar{P} \quad (33)$$

Using the boundary conditions

$$\left. \begin{aligned} P_i &= P_0 \text{ at } i = 0 \\ P_i &= P_m \text{ at } i = m \end{aligned} \right\} \quad (34)$$

$A_1$  and  $A_2$  may be obtained and we get finally

$$P_i = \left(\frac{\mu}{\beta}\right)^2 \bar{P} - \frac{\eta^{(m-i)} - \eta^{-(m-i)}}{\eta^m - \eta^{-m}} \left\{ \left(\frac{\mu}{\beta}\right)^2 \bar{P} - P_0 \right\} - \frac{\eta^i - \eta^{-i}}{\eta^m - \eta^{-m}} \left\{ \left(\frac{\mu}{\beta}\right)^2 \bar{P} - P_m \right\} \quad (35)$$

If  $P_m \rightarrow \bar{P}$  as  $m \rightarrow \infty$ , equation (35) becomes

$$P_i = \left(\frac{\mu}{\beta}\right)^2 \bar{P} - \eta^{-i} \left\{ \left(\frac{\mu}{\beta}\right)^2 \bar{P} - P_o \right\} \quad (36)$$

substituting  $P_i$  from eq (36) in eq (15) gives

$$F_i = \{ \eta^{-i} - \eta^{-(i+1)} \} \left\{ \left(\frac{\mu}{\beta}\right)^2 \bar{P} - P_o \right\} \quad (37)$$

### 3 PANEL CHARACTERISTICS

The relation between the load in a panel and the average panel strains may generally be idealized as shown in Fig 4. The linear portion is characterized by the slope C (as defined by eq (17)). On unloading, it is also assumed that the curve is characterized by the slope C.

Clearly, if we assume that the panel material is ideally elastic-plastic, then for a perfectly flat panel of unit width,

$$C = \frac{E h}{1 - \nu^2}$$

for the elastic behaviour

$$\text{and } P_y = h \sigma_y$$

where E = Young's modulus

and  $\sigma_y$  = Yield stress

In practice, panels have imperfections in geometry due to the manufacturing process of the liner. Some panels include welded seams which may have produced peaking while other panels may include some initial buckling. These initial imperfections can result in the panel deflecting locally away from the concrete surface due to the in-plane or membrane forces producing bending in the panel. This local behaviour has generally been referred to as liner or panel buckling, although it is not a buckling phenomena in the instability sense. It is, however, possible for a 'snap through' buckling phenomena to occur in curved panels if the tie pitch to panel thickness ratio is large (Chan and McMinn [1]).

While panel buckling can be minimised by increasing the liner thickness and/or reducing the pitch of the liner ties, it is not necessary to eliminate it completely. The influence of a buckled panel on the integrity of the liner must be investigated of course and this particular problem is discussed in section 6 where the ideas developed in section 2 are applied. It is fundamental, of course, to establish the load-strain characteristics of any buckled panels that are to be expected in a particular design of liner and this can only be done by experimental test work. Typical liner buckling test work has been described previously (Hardingham, Parker and Spruce [2], Young and Tate [3]) and we only refer to the results of such work in this paper. Examples of panel characteristics obtained from tests carried out for the Hinkley Point B PCPV cavity liner are shown in Fig 5. Naturally, for a buckled panel,



$$c < \frac{E h}{1 - \nu^2}$$

$$\text{and } P_y < h \sigma_y$$

#### 4 SHEAR CONNECTOR CHARACTERISTICS

As in the case of panels with initial imperfections, the simplest way of obtaining the important relation between load and deflexion of a shear connector is by experimental test work. Such tests are carried out on composite specimens and have been described by Hardingham, Parker and Spruce [2], Young and Tate [3], Chapman and Carter [4], Hughes [5], Mainstone and Menzies [6] and Davies [7]. The type of load-deflexion curve which is typical of most shear connectors is shown in Fig 6. In this idealized curve the unloading characteristic is shown to have the same slope as the initial loading characteristic. In most practical cases, the shear connectors will operate over this initial part of the curve only and the linear elastic relationship given by eq (18) applies. However, it is possible, in regions adjacent to gross structural discontinuities such as corners between cavity walls and floors or roofs, for larger deformations to take place and consequently the non-linear characteristic applies. In such cases, the test work should establish the form of the unloading and load reversal as well as the loading characteristics. Examples of shear connector characteristics obtained from some of the tests carried out for the Hinkley Point B PCFV liner are shown in Fig 7 and 8.

This is perhaps also an opportune time to refer to the design of the transition from the cavity floor and roof liners to the cavity wall liner at Hinkley Point B. This transition piece, or corner detail, consists of a solid bar of circular cross-section. The diameter of this 'corner bar' is such that the yield load in the liner is equilibrated by the nominal design bearing pressure on the concrete. However, using the ideas discussed in section 2, this corner bar is regarded in the analysis as a shear connector. Load-deflexion characteristics for this shear connector are shown in Fig 9, these curves being obtained from experimental test work. The tests show, as indicated by the characteristics (Fig 9), that there may exist an effective initial gap between the corner bar and the concrete prior to prestressing the vessel. Furthermore, we also assume that this corner bar cannot carry a reversal of load. Thus, the load-deflexion characteristic is of the form

$$F = K (r - r_g)$$

for  $r > r_g$  only

in which  $r_g$  is the effective initial gap.

#### 5 APPLICATIONS TO THE BEHAVIOUR OF A LINER AT A RIGHT-ANGLED CORNER

In order to investigate the behaviour of a liner at a right-angled corner and to establish the adequacy of the shear connectors and other liner anchorage details, the simple idealization shown in Fig 10 is applicable. Clearly the system is governed by the equations developed in section 2. Furthermore, if we select the origin (i.e. shear connector O) to be at the corner, then each branch, OA and OB (Fig 10), can be treated in an identical manner. The two systems OA and OB are uncoupled in the case of a right-angled corner, but for any

other angle, coupling occurs by means of eq (10).

For the branch under consideration, say OA, we have the boundary condition

$$P_0 = 0$$

If the behaviour of the system is elastic, then eq (20) can be used which may in some cases simplify to give eq (29). If the shear connector behaviour is inelastic, then eq (28) must be used.

This form of application of the ideas developed in section 2 is illustrated by the following example which is part of an investigation of the behaviour of the Hinkley Point B PCPV cavity liner in the region of the wall to floor corner. In this example we consider the wall liner only. This liner consists of  $\frac{1}{2}$ -in. thick mild steel plate with cooling pipes welded on vertically at approximately  $5\frac{1}{2}$ -in. pitch.  $\frac{3}{8}$ -in. diameter studs act as ties and shear connectors on a vertical pitch varying from 6 in. near the top and bottom corners to 9 in. over the central region and placed mid-way between alternate cooling pipes to form a staggered array. The top and bottom of the liner terminate with the solid corner bar detail discussed in section 4.

The corner bar is considered as shear connector 0 having characteristics based on the curve shown in Fig 9. The shear connector characteristics of the studs are based on curve (i) in Fig 8. The panels are defined by the vertical pitch of the shear connectors and the load-strain characteristics assessed by assuming pure membrane deformation, the material being assumed to be perfectly elastic-plastic and with the cooling pipe stiffness included in the vertical (x) direction.

We show here the results of analysis based on eq (28) and using the two extreme design strain distributions shown in Fig 11. These two design conditions correspond to:

- i) vessel subjected to prestress only but with the liner at its design temperature
- ii) vessel subjected to prestress plus operating pressure with the concrete at its steady state temperature distribution but with the liner cold.

Corresponding to these design conditions, the load distributions in the liner panels and shear connectors together with the shear connector displacements are shown in Fig 12 and 13. Fig 12 shows the response when there is significant initial gap between the corner bar and the concrete while Fig 13 shows the response when there is no initial gap.

## 6 APPLICATION TO THE EFFECT OF PANEL BUCKLING

Due to the so-called buckling, the load-strain characteristics of panels may vary throughout the liner in an arbitrary manner. From a design point of view it is convenient, and usually sufficient, to consider the extreme case in which a buckled panel, with the least load carrying capacity, is completely bounded by identical ideally flat panels with the highest load carrying capacity. In such a case, strains tend to accumulate in the buckled panel giving rise to increased displacements of the panel and increased duty of the shear connectors and ties.

This panel buckling effect is quite local and may be represented by the idealization shown in Fig 14. It is reasonable to consider uniform design strain conditions only and to achieve a

design in which the shear connectors are identical and loaded in the linear part of the characteristic. Consequently the analysis becomes very simple, since if we let  $P_0$  be the load in the buckled panel, the load in any other panel  $P_i$  is given generally by eq (35), with  $\beta = \mu$ .

If the flat panels remain elastic, then eq (35) can be simplified, or replaced by eq (36) to give

$$P_i = \bar{P} - \eta^{-i} (\bar{P} - P_0) \quad (38)$$

in which, (eq (22)),

$$\bar{P} = C (\epsilon_x + \nu \epsilon_y) = \text{constant}$$

If  $\bar{P} > P_y$ , the yield load for the flat panels, then some of the flat panels will yield, say  $i \geq m$ , in which case, for  $i \leq m$ , eq (35) gives

$$P_i = \bar{P} - \frac{\eta^{(m-i)} - \eta^{-(m-i)}}{\eta^m - \eta^{-m}} (\bar{P} - P_0) - \frac{\eta^i - \eta^{-i}}{\eta^m - \eta^{-m}} (\bar{P} - P_y) \quad (39)$$

while, for  $i \geq m$

$$P_i = P_y \quad (40)$$

the elastic-plastic boundary being defined by, using eq (23b)

$$r_m = 0 \quad (41)$$

For the buckled panel, i.e. panel 0 (Fig 14), the compatibility condition, eq (16) becomes, because of symmetry of the system,

$$2 r_0 = (\gamma_{x_0} - \epsilon_x) l_0 \quad (42)$$

Using eq (15) and (18), eq (42) can be rewritten as

$$2 (P_1 - P_0) = (\gamma_{x_0} - \epsilon_{x_0}) K l_0 \quad (43)$$

Eq (43), together with the load-strain characteristic of the buckled panel and either eq (38) or (39) and (41) provide the solution to the problem. Generally, as discussed in section 3,  $P_0$  will reach a constant value if the strain is large enough in which case eq (43) is not required in order to determine  $P_i$ .

In order to illustrate this particular application of the ideas of section 2, we again consider as an example, the Hinkley Point B PCFV cavity liner. The effect of a buckled panel on the floor liner is shown for two levels of strain in Fig 15. At the lower strain level, the flat panels remain elastic while at the higher strain level, all but one panel on each side and adjacent to the buckled panel have yielded. In this example the direction (x) under consideration is circumferential, the radially welded on cooling pipes being considered as

shear connectors. The shear connector characteristics for these cooling pipes are shown in Fig 7 and the load-strain characteristics of both the flat and buckled panel are shown in Fig 5. The buckled panel is taken to occupy a length equal to three cooling pipe pitches.

#### 7 APPLICATION TO THE EFFECT OF CHANGES IN PLATE THICKNESS AND YIELD STRENGTH

A further interesting problem to which our ideas are readily applicable is the effect of a change in plate thickness or in yield stress of the material such as might occur at a welded seam. This effect, like the panel buckling effect discussed in section 6, is local and may be investigated using the idealization shown in Fig 16. Indeed, the similarity with the panel buckling problem is obvious if we choose for panel 0, a panel incorporating the transition from one thickness or material to another. For each of the two systems  $Ox$  and  $Ox'$  (Fig 16), eq (38) to (41) may be applied. For panel 0 (Fig 16), however, the compatibility condition eq (16) becomes

$$r_o + r_o' = (\gamma_{x_o} - \epsilon_x) l_o \quad (44)$$

while the load-strain relationship is given by, for elastic behaviour

$$P_o = C_o (\gamma_{x_o} + \nu \epsilon_y) \quad (45)$$

in which

$$C_o = \frac{2E}{1-\nu^2} \left/ \left( \frac{1}{h} + \frac{1}{h'} \right) \right. \quad (46)$$

or, if yielding occurs,

$$P_o = h\sigma_y \text{ or } h'\sigma_y' \quad (47)$$

whichever is the least.

This application is illustrated in Fig 17 which shows the effects of both change in thickness and change in yield stress for the roof liner of Hinkley Point B PCPV cavity. The  $x$ -direction has been taken circumferentially with the cooling pipes acting as shear connectors having the characteristics shown in Fig 7.

#### 8 CONCLUSIONS

The uncoupled approach outlined in section 2 for the stress analysis of PCPV liners can be applied readily and with little expense to most of the problems facing the liner designer. The method has been used for the analysis of the liners to the Oldbury, Hinkley Point B and Hunterston B PCPVs.

Test work, or access to test results, is essential in order to provide the necessary load-deflexion characteristics of shear connectors and to establish the characteristics of particular design features such as corner bars. The application of the method to toroidal corners is, of course, quite simple. Other useful applications in addition to those outlined in sections 5, 6 and 7 are to the investigation of local variations in liner temperature and local loading effects.

9 ACKNOWLEDGEMENTS

The author wishes to express his thanks to The Nuclear Power Group Ltd for permission to present this paper and also to Whessoe Ltd., Darlington, who carried out the experimental test work referred to in the text.

10 NOTATION

- a radius of curvature
  - h panel thickness
  - l panel length
  - s meridional direction
  - $\theta$  angle between panel and shear connector
  - r shear connector displacement
  - u,w panel, or concrete displacements
  - F shear connector force
  - P panel force
  - T tie force
  - p normal pressure between panel and concrete
  - q shear between panel and concrete
  - C panel characteristic
  - K shear connector characteristic
  - k concrete bearing stiffness
  - $P_y$  yield load in panel
  - $\sigma_y$  yield stress
  - E Young's modulus
  - $\nu$  Poissons ratio
  - $\gamma$  membrane strain in panel
  - $\epsilon$  design strain
  - $\alpha T$  thermal strain
  - $\bar{P}$   $C(\epsilon_x + \nu\epsilon_y)$
  - $\mu$   $\sqrt{\frac{Kl}{C}}$
  - $\beta$   $\mu \sqrt{1 - \frac{C}{ka^2}}$
- Subscripts x,y directions
- c concrete
  - s steel or step
  - i,j,k,m panel or shear connector identifiers

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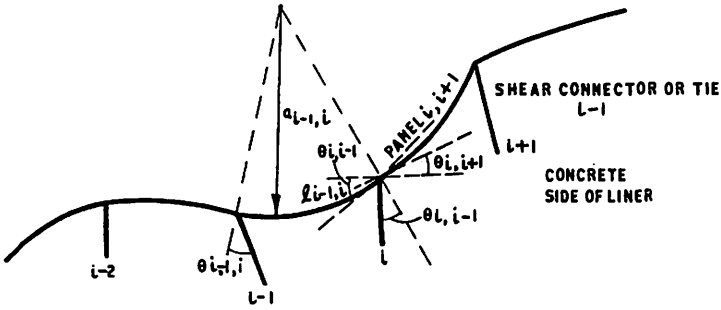


FIG 1 GENERAL IDEALIZATION OF A P.C.P.V. LINER

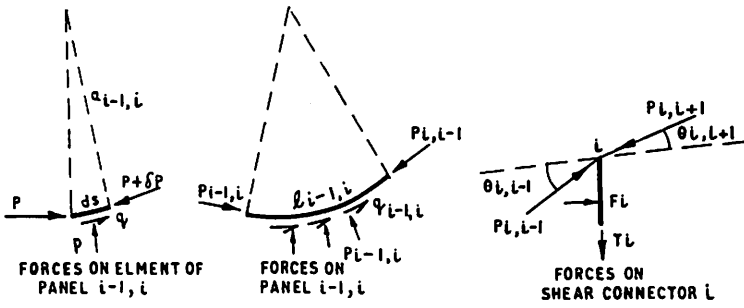


FIG 2 FORCES

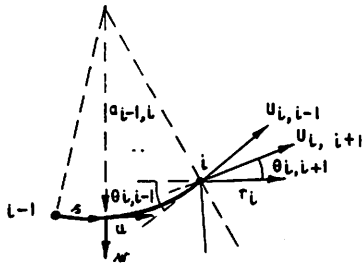


FIG 3 DISPLACEMENTS

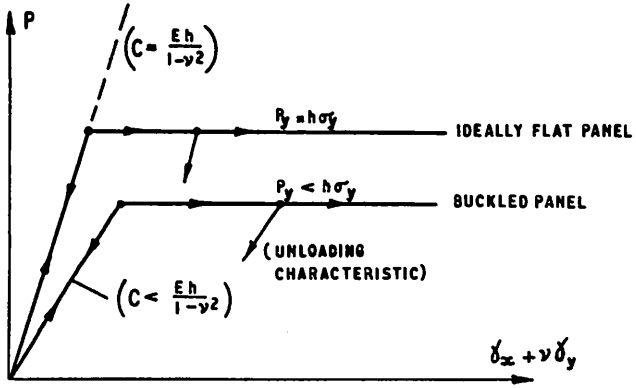


FIG 4 IDEALIZED PANEL CHARACTERISTICS

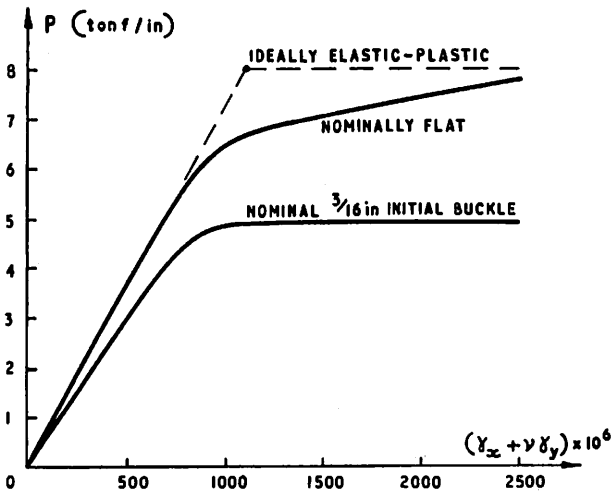


FIG 5 PANEL TEST RESULTS



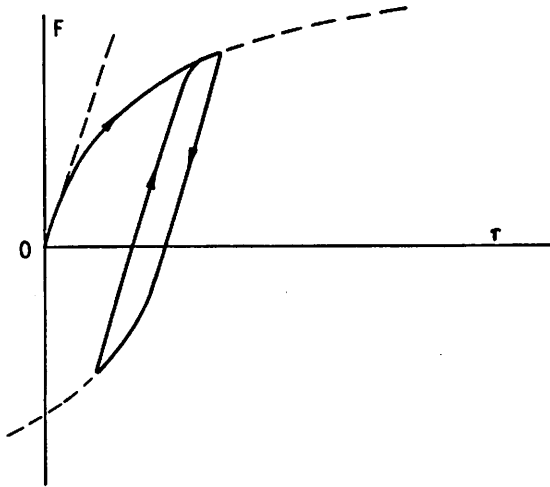


FIG 6 GENERAL FORM OF SHEAR CONNECTOR CHARACTERISTICS

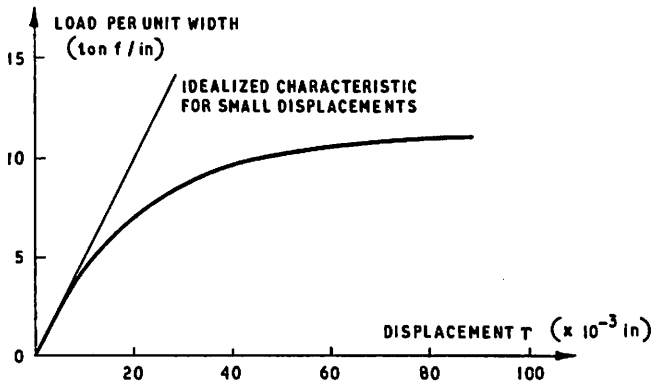


FIG 7 SHEAR CONNECTOR CHARACTERISTIC FOR COOLING PIPES

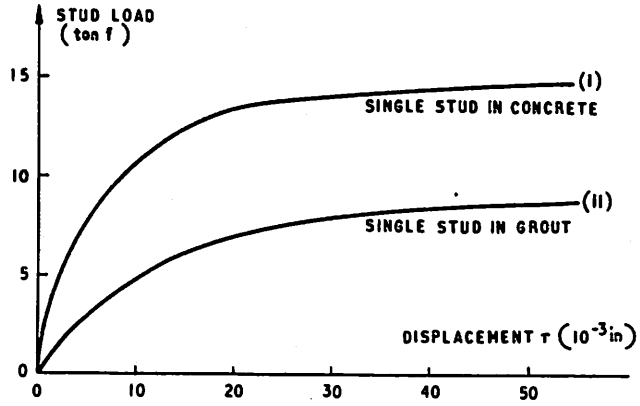


FIG 8 SHEAR CONNECTOR CHARACTERISTICS FOR SINGLE STUDS

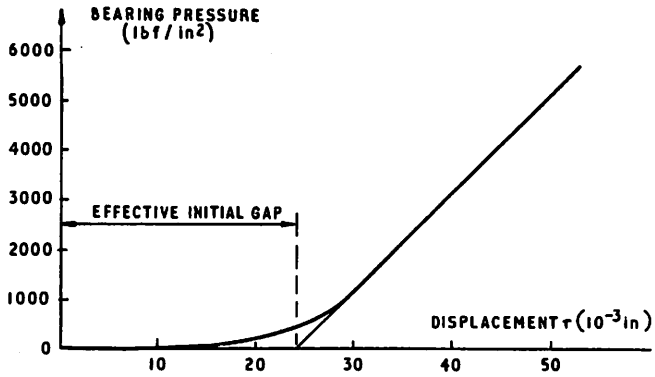


FIG 9 CORNER BAR CHARACTERISTICS

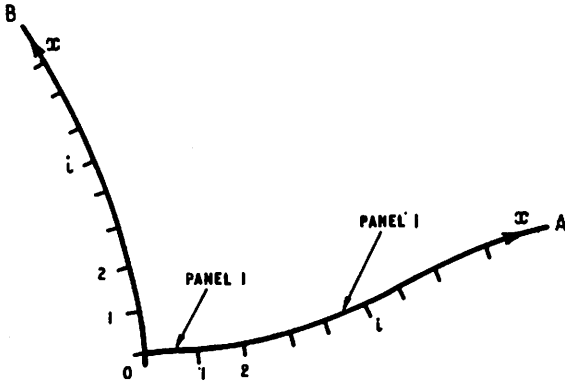


FIG IO IDEALIZATION OF LINER AT A RIGHT ANGLED CORNER

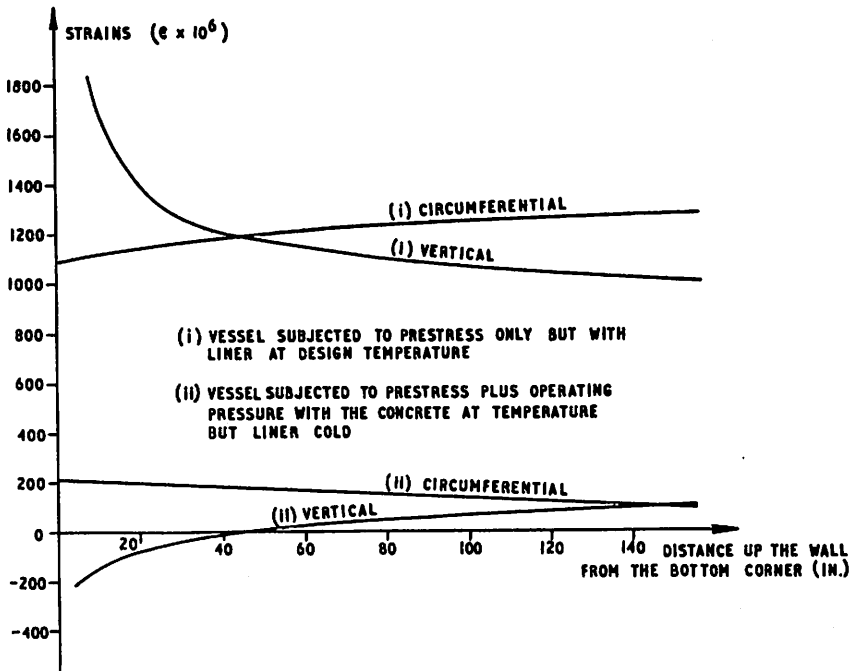


FIG II HINKLEY POINT B - CAVITY WALL LINER DESIGN STRAINS

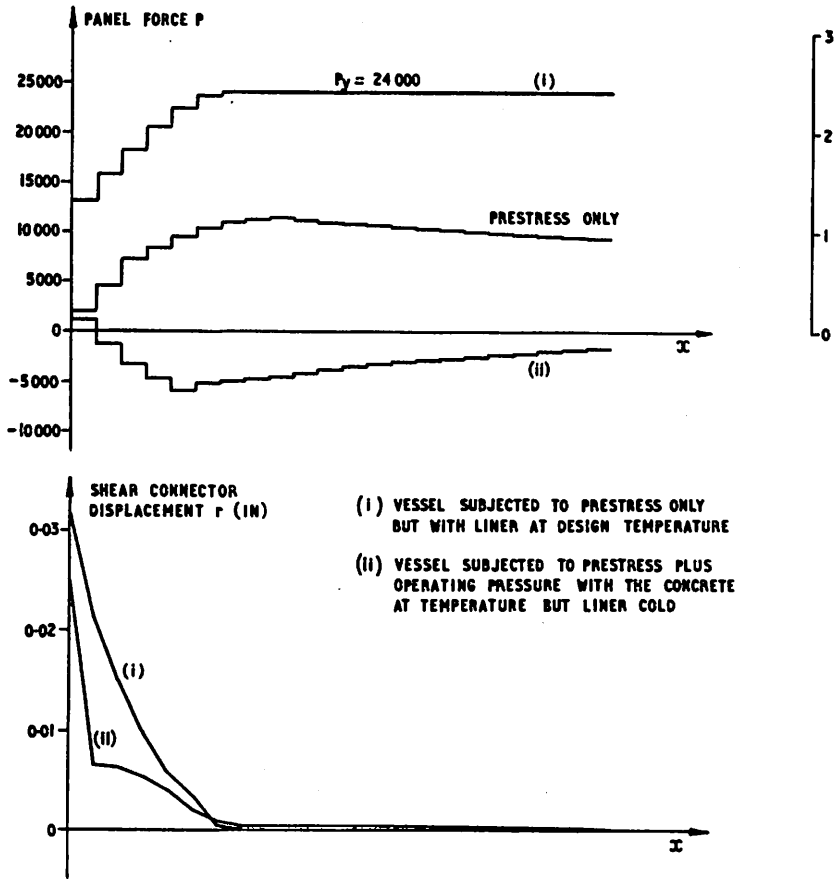
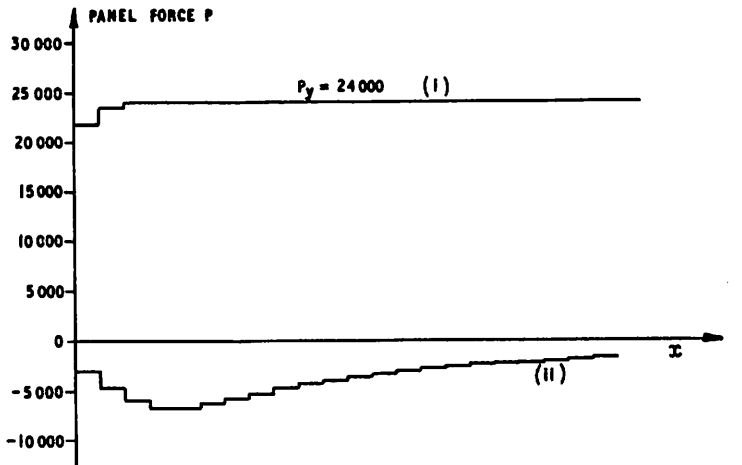


FIG 12 HINKLEY POINT B - LOAD DISTRIBUTION IN THE WALL LINER AND THE SHEAR CONNECTOR DISPLACEMENTS (CORNER BAR INITIAL GAP = 0.024 IN)



- (i) VESSEL SUBJECTED TO PRESTRESS ONLY BUT WITH LINER AT DESIGN TEMPERATURE
- (ii) VESSEL SUBJECTED TO PRESTRESS PLUS OPERATING PRESSURE WITH THE CONCRETE AT TEMPERATURE BUT LINER COLD

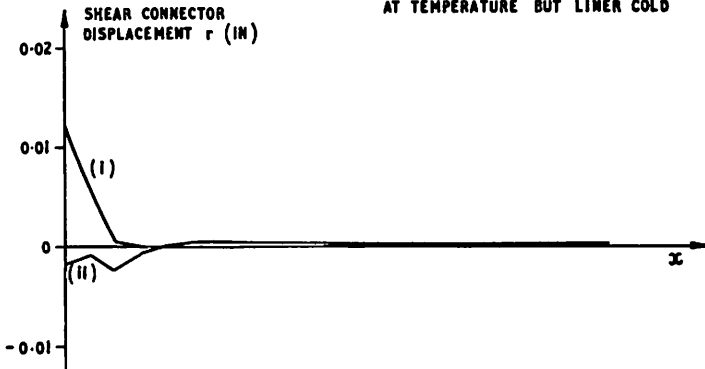


FIG 13 HINKLEY POINT B - LOAD DISTRIBUTION IN THE WALL LINER AND THE SHEAR CONNECTOR DISPLACEMENTS (CORNER BAR INITIAL GAP = 0)

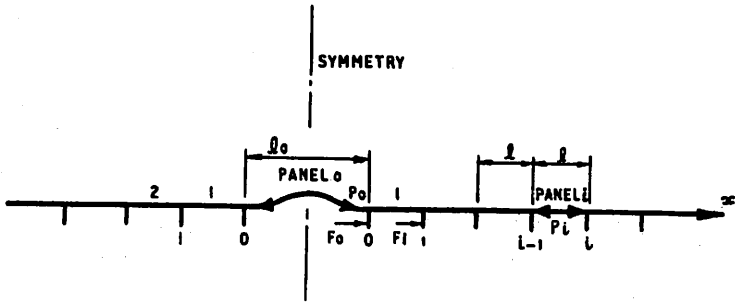


FIG 14 IDEALIZATION FOR INVESTIGATING THE EFFECT OF PANEL BUCKLING

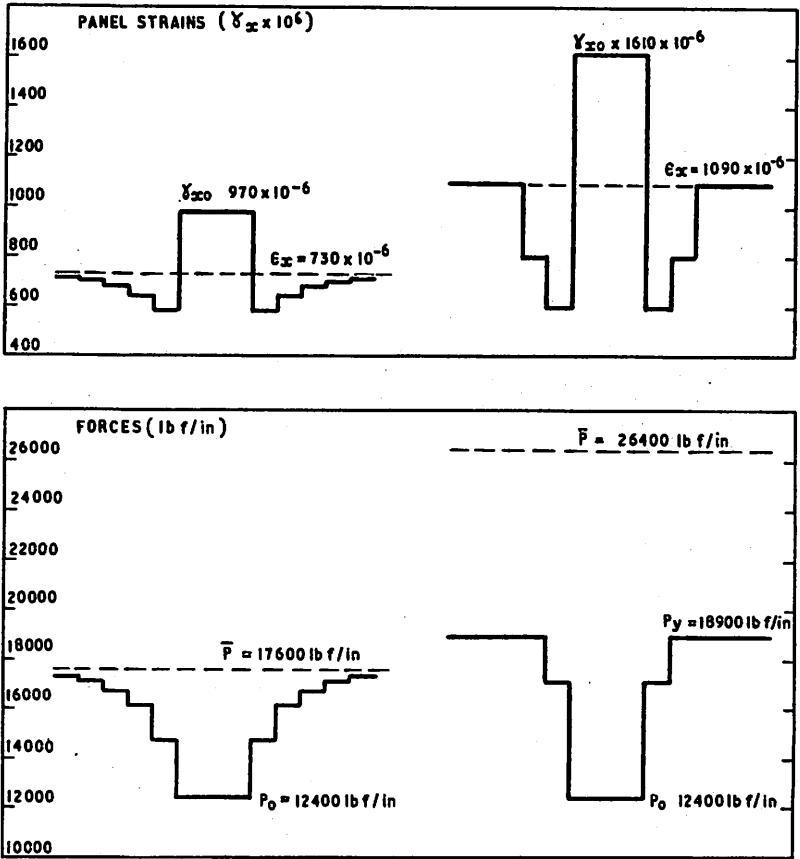


FIG 15 HINKLEY POINT 'B' FLOOR LINER - THE EFFECT OF A BUCKLED PANEL

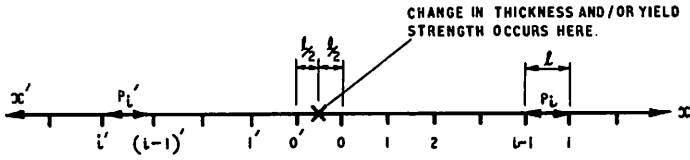


FIG 16 IDEALIZATION FOR INVESTIGATING THE EFFECT OF CHANGES IN THICKNESS AND YIELD STRENGTH

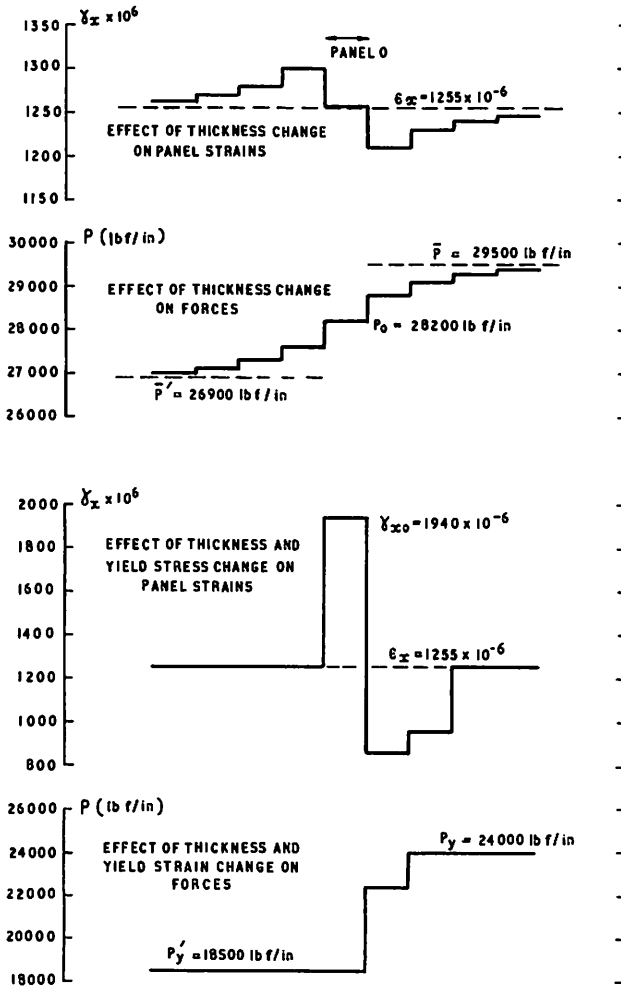


FIG 17 HINKLEY POINT 'B' ROOF LINER—THE EFFECT OF CHANGE IN PLATE THICKNESS AND YIELD STRESS

DISCUSSION

Q

D. COSTES, France

Do you take fatigue into account ?

A

J. V. PARKER, U. K.

We always take fatigue into account when concerning a design. By using the stress analysis method discussed in the paper, the cyclic loads and displacements, or cyclic stresses and strains in the liner and anchorages can be calculated. The fatigue life is then checked by reference to fatigue test data.

Q

W. ALBRECHT, Germany

Can low-cycle fatigue occur in liners ?

A

J. V. PARKER, U. K.

Low-cycle fatigue is not usually a problem with conventional liners unless there are badly designed details which give rise to high strain concentrations. Naturally we avoid much details.