

LINER PLATE BUCKLING AND BEHAVIOR OF STUD AND RIP TYPE ANCHORS

J.M. DOYLE,

*Department of Materials Engineering, College of Engineering,
University of Illinois at Chicago Circle, Chicago, Illinois,*

S.L. CHU,

*Analytical & Computer Division,
Sargent & Lundy Engineers, Chicago, Illinois, U.S.A.*

ABSTRACT

The evaluation of forces and deformations of liner plate anchors in containment structures is studied. These forces and deformations are due to buckling, and hence, reduction in load carrying capacity of a panel of the liner plate. Specifically, the behavior of headed studs and of continuous rib type anchors is investigated. The stiffness characteristics of the anchors are based on experimental evidence, and are nonlinear. Parameters which are varied include the plate thickness, yield stress and anchor spacing.

The results of several representative configurations are presented from which behavior trends can be observed. It is found that an increase in yield stress of an amount often encountered in practice causes a large increase in anchor force and deformation. On the other hand, a plate thickness increase of 15% results in a much smaller increase of anchor force and displacement.

1. INTRODUCTION

Thin steel plate liners attached directly to the interior wall of prestressed concrete reactor vessels or prestressed concrete containment structures provide an air tight seal. As such, their structural integrity must be maintained under design accident conditions. Since these liners are cast into the concrete, they are subject to all the strains experienced by the concrete subsequent to placement. These strains include those due to shrinkage and creep of concrete, and strains due to post-tensioning forces.

In addition, if there is an increase in temperature inside the vessel, the liner temperature will also rise. However, since the liner is rigidly attached to the concrete wall, it will be prevented from expanding. All of these effects cause compressive stresses in the liner. Quite often, the summation of all of them gives a resultant stress very near the yield stress of the material.

Due to its method of attachment to the interior wall, the liner is divided into a series of rectangular panels. These panels can then be idealized to rectangular plates with the boundaries restrained against lateral deflection and rotation. Due to the magnitude of the compressive stresses in these plates, some buckling must be anticipated. Since the induced stresses are caused by displacement of the edges of the panel relative to each other, the buckling itself is a relatively minor matter because the lateral deflections are self-limiting. A more serious problem, however, arises due to the decreased load carrying capacity of the buckled panel.

At the various points of attachment, or anchors, the compressive force in panels on either side of the anchor will be equal, provided that neither panel is buckled. Therefore, the net force acting on the anchor itself will be zero. On the other hand, if one of the panels adjacent to a given anchor is buckled, its load carrying capacity will be reduced. This reduction is quite significant in the event the stress in the panel is close to the yield prior to buckling. The net result in a situation of this type would be a force differential at the anchorage. In the event this difference in force exceeded the shear capacity of the anchor, failure between the plate and anchor would occur. Further consequence of failure of one anchor would be a doubling of the span of the buckled panel. This would cause an even lower post-buckling capacity and a higher force differential at the next anchor. In the most severe instance, a chain reaction of anchor failures could occur.

The liner-anchor system should be designed to prevent any failure between the plate and anchors. As an additional safeguard, the design should insure that if a single anchor does fail it will not result in rupture of the plate and that the incident will not result in propagation.

A number of papers [1-4] have discussed the design of liners, with emphasis on the liners for prestressed concrete reactor vessels (PCRV). Tan [5] states many of the problems which have arisen and gives an indication of the method of solution used in several cases. He points out that one of the difficulties connected with analysis of a liner anchor system is the inclusion of the behavior of the weld between anchor and plate and the concrete surrounding the anchor into the overall force deformation picture.

It is the purpose of this paper to investigate several alternate liner-anchor configurations for containment structures, and to evaluate their reliability under design accident conditions. Headed stud anchors of 1/2, 5/8 and 3/4 inch diameters, and 3x3x1/4 angle continuous ribs are studied. Liner plates of 1/4, 5/16 and 3/8 inch thickness, with varying yield stress, are considered.

Experimentally obtained force-displacement relationships are used for the headed studs [6]. Therefore, the concrete and weld actions are included. A conservative approximation, based on analytical and experimental results, is used for the stiffness relationship of the continuous angle ribs. The post-buckling capacity for plates strained beyond the yield point is conservatively estimated on the basis of analyses and experiments performed on struts.

2. ANCHOR FORCES

In order to estimate the forces and displacements at anchors, it is necessary to assume a pattern of buckling. The configuration considered in this study is that of one panel being buckled and several adjacent panels remaining straight, as shown in Figure 1. The configuration of this type would result in some movement in many of the anchors in the adjacent sections, with the most severe movement taking place right next to the buckled plate. The force and deformation at each anchor, of course, depends on the relative stiffness of the panels before any buckling occurs, and on the post-buckling capacity of the buckled section.

Regardless of the anchorage scheme used, the liner will be divided into a series of rectangular panels. For analysis, representative strips are considered. On continuous ribs, a unit width is assumed; while with the headed studs, a width equal to the least pitch is used. Typical configurations and the corresponding models are shown in Figure 2.

The plate material was assumed to be characterized by an elastic-perfectly plastic stress-strain law. Therefore, the stiffness of the unbuckled plates can be calculated readily. It follows that the force in any unbuckled plate is given by

$$F_p = \sigma_o wt - E \frac{U_n - U_{n+1}}{a} wt, \quad (2.1)$$

where,

- F_p = compressive force in strip
- σ_o = initial stress
- w = strip width
- t = plate thickness
- E = modulus of elasticity
- a = span of strip
- U_n = displacement of nth anchor.

On the other hand, the stiffness of the anchors, $f(U_n)$ is a nonlinear function obtained by experiment. For the headed studs, the data are available in Ref. [6] and the curves are shown in Figure 3.

There is no complete force-deformation data in the literature for 3x3x1/4 ribs. However, based on a finite element analysis of this type of rib, welded with 4 inches per foot of 3/16 inch fillets, a relationship which is valid prior to the onset of yield in the concrete was obtained.

In addition, Ref. [5] gives ultimate values of load and deformation for angles imbedded in concrete. The results of both of these sources are combined to give an approximate stiffness relationship for the angles (See Fig. 4).

It was found during the course of study that the most severe case of initial strain is when all panels are strained beyond their yield point prior to buckling. This state of strain is the limiting case which could occur under design accident condition loading. Therefore, it was utilized throughout.

The post-buckling capacity of a plate initially strained beyond the yield point does not appear to have been calculated. However, Young and Tate [2] have developed a method of computing post-buckling loads for struts of rectangular cross-section. These authors and others [3, 4] have also obtained post-buckling loads experimentally. The test results show the same qualitative behavior; however, the analytical method gives lower values than those obtained by experiment and therefore is conservative. A comparison of theoretical with experimental results is shown in Figure 5. It is noted that in these tests, the strip had an initial lateral deformation at the center of 0.6 of the thickness. All experimental evidence indicates that initial lateral deformation decreases the post-buckling capacity.

While the post-buckling behavior of a plate is not known, certainly its load carrying ability per unit width would not be less than that of a narrow strut of the same thickness. In this analysis, the theoretical results for a strip were used to obtain the post-buckling capacity of the panels. Since these predicted values are below the experimental values, and since the strut is weaker than the entire plate, the approach is considered quite conservative.

Equilibrium considerations applied at each anchor give the following set of equations for the determination of the anchor displacements U_n :

$$U_1 = U_2 + \frac{a(\sigma_0 - \sigma_{pb})}{E} - \frac{a}{Ewt} f(U_1) \quad (2.2)$$

$$U_n = \frac{U_{n+1} + U_{n-1}}{2} - \frac{a}{2Ewt} f(U_n) \quad n = 2, 3, \dots, N.$$

In these equations, $f(U_n)$ is the force-displacement relationship of the anchors, σ_{pb} is the stress in the buckled panel, and N is the total number of anchors considered. In order to solve the set of equations, let

$$U_{N+1} = 0. \quad (2.3)$$

In the cases treated here, 10 anchors were considered to act ($N = 10$). The solution showed that over the range of parameters covered the ratio of U_{10} to U_1 was 0.02 or less. As a check, a few cases were computed with $N = 20$. However, no significant change in maximum anchor

force or displacement was observed.

Due to the nature of the anchor stiffness, the equations are not linear. They were solved by a multi-iterative technique. As may be observed, a scheme can be devised to obtain values of U_n for a given σ_{pb} . However, σ_{pb} itself depends on the displacement of the anchors next to the buckled panel, and so σ_{pb} must be adjusted after each new set of U_n 's is found.

After obtaining a first solution to Equation (2.2), by relaxation, for an assumed value of σ_{pb} , a check was made to see if the assumed value corresponded to the resulting strain in the buckled panel; the strain in the buckled panel being given by:

$$\epsilon_a = \frac{2U_1}{a}. \quad (2.4)$$

If it did not, a new value of σ_{pb} was calculated, based on ϵ_a given by Equation (2.4), and the process repeated. Experience has shown that if σ_{pb} is assumed to be zero initially, convergence can be obtained in four or five iterations.

3. RESULTS

The maximum forces and deformations have been calculated for several possible configurations. These results along with the appropriate ultimate values are given in Table I.

It is noted that in all cases where headed stud anchors were used, the plate thickness was not less than one-half of the stud diameter. The reason for this is that when the diameter to thickness ratio is greater than about 2.7, failure will occur by tearing of the plate [7]. To avoid this, and to remain conservative, the ratio is usually restricted to 2.0.

Several combinations must be investigated for any liner design. Allowances must be made for variation in yield stress and thickness in neighboring plates, misalignment at joints, initial imperfections and for the possible failure of an anchor when the analysis is made. In general, all of the irregularities mentioned cause a reduction in the post-buckling capacity of the buckled plate, an increased stiffness in the unbuckled plate, or both. Consequently, larger values of force and deformation are experienced at the anchors.

Two inconsistencies which commonly occur and which are easy to detect, are the variations in yield stress and in plate thickness. Tables II and III illustrate the trend in maximum anchor force and deformation due to these two factors. The range of values covered are typical of those encountered for a nominal 1/4 inch plate with a specified yield stress of 36,000 psi.

The limiting case, in a situation of this type, is that of a zero post-buckling capacity. Table IV lists maximum anchor load and deformation

for a few typical configurations assuming no post-buckling capacity in the plate. In all cases, the computed quantities are considerably below ultimate.

4. DISCUSSION

Here, the results of a study concerned with the problems encountered in the design of liner plate anchors are presented. A number of typical wall anchor configurations were investigated. These included continuous angle rib type anchors as well as the headed stud type anchor. In particular, 3x3x1/4 angles with 4 inches of weld per foot and three diameters of studs, 1/2, 5/8 and 3/4 inch were studied. In addition, several different plate thicknesses, yield stresses and spacing patterns were considered. The maximum forces and displacements at the anchors in all cases showed good margins of safety when compared with ultimate values.

The results of this study include some important information in regard to yield stress and plate thickness. It has been shown that an increase in yield stress, by an amount often found in actual practice, gives rise to a significant increase in anchor forces and movements. On the other hand, a 15% increase of plate thickness produces only a slight increase in the anchor reactions and deformations. Therefore, in order to reduce the danger of anchor failure, low yield stress should be specified for the liner plate. In addition, close control should be exercised to insure that variations are reduced to a minimum.

In an investigation of this type, it is difficult to properly evaluate a number of factors. The inelastic post-buckling capacity of a panel, the effect of eccentricities due to misalignment and the effect of initial bulges are just a few items which ought to be incorporated. To obtain a bound on the anchor forces and movements, several configurations were analyzed assuming no post-buckling capacity in the panels. Clearly, this would represent the extreme case. Even under these conditions, it appears that failure would not result for liner-anchor configurations typical of those in current use.

The determination of a factor of safety should be examined. For example, if the existing load is compared to the ultimate load, the difference will generally be quite small. On the other hand, if a comparison is made between a given state of deformation and the ultimate deformation, the margin will be considerably better. This fact is due to the ductility exhibited by all types of anchors. One method for computing factor of safety which has been suggested by Tan [5], is to compare the work done in bringing an anchor to a particular stage of deformation with the work required to cause a failure in the anchor. Generally speaking, if this type of computation is used, the factor of safety will be approximately the same as if the deformation is compared to the ultimate deformation. Either one would appear reasonable to use.

When headed studs are used, care must be exercised in choosing plate thickness. In case a failure should occur, it must not involve rupture of the plate itself. To provide for this a maximum diameter to thickness ratio of 2 seems reasonable.

The results of this work give considerable quantitative information for assessing the behavior of liner-anchor systems in nuclear containment structures. While it is impossible to include all possible configurations and types of anchors, those treated are fairly representative. Trends in anchor forces and deformations due to changes in the various parameters can be observed from the data presented. If more detailed information is required, the analytical procedures are readily adaptable to machine computation.

5. ACKNOWLEDGMENT

This investigation was supported by Sargent & Lundy, Engineers, Chicago, Illinois. We express our appreciation to Mr. M. Zar, Manager of the Structural Department and to Mr. C. F. Beck, Head of the Computer Services Division for their interest and encouragement. In addition, we wish to thank Mr. Adolph Walser for many helpful suggestions.

REFERENCES

- [1] Hardingham, R. P., J. V. Parker, and T. W. Spruce, "Liner Design and Development for the Oldbury Vessels." Conference on Prestressed Concrete Pressure Vessels. Inst. of Civil Engineers. London. Paper J56. 1967.
- [2] Young, A. G. and L. A. Tate, "Design of Liners for Reactor Vessels." Conference on Prestressed Concrete Pressure Vessels. Inst. of Civil Engineers. London. Paper J57. 1967.
- [3] Chapman, J. C. and A. Carter, "Interaction Between a Pressure Vessel and Its Liner." Conference on Prestressed Concrete Pressure Vessels. Inst. of Civil Engineers. London. Paper J58. 1967.
- [4] Bishop, R. F., R. W. Horseman, and C. M. White, "Liner Design and Construction." Conference on Prestressed Concrete Pressure Vessels. Inst. of Civil Engineers. London. Paper J59. 1967.
- [5] Tan, C. P., "A Study of the Design and Construction Practices of Prestressed Concrete and Reinforced Concrete Containment Vessels." Clearinghouse for Federal Scientific and Technical Information. Report No. TID-25176. 1969.
- [6] Nelson stud welding applications in power generating plants. Nelson Stud Welding Company., Lorain, Ohio.
- [7] Goble, G. G., "Shear Strength of Thin Flange Composite Specimens." AISC Engineering Journal. April, 1968. pp. 62-65.

TABLE I. MAXIMUM ANCHOR FORCES AND DEFORMATIONS

Thickness, in.	Width, in.	Span, in.	Yield Stress, psi.	Anchor Type	Maximum Force, lb.	Computed Deformation, in.
0.25	7.0	15.0	36,000	1/2" stud	10,631	0.0449
			43,000		11,343	0.0613
			50,000		11,907	0.0801
			54,000		12,264	0.0920
0.3125	9.0	17.5	36,000	5/8" stud	11,795	0.0764
			43,000		12,623	0.1040
			50,000		13,498	0.1332
			54,000		13,960	0.1486
			36,000		15,905	0.0528
			43,000		17,092	0.0732
0.375	10.5	21.0	50,000	3/4" stud	18,350	0.0949
			54,000		19,109	0.1080
			36,000		22,188	0.0646
			43,000		24,017	0.0874
			50,000		25,592	0.1143
0.25	1.0	15.0	54,000	3x3x1/4 angles	26,270	0.1313
			36,000		3,378	0.0154
			43,000		3,520	0.0225
			50,000		3,712	0.0321
			54,000		3,820	0.0375

Ultimate Values
Force, lb. Deformation, in.

1/2" studs	14,200	0.167
5/8" studs	23,100	0.299
3/4" studs	33,000	0.341
3x3x1/4 angles	6,670	0.180

IN ALL CASES, DIMENSIONS AND YIELD STRESS OF BUCKLED AND UNBUCKLED PANELS ARE EQUAL.

TABLE II

VARIATION OF MAXIMUM ANCHOR FORCES AND DEFORMATIONS WITH YIELD STRESS

Yield Stress, psi	Maximum Computed		Ultimate	
	Force, lb.	Def, in.	Force, lb.	Def, in.
36,000	10,631	0.0449	14,200	0.167
43,000	11,343	0.0613		
50,000	11,907	0.0801		
54,000	12,264	0.0920		

Dimensions: Thickness = 0.25 in.; width = 7 in.; span = 15 in.

Anchors: $\frac{1}{2}$ -in headed studs

TABLE III

VARIATION OF MAXIMUM ANCHOR FORCE AND DEFORMATION
WITH UNBUCKLED PLATE THICKNESS

Unbuckled Plate Thickness, in.	Maximum Computed		Ultimate	
	Force, lb.	Def, in.	Force, lb.	Def, in.
0.25	10,631	0.0449	14,200	0.167
0.26	10,803	0.0473		
0.27	10,964	0.0494		
0.28	11,052	0.0516		
0.29	11,114	0.0537		

Dimensions: Buckled plate thickness = 0.25 in.; width = 7 in.;
span = 15 in.

Anchor Type: $\frac{1}{2}$ -in. headed studs

Yield Stress: 36,000 psi. for all panels

TABLE IV. MAXIMUM ANCHOR FORCES AND DEFORMATIONS; POST-BUCKLING

CAPACITY = 0

Thickness, in.	Width, in.	Span, in.	Yield Stress, psi.	Anchor Type	Maximum Force, lb.	Computed Deformation, in.
0.25	7.0	15.0	43,000	1/2" stud	11,795	0.0764
0.3125	9.0	17.5	43,000	5/8" stud	18,186	0.0921
0.375	10.5	21.0	43,000	3/4" stud	25,433	0.1103
0.25	1.0	15.0	43,000	3x3x1/4 angles	3,520	0.0225

Ultimate Values
Force, lb. Deformation, in.

1/2" stud	14,200	0.167
5/8" stud	23,100	0.299
3/4" stud	33,000	0.341
3x3x1/4 angles	6,670	0.180

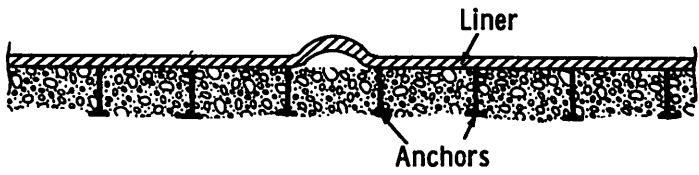


FIGURE 1. Assumed Buckling Pattern.

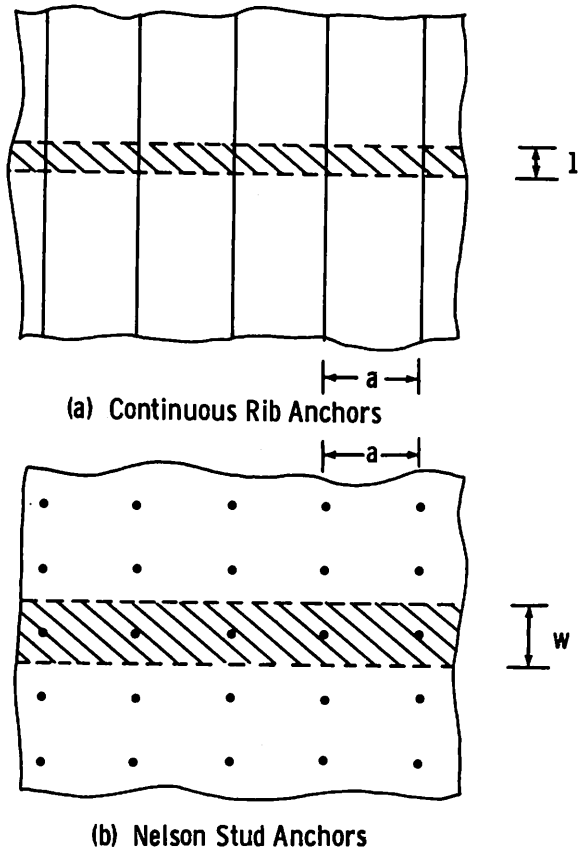


FIGURE 2. Anchor Layout with Strip Used for Analysis.

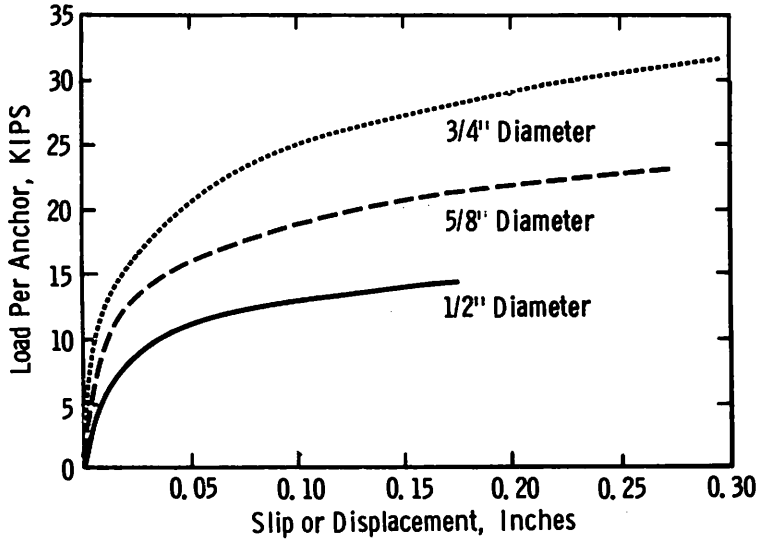


FIGURE 3. Load-Displacement Curves for Headed Studs.

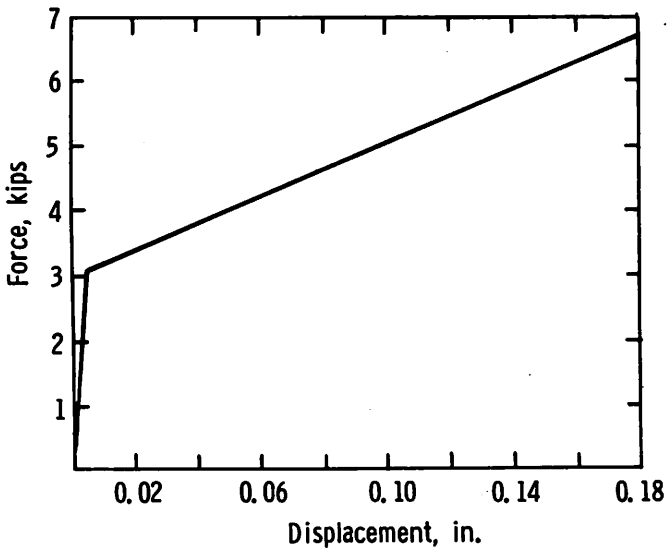


FIGURE 4. Load-Displacement Relation for 3x3x1/4 Angle Ribs.

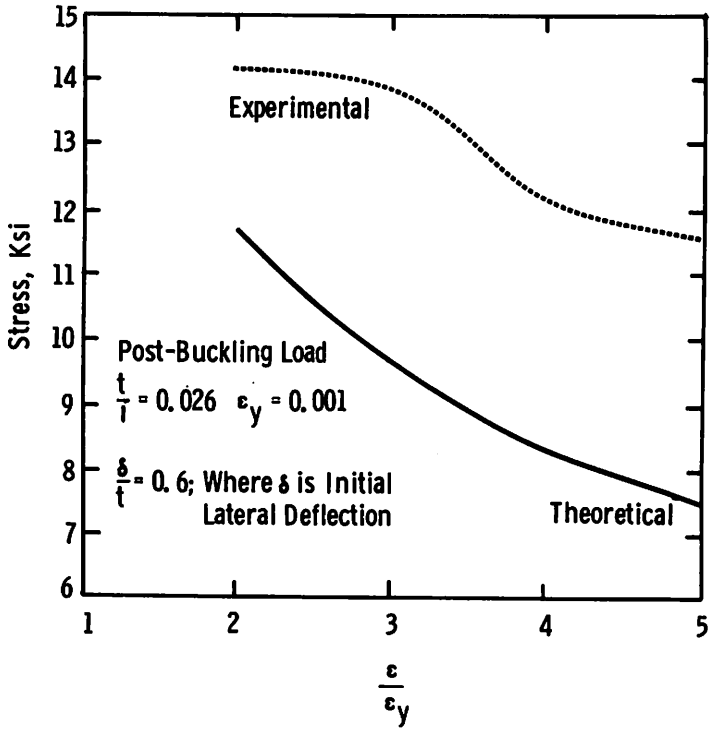


FIGURE 5. Post-Buckling Load Vs. Overall Strain.

DISCUSSION

M. BENDER, U. S. A.

Q

Could you describe the structural assumptions concerned with weld mismatch when the mismatch occurs in the buckled section. Does this deserve to be investigated ?

J. M. DOYLE, U. S. A.

A

The only way we treated the weld mismatch was to assume that such an imperfection would reduce the post-buckled capacity of a plate. Then the limiting case of zero post-buckled capacity was calculated. For such a case, the maximum anchor deformations were found to be considerably less than ultimate values. As far as the liner-anchor system is concerned, then such a mismatch would not be too bad. However, as far as the strength of the seam is concerned, the problem might be significant.

H. D. von der WEYDEN, Germany

Q

Has the thickness of the liner an important influence on the deformation characteristic of the anchors ?

J. M. DOYLE, U. S. A.

A

To my knowledge, it does not. However, the ratio of stud diameter to plate thickness should be kept to 2 or less to insure that failure, should it occur, will be in the weld and not in the plate itself.

C. M. WHITE, U. K.

C

May I add a comment. The stiffness characteristic of liner attachments does vary with liner thickness. From experimental work it is known that the deformation of the anchor all occurs in the short length of the anchor adjacent to the liner. With a thinner liner the intersection point of the attachment and the liner can flex more readily and hence affect the stiffness characteristic.

J. M. DOYLE, U. S. A.

A

Thank you, I was not aware of the point you made. The stiffness relationship for any anchor system should be obtained for a specific case. The curves used in this paper are fairly typical, I would say, and therefore, similar results could be expected with other liner-anchor schemes.