CALCULATION METHODS OF MULTI-LAYER INSULATIONS FOR HIGH TEMPERATURE GAS REACTORS

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ABSTRACT

This paper deals with the presentation of calculation methods for the evaluation of the heat losses in multilayer insulation systems with an emphasis on the natural convection aspect and, with the determination of the optimal dimensions of the cells and number of layers.

First, the case of a single cell will be considered before dealing with the case of cells in series-parallel.

The validity of the mathematical method is checked with the results of experimental basic and global tests.

1. INTRODUCTION

Thermal insulation of prestressed concrete vessels of high temperature gas reactors represents an important percentage of the cost of the concrete vessel of the order of 25 % by some estimates. As nominal temperatures become higher it is essential to attain a good comprehension of the heat transfer phenomena involved in these insulations in order to make optimization calculations. Heat is transferred from the hot gases surrounding the core to the cold concrete by three modes; natural convection, radiation and thermal conduction. The relative importance of each of these modes depends not only on temperatures but also on the nature of the gas used. For instance, the solutions optimized for CO_2 cannot be extrapolated directly to helium because of the great differences in density and thermal conductivity between the two gases and the design should specifically take these differences into account. This paper deals mainly with the metallic cellular insulating structure types as represented on Fig.1.

2. NATURAL CONVECTION IN A VERTICAL CELL

Let us consider a vertical cell defined by its thickness "d" and its height "l", where the ratio l/d is in the range from 2 to 50 - Fig 2 - The cell is supposed deep enough so that the problem can be treated in a two dimensions system.

Dimensionless numbers are used to characterize the phenomenon

$\frac{\lambda_a}{\lambda_f}$ = the apparent relative thermal conductivity

$Gr_\ell = \frac{g\,\ell^3 . \Delta T_H}{\nu^2 . T_m}$ and $Gr_d = \frac{g\,.\,d^3 \Delta T_H}{\nu^2 . T_m}$, GRASHOF numbers.

ℓ/d = ratio of height over thickness, $Pr = \frac{\mu\, C_p}{\lambda_f}$ = PRANDTL number

2.1. General equations

Only the case of laminar free convection, which is the field of practical applications for insulations, will be considered here. For a given geometry, as the GRASHOF number increases three different types of laminar flows can be successively observed /‾1_7 /‾2_7. Fig.3 - The passage of one type of flow to the other depends on the values of the dimensionless parameters. The first two types of flows which cover the range of optimization calculations can be represented by the following set of equations (motion, continuity and energy balance).

$$\frac{\partial u}{dt} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = - g\beta\ (T-T_m) - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) \qquad (1)$$

$$\frac{\partial v}{dt} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = - \frac{1}{\rho}\frac{\partial p}{\partial y} + \nu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) \qquad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (3)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\lambda}{\rho C_p}(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) \qquad (4)$$

The fluid properties are supposed constant except for the temperature variation of density which is creating the natural convection. Viscous dissipation and compressibility are neglected in the energy balance equation.
This set of equations has been solved for different boundary conditions by an alternating direction finite difference method as the one used by J.O. WILKES and S.W. CHURCHILL [3].

2.2. Numerical solutions

The numerical solutions give the values of p u,v,T. in transient and in steady state conditions. By a subsequent integration it is possible to calculate the heat flux which is transmitted from the hot wall to the cold wall. From an engineering point of view, we are mainly interested in the values of the apparent thermal conductivity λ_a which can be deducted from this integration.

The numerical results presented here give the values of $\frac{\lambda_a}{\lambda_f}$ for different values of l/d and Gr_d with the PRANDTL number constant at 0.733. Two different boundary conditions are considered. Numerical results are tabulated in appendix 1.

Boundary conditions A : The vertical walls are at uniform temperatures T_1 and T_2 and the horizontal walls are insulated. The results are represented in Fig.4, where $\frac{\lambda_a}{\lambda_f}$ is plotted versus the GRASHOF number Gr_d for values of l/d ranging from 2 to 50. They can be expressed in the following analytical form with a good approximation when compared to numerical values (within 5 %).

$$\frac{\lambda_a}{\lambda_f} = 1 + 1.89.10^{-4}.(\frac{l}{d})^{-3/4}.Gr_d \quad \text{for} \quad Gr_d < 1.32.10^3.(\frac{l}{d})^{3/4} \qquad (5)$$

$$\frac{\lambda_a}{\lambda_f} = 0.114 \left(\frac{\ell}{d}\right)^{-1/4} (Gr_d)^{1/3} \quad \text{for} \quad Gr_d > 1.32.10^3 \left(\frac{\ell}{d}\right)^{3/4} \qquad (6)$$

For the same values of the GRASHOF number Gr_d the apparent thermal conductivity diminishes when the ratio l/d increases.

Boundary conditions B : The vertical walls present linear temperature variations and the horizontal walls an insulated. The temperature distributions along the vertical walls are represented by the equations :

$$T_1(x) = T_1 + \Delta T_v . \frac{x}{\ell} \quad \text{with} \quad -\frac{\ell}{2} < x < \frac{\ell}{2} \qquad (7)$$

$$T_2(x) = T_2 + \Delta T_v . \frac{x}{\ell} \quad \text{"} \quad -\frac{\ell}{2} < x < \frac{\ell}{2} \qquad (8)$$

If we call $T_1 - T_2 = \Delta T_H$, the apparent thermal conductivity $\frac{\lambda_a}{\lambda_f}$ is a function of the three dimensionless parameters, Gr_d, l/d and $\left(\frac{\Delta T_v}{\Delta T_H}\right)$. In practical applications boundary conditions B are found more often than boundary conditions A.

For constant values of Gr_d and l/d, the apparent thermal conductivity $\frac{\lambda_a}{\lambda_f}$ increases when the ratio $\frac{\Delta T_v}{\Delta T_H}$ increases and can be represented by the formula

$$\left(\frac{\lambda_a}{\lambda_f}\right)_B \simeq \left(1 + 0.25 \frac{\Delta T_v}{\Delta T_H}\right) \left(\frac{\lambda_a}{\lambda_f}\right)_A \quad \text{for} \quad Gr_d > 4.000 \qquad (9)$$

A and B refer to boundary conditions A and B.

A third important boundary condition concerns the case of a uniform heat flux being imposed on the vertical walls. From a computer point of view, it is easier to impose temperature distributions at the walls than flux distributions. Once the GRASHOF number Gr_d and the ratio l/d are fixed, conditions of practically uniform heat flux at the walls can be obtained by choosing the value of $\frac{\Delta T_v}{\Delta T_H} \simeq 2$.

In all these different cases, the horizontal walls have been supposed insulated. As we are considering values of l/d >> 1 , if heat is exchanged along the horizontal walls, it will represent a small fraction of the total heat exchange and can be treated as a correction factor. See paragraph 5.

2.3. Comparaison of numerical results with experimental results

Natural convection in vertical cells with boundary conditions A have been studied experimentally by various authors [4] [5] [6] [7] [8] who have proposed different correlations of which three are considered here :

Results of DE GRAAF and VAN DER HELD

$$\frac{\lambda_a}{\lambda_f} = 1 \qquad \text{for} \qquad Gr_d < 7.10^3 \qquad (10)$$

$$\frac{\lambda_a}{\lambda_f} = 0.0384\ Gr_d^{0.37} \qquad " \qquad 10^4 < Gr_d < 8.10^4 \qquad (11)$$

$$\frac{\lambda_a}{\lambda_f} = 0.0317\ Gr_d^{0.37} \qquad " \qquad Gr_d > 2.10^5 \qquad (12)$$

Results of ECKERT and CARLSON

$$\frac{\lambda_a}{\lambda_f} = 1 + \frac{d}{\ell}\{0.00292\ Gr_d^{0.857} - 0.00144\ Gr_d^{0.75}\} \qquad (13)$$

$$\text{for} \qquad Gr_d < (\frac{\ell}{d})^{0.363}.1520$$

$$\frac{\lambda_a}{\lambda_f} = 0.119\ (\frac{\ell}{d})^{-0.1}\ .\ Gr_d^{0.3} \qquad " \qquad 2.72.10^4.(\frac{\ell}{d})^{0.363} < Gr_d < 10^5 \qquad (14)$$

Results of MULL and REIHER (with interpretation by M.JAKOB)

$$\frac{\lambda_a}{\lambda_f} = 1 \qquad \text{for} \qquad Gr_d < 2.10^3 \qquad (15)$$

$$\frac{\lambda_a}{\lambda_f} = 0.18\ (\frac{l}{d})^{-1/9}\ Gr_d^{\ 1/4} \qquad \text{for} \qquad 2.10^4 < Gr_d < 2.10^5 \qquad (16)$$

In these results the PRANDTL number is taken equal to 0.71. When passing to other values of the PRANDTL number Gr_d should be replaced by $Gr_d.Pr.(0.71)^{-1}$ In Fig.5 these correlations are compared with the theoretical results for different values of the ratio l/d. The agreement between theoretical and experimental results is satisfactory for values of l/d in the range of 5 to 30, for Gr_d numbers up to 30.000. For higher GRASHOF number, theoretically, $\frac{\lambda_a}{\lambda_f}$ varies as $(\frac{1}{d})^{-0,25}$, experimentally it varies about as $(\frac{1}{d})^{-0,10}$. This discrepancy might be explained by the existence of secondary flows which are not accounted in the mathematical model [2]. The reason that DE GRAAF and VAN DER HELD did not find the values of $\frac{\lambda_a}{\lambda_f}$ to be dependant upon the ratio l/d was probably due to the fact that they carried out their investigation for values of l/d in the range 18 to 65. Experimental results by JANNOT, MORDCHELLES- REGNIER [7] confirmed ECKERT and CARLSON's.

It should be noted that generally the experimental correlations are not well defined analytically in the critical range of GRASHOF numbers from 2000 to 15.000 which is the field of applications for our problem, while theoretical results are consistent in this range.

2.4 Determination of the minimal heat losses in a single cell

Temperature conditions are fixed and so is the height "l" of the cell which is generally determined by mechanical resistance reasons. Then the GRASHOF number Gr_l is known. The problem is to determine the optimum thickness "$d_{opt.}$" of the cell which will give the minimal heat losses. Boundary conditions A are considered.

The heat loss per unit surface is :

$$\phi = \lambda_a.\frac{T_1-T_2}{d} = \frac{\lambda_a}{\lambda_f}\ .\{\frac{\lambda_f.(T_1-T_2)}{l}\}\ \frac{l}{d} \qquad (17)$$

For these optimization calculations, the function $\frac{\lambda_a}{\lambda_f}$ is simplified. Fig.6 Instead of equations (5) and (6), the following equations are used :

$$\frac{\lambda_a}{\lambda_f} = 1 \qquad \text{for} \qquad \frac{d}{l} < 5.68.(Gr_l)^{-4/15} \qquad (18)$$

$$\frac{\lambda_a}{\lambda_f} = 0.114.(\frac{\ell}{d})^{-1/4}.(Gr_d)^{1/3} = 0.114.(\frac{\ell}{d})^{5/4}(Gr_\ell)^{1/3} \text{ for } \frac{d}{\ell} > 5.68.(Gr_\ell)^{-4/15} \quad (19)$$

Going back to equation (17), the reduced heat flux can be written :

$$\frac{\phi.\ell}{\lambda_f(T_1-T_2)} = (\frac{d}{\ell})^{-1} \quad \text{for} \quad \frac{d}{\ell} < 5.68.(Gr_\ell)^{-4/15} \quad (20)$$

$$\frac{\phi.\ell}{\lambda_f(T_1-T_2)} = 0.114\ (\frac{d}{\ell})^{1/4}.(Gr_\ell)^{1/3} \quad \text{for} \quad \frac{d}{\ell} > 5.68\ (Gr_\ell)^{-4/15} \quad (21)$$

If we plot the reduced heat flux versus the reduced thickness "$\frac{d}{l}$" of the cell, we obtain curves which are shown in Fig 7. For a given value of Gr_1, there is optimum thickness "$d_{opt.}$" corresponding to minimal heat losses :

$$(\frac{d}{\ell})_{opt} = 5.68.(Gr_\ell)^{-4/15} \quad \text{then} \quad (Gr_d)_{opt} = 183(Gr_\ell)^{1/5} \quad (22)$$

$$(\frac{\phi.\ell}{\lambda_f(T_1-T_2)})_{mini} = (\frac{d}{\ell})^{-1}_{opt}.\Delta \quad \text{with } \Delta = 1.13 \text{ and } 10^4 < Gr_\ell < 10^9 \quad (23)$$

Δ accounts for a correction due to the simplification of the formulas. For values of d, smaller or greater than $d_{opt.}$ either the molecular thermal conduction or the natural convection will increase the heat losses.

III HEAT TRANSFER BY RADIATION IN A CELL*

The following assumptions are made :

- there is no absorption in the gas (valid for helium but not for CO_2)
- the emissivity, absorptivity and reflectivity of the walls are not depending on temperatures
- the direct emissivity is diffuse and the specular reflection is not considered

In the case of two parallel plates, of infinite dimensions, at temperatures T_1 and T_2 the radiation heat transfer by unit surface is given by :

$$\phi_r = \frac{\sigma.K.(T_1^4-T_2^4)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}-1} \qquad K = 1 \quad (24)$$

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*These results are extracted from a study by Ste BERTIN. Mai-1971 Echanges radiatifs dans une cellule à parois métalliques non isothermes" by M.JANNOT - P.NAUDIN - S.VIANNAY - Extension of contract in réf.[8].

When considering walls with vertical temperature distributions $T_1(x)$ and $T_2(x)$ it is convenient to use a formula of the same type by introducing a correction factor K. If l/d is very great, the horizontal walls can be neglected and K is equal to :

$$K = \frac{\frac{1}{\ell}\{\int T_1^4 dx - \int T_2^4 dx\}}{T_1^4 - T_2^4} \tag{25}$$

If $T_1(x)$ and $T_2(x)$ are linear and given by equations (7) and (8)

$$K = 1 + \frac{\Delta T_v^2}{2(T_1^2 + T_2^2)} \tag{26}$$

If the influence of the horizontal walls cannot be neglected, the 4 surfaces are divided into bands (dAi) at uniform temperatures T_i Fig.8; an energy balance gives :

$$R^-_{(dA_1)} = \frac{1}{dA_1}\{\sum_{dA_2} . R^+_{(dA_2)} . F_{dA_1 . dA_2} . dA_2 + \sum_{dA_3} \ldots + \sum_{dA_4} \ldots\} \tag{27}$$

$$R^+_{(dA_1)} = \sigma . \varepsilon_{(dA_1)} . T^4_{(dA_1)} + (1 - \varepsilon_{(dA_1)}) . R^-_{(dA_1)} \tag{28}$$

The resolution of this system of 2n linear equations (n is the total number of bands) gives the values of the 2n unknowns, R^+ and R^-.
The heat transmitted from wall (1) to the three other walls is then

$$\Phi_r(1) = \sum_{dA_1} \left[\sigma . \varepsilon_{(dA_1)} T^4_{(dA_1)} - R^-_{(dA_1)} . \varepsilon_{1(dA_1)} \right] dA_1 \tag{29}$$

A similar approach can be used in the case of specular reflection.

IV CELLS IN SERIES AND PARALLEL

The general statement of the problem is the following : the permissible heat losses are fixed between a hot wall and a cold wall which are at known temperatures Fig.10. The optimization of the insulation consists in the determination of the minimum number of cells in series which will meet the requirements with the minimum total thickness. Heat is transfered by natural convection, thermal conduction in the horizontal walls and by radiation. The height l of the cells is known. Boundary conditions A are considered, which means that the intermediate vertical walls are supposed to be at uniform temperatures.

Let us consider the case of cell (n). The permissible heat flux Φ is equal to:

$$\phi = \frac{\lambda_a(T_{n-1}-T_n)}{d_n} + \lambda_m.\frac{e}{\ell}\,\frac{(T_{n-1}-T_n)}{d_n} + \sigma\{\frac{1}{\varepsilon_n} - \frac{1}{\varepsilon_{n-1}} - 1\}^{-1}.K_n(T_n^4-T_{n-1}^4) \qquad (30)$$

natural convection — metallic thermal conduction — radiation

For each cell we have to determine $\frac{d_n}{l}$ and T_n. Equation (30) is rewritten in the following form :

$$1 = (\frac{\lambda_f.T_{n-1}}{\phi.\ell}).(\frac{\lambda_a}{\lambda_f} + \frac{\lambda_m}{\lambda_f}.\frac{e}{\ell}).(\frac{\ell}{d_n}).\Delta\theta_n + (\frac{W_n.T_{n-1}^4}{\phi}).\{1-(1-\Delta\theta_n)^4\} \qquad (31)$$

where $\Delta\theta_n = \frac{T_{n-1} - T_n}{T_{n-1}}$ and $W_n = \sigma.\{\frac{1}{\varepsilon_n} + \frac{1}{\varepsilon_{n-1}} - 1\}^{-1}.K_n$

The total number of cells N will be minimized, if for each cell the value of $\Delta\theta_n$ is maximum. If from equation (31) we plot $\Delta\theta_n$ versus $(\frac{d_n}{l})$ where $\frac{\lambda_a}{\lambda_f}$ is represented by the simplified profile shown in Fig.6 the maximum value of $\Delta\theta_n$ is obtained for $(\frac{d_n}{\ell})$ equal to $(\frac{d}{\ell})_{opt.}$ as in equation (22) that is :

$$(\frac{d_n}{\ell})_{opt} = 5.68.(Gr_\ell)_n^{-4/15} = 5.68.\{\frac{g.\ell^3.(T_{n-1}-T_n)}{\nu^2.\frac{T_{n-1}+T_n}{2}}\}^{-4/15} \qquad (32)$$

$$(\frac{d_n}{\ell})_{opt} = 5.68.(\frac{g.\ell^3}{\nu^2})^{-4/15}.\{\frac{\Delta\theta_n}{(1-\frac{\Delta\theta_n}{2})}\}^{-4/15} \qquad (33)$$

$$(\frac{\lambda_a}{\lambda_f})_{opt} = 1.13$$

Finally the two unknowns $(\frac{d_n}{l})_{opt.}$ and $\Delta\theta_n$ are obtained by resolution of equations (31) and (33). By eliminating $(\frac{d_n}{l})_{opt}$ between these two equations, it can be written

$$1 = a_n.\,F_{an}(\Delta\theta_n) + b_n.F_{bn}(\Delta\theta_n) \qquad (34)$$

$$a_n = 0.176.(\frac{\lambda_f.T_{n-1}}{\phi.\ell}).(1.13 + \frac{\lambda_m}{\lambda_f}.\frac{e}{\ell})(\frac{g.\ell^3}{\nu^2})^{4/15} \qquad (35)$$

$$b_n = \frac{W_n \cdot T_{n-1}^4}{\Phi} \tag{36}$$

$$F_{an} = \frac{\Delta\theta_n^{19/15}}{(1-\frac{\Delta\theta_n}{2})^{4/15}} \tag{37}$$

$$F_{bn} = \{1.(1-\Delta\theta_n)^4\} \tag{38}$$

The coefficients a_n and b_n are known when the cell (n-1) has been calculated. The equation (34) can be solved numerically, giving $\Delta\theta_n$ as a function of a_n and b_n. Such numerical solutions are presented in Fig.9. In the expression of the coefficient a_n and b_n the kinematic viocosity and the thermal conductivity λ_f should be defined for the temperature$(\frac{T_{n-1}+T_n}{2})$; this can be done by iterations, using T_{n-1} as an initial value. When $\Delta\theta_n$ is known, $(\frac{d_n}{l})_{opt}$ is obtained by equation (33) and the cell (n) is determined and so on, until T_{n+1} is equal or inferior to the hot wall temperature. N is then equal to the number of cells to be put in series. In Fig.10, a numerical example is presented.

V - CORRECTION FACTORS

When using this method, correction factors have to be introduced to account for certain simplifications in the mathematical model.

a) walls_at_non_uniform_temperatures : In the formulation we have used numerical results concerning boundary conditions A, that is, walls at uniform temperatures, because practically all available experimental and theoretical results concerning the single cell, deal with these boundary conditions. When passing to the configuration shown in Fig.1, the walls are generally in thin metallic foils, presenting important vertical temperature gradients, due to the uneffective thermal conduction in the foils. The boundary conditions of one cell, in this case, are nearer to conditions of uniform heat flux than of uniform temperatures. Along a vertical wall, heat is transferred between a rising layer which is warming and a descending layer which is cooling, with a temperature difference between the two layers which is about constant. For the same temperature difference (T_1-T_2)in a cell, the apparent thermal conductivity λ_a is greater with boundary conditions of uniform heat flux than of uniform temperatures, as can be seen from formula (9), from which a correction factor $\alpha = \frac{(\lambda_a)_B}{(\lambda_a)_A}$ can be deducted.

b) vertical heat flux phenomenon - along the horizontal walls from the hotter layer under the wall to the colder layer above the wall. To approximate the value of this heat flux the assumption has been made that the average heat transfer coefficient along the horizontal wall is of the same order as the average heat transfer coefficient $\bar{h}$ along the vertical wall and the heat ψ_v transferred vertically is proportional to $\frac{\bar{h}}{2}$, "d", and to the average difference temperature between the two laminar layers on both parts of the horizontal wall.

Numerical results show that the ratio $\frac{\psi_v}{\psi_H}$ of heat transmitted vertically over heat transmitted horizontally varies little with the GRASHOF number, but essentially with the ratio l/d:

$$\frac{\psi_v}{\psi_H} = \beta . \frac{d}{\ell} \tag{39}$$

β is of the order of 0,45 in the case of walls with uniform heat flux. For an insulation configuration as represented in Fig.1, the upper cells will transmit more heat than the lower cells. With n cells in series, the total amount of vertical heat transmitted in a row will be

$$\psi_{v.T} = \sum_1^N . \psi_{v.n} = \sum_1^N \beta(\psi_H . \frac{d}{\ell})_n \tag{40}$$

If we make the assumption that in the upper row of cells the heat $\psi_{v.T}$ is added to the heat transmitted horizontally and if we also take into account the correction factor $\alpha = \frac{(\lambda_a)_B}{(\lambda_a)_A}$, then the permissible heat flux Φ used in equation (31) must be corrected by a factor

$$1 + \sum_1^N . \frac{(\alpha . \beta . \frac{d}{\ell})_n}{\phi . \ell} \tag{41}$$

Another correction factor which will not be dealt with in this paper concerns: a "macroscopic" convection factor. Because of safety reasons the cells cannot be fully leaktight and must allow for the possibility of rapid emergency depressurisations through communications between the cells. To the "microscopic" natural convection involving a single cell is superimposed a "macroscopic" natural convection concerning the whole insulation system.

EXPERIMENTAL RESULTS

Presentation of experimental results have been kept to a minimum, only as illustrations of the mathematical model.
A certain number of experimental tests, of which some partial results are presented here, have been performed or are under way, in order to check the validity of the method. Two kinds of tests are being carried out; (a) basic tests on a multilayer configuration with rectangular cells, (b) global tests on geometrical configurations which could be assimilated to rectangular cells with the adjonction of form coefficients.

basic tests the aims of these tests, performed with helium and nitrogen, are to evaluate the influence of vertical temperature distribution over the apparent thermal conductivity and to verify the limits of validity of the GRASHOF similitude. A schema of the experimental test section is represented in Fig.11. It consists essentially of four cells (500 x 500 x 50 mm) in series. The hot wall is heated by an electrical system and the cold wall is maintained at constant temperature by a water circulation system. Special care has been taken to avoid heat losses by thermal conduction, or by parasitical radiation or natural convection. Important vertical temperature gradients were observed on the intermediate vertical walls, made up of thin stainless-steel foils. The values of $\frac{\Delta T_v}{\Delta T_H}$ were in the range of 0,8 to 1.7 and the apparent conductivity measured from energy balances, agreed in a rather satisfactory manner with theoretical results, in the laminar field. In Fig.12, the scattering of the laminar points is notable. In Fig.13 these points have been corrected with formula (9) and the scattering is much less important.

global tests - [9] [10] [11] [12]
global thermal tests have been performed on two different kinds of insulations : (a) a special design which is being developped at the Ispra center, for which are equivalent length l and width d were defined Fig.(14).(b) a mixed solution called "knitt-mesh" consisting of cells filled with a metallic lattice with a "porosity" of 95%. Tests were done with nitrogen at atmospheric pressure up to temperatures of 400°C. One of the two test facilities is shown in Fig.(15).
For the first type of insulation, after having eliminated the heat transfer by radiation and metallic thermal conduction, the heat transfer by natural convection can be represented by a formula of the type

$$\frac{\lambda_a}{\lambda_f} = 0.09\ (Gr_d)^{1/3}$$

formula to be compared with formula (6). Taken into account the equivalent ratio (l/d) and correction factors due to the vertical heat flux phenomenon and non isothermal walls, but not including the correction due to the macro-convection, formula (42) gives results which are about 25% higher than results given by formula (6). A similar approach with the "knitt-mesh" insulation was also showing a difference of 20 % with the theoretical results concerning the "ideal" multi-layer.

CONCLUSIONS

A method has been presented for the calculations of multi-layer insulation systems in the case of heat transfer by natural convection, radiation and thermal conduction. For the single cell, the basic equations of motion, continuity and energy, in laminar free convection, have been solved numerically for different boundary conditions and it has been possible to obtain rather simple formulas to represent the apparent conductivity in the intermediate range of the GRASHOF numbers Gr_d from 2000 to 15.000 where available formulation was scarce, and which is precisely the range of practical applications.

Results concerning the single cell, with ideal boundary conditions, should be transposed with care to the configuration of cells in series-parallel where the boundary conditions are somewhere in between conditions of uniform temperatures and uniform heat flux at the walls.

A field of nuclear application of the method concerns the insulation of prestressed concrete vessel for High Temperature Gas Reactors with helium as coolant. An illustrative example shows the optimum thicknesses of the cells to vary by a coefficient 1 to 2,5 from the cold side to the hot side suggesting that cells of variable thicknesses should be adopted in industrial applications.

The influence of vertical temperature gradients was measured experimentally in a test section consisting of four cells in series. An increase of about 30 % in the apparent thermal conductivity was observed because of these gradients.

Global thermal tests were performed on two insulation systems, one presenting a special cellular geometry which can be only approximately likened to rectangular cells, and the other one filled with a metallic lattice with a porosity of 95 %.

In the two cases the measured apparent thermal conductivity was about 25% in excess of predictions given by equation (6). Due to the inherent experimental imprecision and to the fact that the macroconvection phenomenon was not accounted for, it can be concluded that the method can be transposed to non ideal geometries with a reasonable accuracy.

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APPENDIX 1

Numerical values of $\frac{\lambda_a}{\lambda_f}$ as a function of l/d, Gr_d, $\frac{\Delta T_H}{\Delta T_v}$.
(Resolution of set of equations (1) (2) (3) (4))

$\frac{l}{d}$	Gr_d	$\frac{\Delta T_r}{\Delta T_H}$	$\frac{\lambda_a}{\lambda_f}$
1	10.000	0	2,01
1	20.000	0	2,50
1	40.000	0	3,08
1	60.000	0	3,48
1	100.000	0	4,06
1	200.000	0	4,99
1	400.000	0	7,64
2	300	0	1,01
2	1.000	0	1,11
2	1.000	1	1,30
2	2.000	0	1,30
2	2.000	1	1,57
2	4.000	0	1,61
2	4.000	1	1,94
2	10.000	0	2,14
2	10.000	1	2,55
2	20.000	0	2,60
2	20.000	0,5	2,85
2	20.000	1	3,10
2	20.000	3	3,89
2	20.000	5	4,44
2	40.000	0	3,14
2	40.000	1	3,78
2	100.000	0	4,06
2	200.000	0	5,00
2	200.000	1	6,05
5	1.000	0	1,04
5	2.500	0	1,17
5	10.000	0	1,73
5	20.000	0	2,10
5	40.000	0	2,50
5	20.000	3	3,38
5	4.000	0	1,31
5	4.000	1	1,66
5	4.000	2	1,94
5	20.000	1	2,64

$\frac{l}{d}$	Gr_d	$\frac{\Delta T_v}{\Delta T_H}$	$\frac{\lambda_a}{\lambda_f}$
10	400	0	1
	1.000	0	1,01
	1.000	1	1,10
	2.000	0	1,04
	2.000	1	1,20
	4.000	0	1,12
	4.000	1	1,39
	4.000	2	1,62
	10.000	0	1,40
	10.000	1	1,80
	20.000	0	1,73
	20.000	1	2,19
	20.000	1,8	2,51
	20.000	2	2,58
	40.000	0	2,10
	40.000	1	2,60
30	4.000	0	1,02
	4.000	1	1,14
	4.000	2	1,25
	10.000	0	1,09
	10.000	1	1,34
	10.000	2	1,55
	20.000	0	1,26
	20.000	1	1,60
	20.000	1,3	1,69
	20.000	2	1,89
	25.000	0	1,35
	40.000	0	1,59
	50.000	0	1,73
50	2.500	0	1
	10.000	0	1,04
	25.000	0	1,17
	40.000	0	1,34

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NOMENCLATURE :

a_n, b_n : dimensionless coefficients related to cell (n)

C_p : specific heat of gas

d : width of cell

d_n : width of cell (n)

$d_{opt.}$: optimum value of d

e : thickness of horizontal wall

f_{an}, f_{bn} : dimensionless coefficients related to cell (n)

$F_{(dA_1\ dA_2)}$: form coef.between elements (dA_1) and (dA_2)

g : acceleration of gravity

Gr_d, Gr_1 : GRASHOF numbers with d and 1 as characteristic length

$\bar{h}$: average heat transfer coef.

K, K_n : correction factor or correct. factor related to cell (n)

1 : height of cell

p : pressure

Pr : PRANDTL number

$R^+(dA_1)$: flux density leaving surf. element (dA_1)

$R^-(dA_1)$: flux density received by surf.element (dA_1)

T : Temperature of gas

T_1 : Temperature of hot wall, or average temp.of hot wall

$T_1(x)$: local temp.of hot wall

T_2 : Temp.of cold wall or average temp.of cold wall

$T_2(x)$: local temp.of cold wall

T_m : average temperature of gas

$$T_m = \frac{T_1+T_2}{2}$$

ΔT_H : average temp.dif.between hot wall and cold wall

ΔT_v : vertical temp.dif.defined by equations (7) and (8)

T_n : Temperature of hot wall of cell (n)

t : time

u, v : components of velocity along x and y axis

x.y : rectangular coordinates.

W_n : dimensionless coefficient related to cell (n)

α, β : correction factors

$\varepsilon_1, \varepsilon_2, \varepsilon_n$: emmissivity coef.

λ_a : apparent thermal conduct. defined by

$$\phi = \lambda_a . \frac{T_1-T_2}{d}$$

λ_f : molecular thermal conduct. of gas

λ_m : thermal conduct.of metallic horizontal wall

ν : kinematic viscosity

μ : dynamic viscosity

ϕ : admissible heat loss per unit surface

ϕ_r : radiative heat loss per unit surface

ψ_H : horizontal component of heat flux

ψ_v, $\psi_{v.n.}$, $\psi_{v.T.}$: vertical component of heat flux, or related to cell (n) or total

$\psi_r(1)$: radiative heat flux trans. from wall (1)

σ : STEFAN-BOLTZMANN constant

$\Delta\Theta_n$: relative temperature dif. related to cell (n)

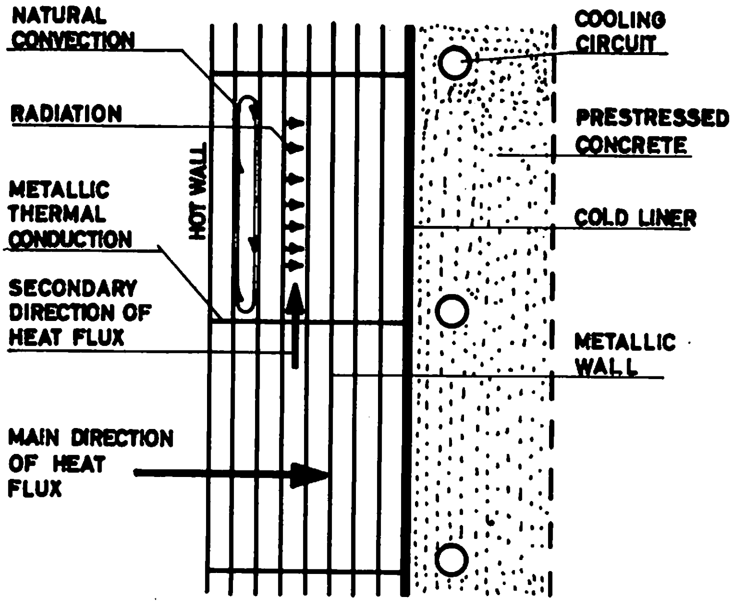


Fig.1-Schema of multi-layer insulation

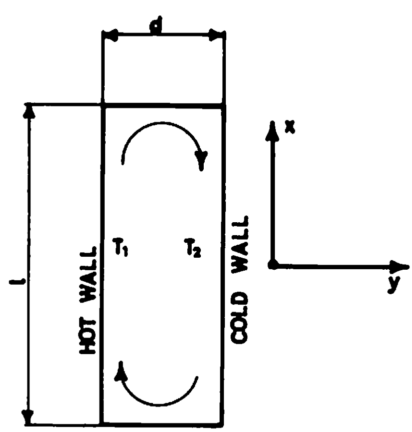


Fig.2 - Schema of vertical cell

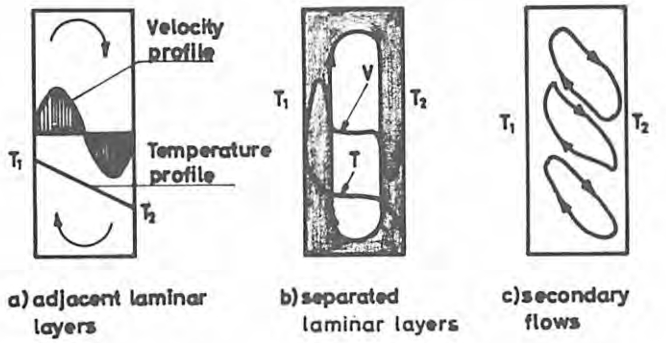


Fig. 3 – Different types of laminar flows

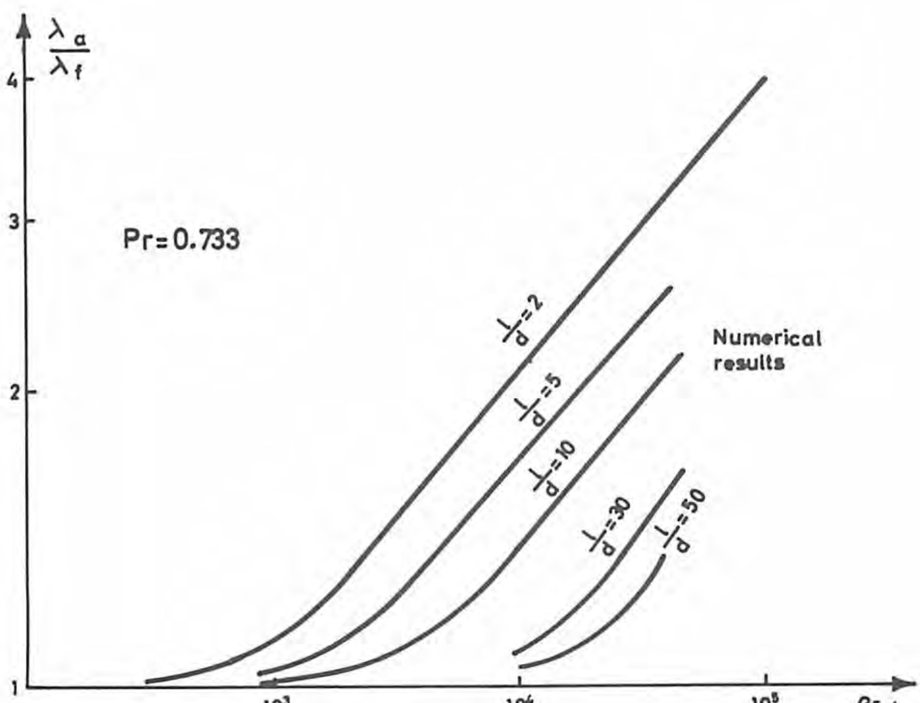


Fig. 4 - Apparent conductivity versus Grashof number

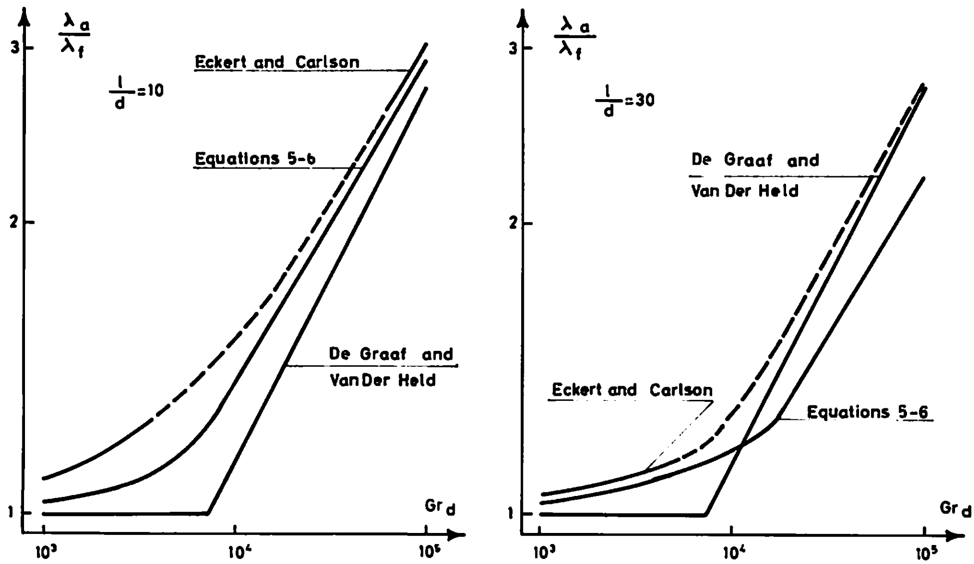


Fig. 5 Comparison with experimental formulas

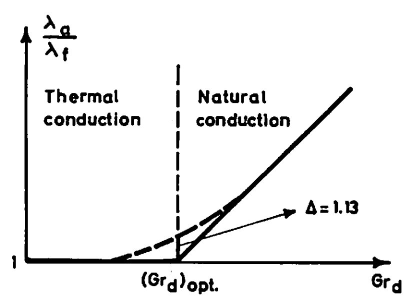


Fig. 6-Simplified profile of apparent conductivity

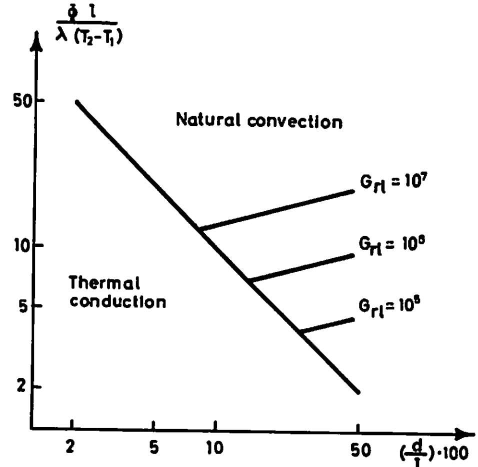


Fig. 7-Heat flux losses versus width of cell

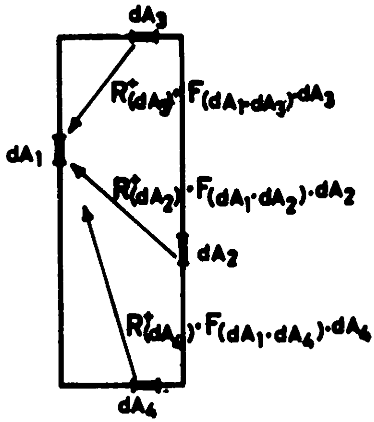


Fig.8-Schema of radiation exchanges

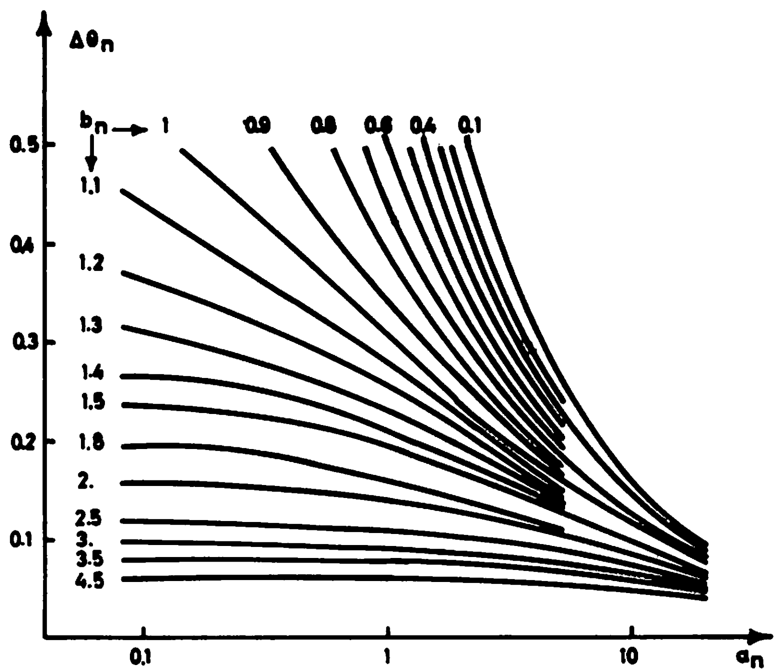
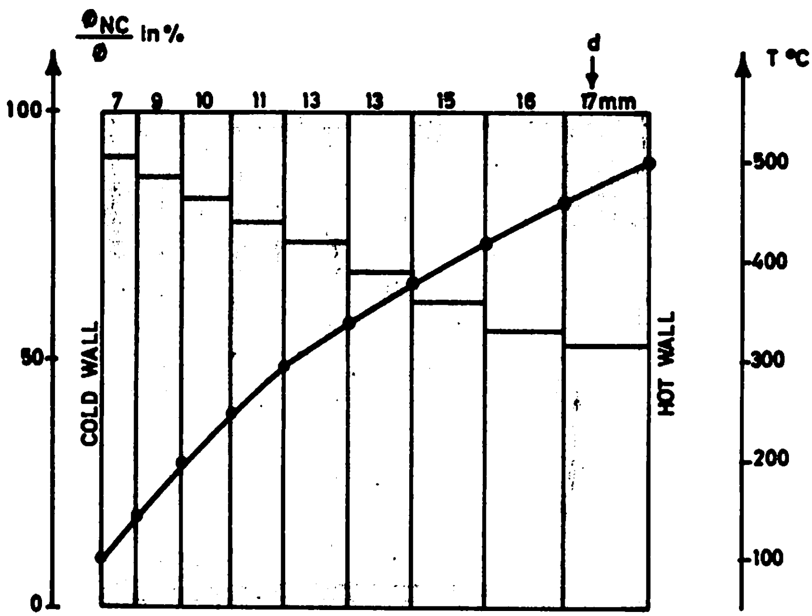


Fig.9-Values of $\Delta\theta_n$ versus a_n and b_n



● Wall temperature

—— % of heat flux transmitted by natural convection

ϕ : 0.15 W/cm^2 ; ε=0.33 ; l=20 cm

Gas: helium at 25 atm

Cold wall: 100 °C ; Hot wall: 500 °C

Fig. 10 - Illustrative example

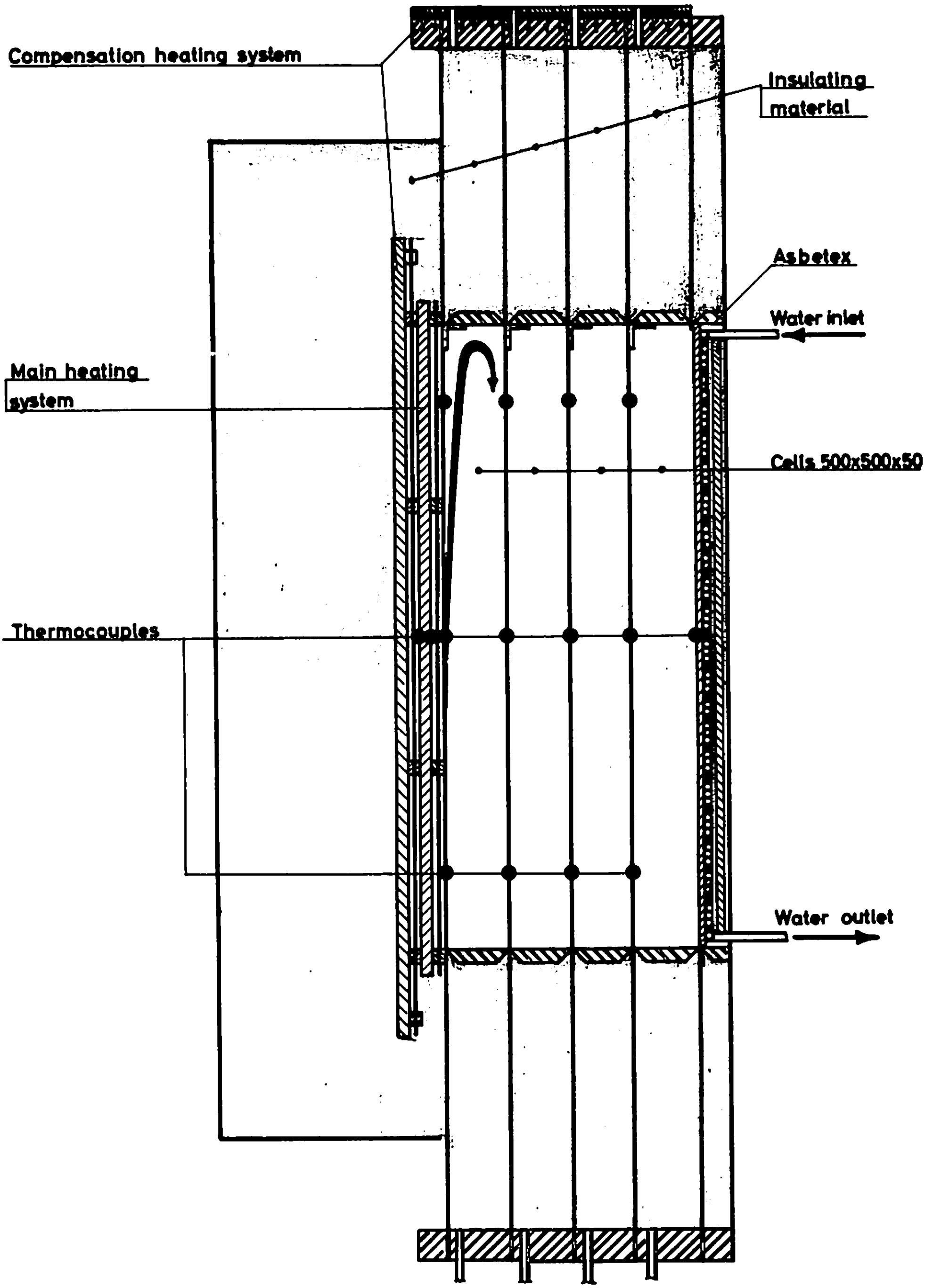


Fig.11- Schema of test section

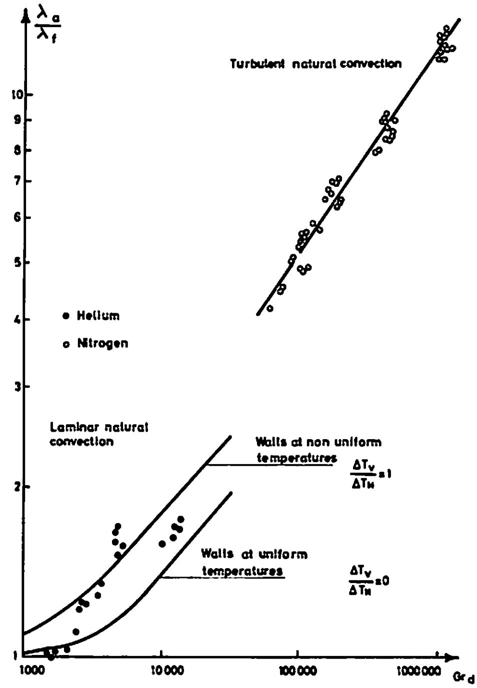


Fig.12 Apparent thermal conductivity with non isothermal walls

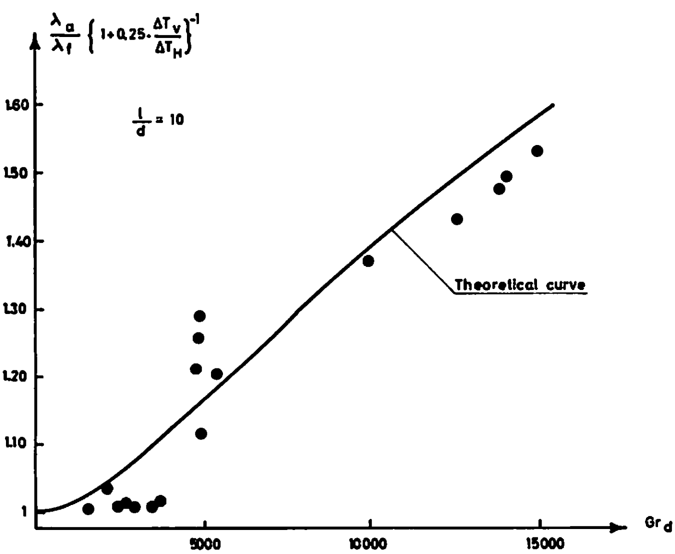


Fig.13 - Apparent thermal conductivity after correction

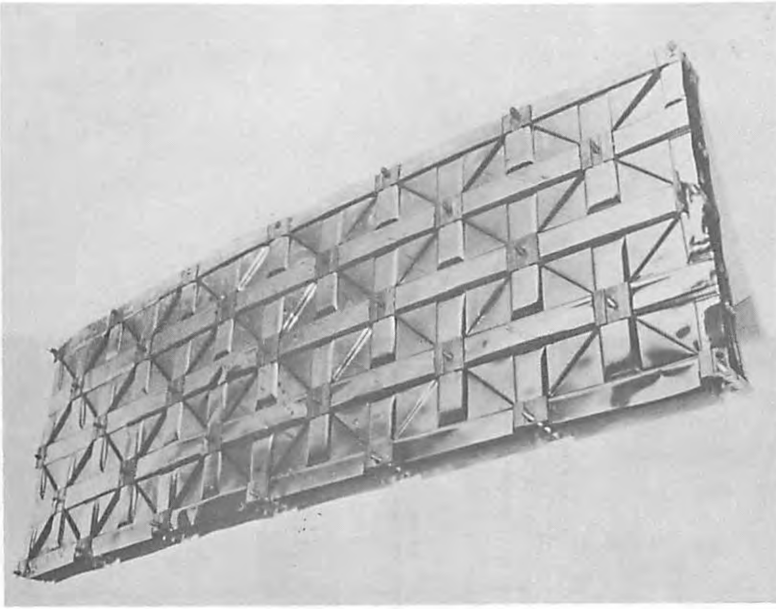


Fig. 14 - a-" Casali" insulation

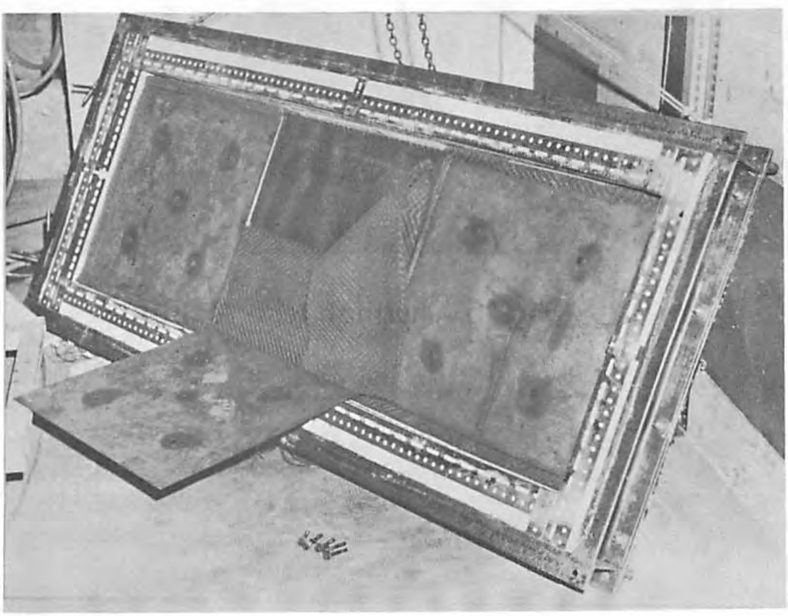


Fig.14 - b-"Knit - Mesh" insulation

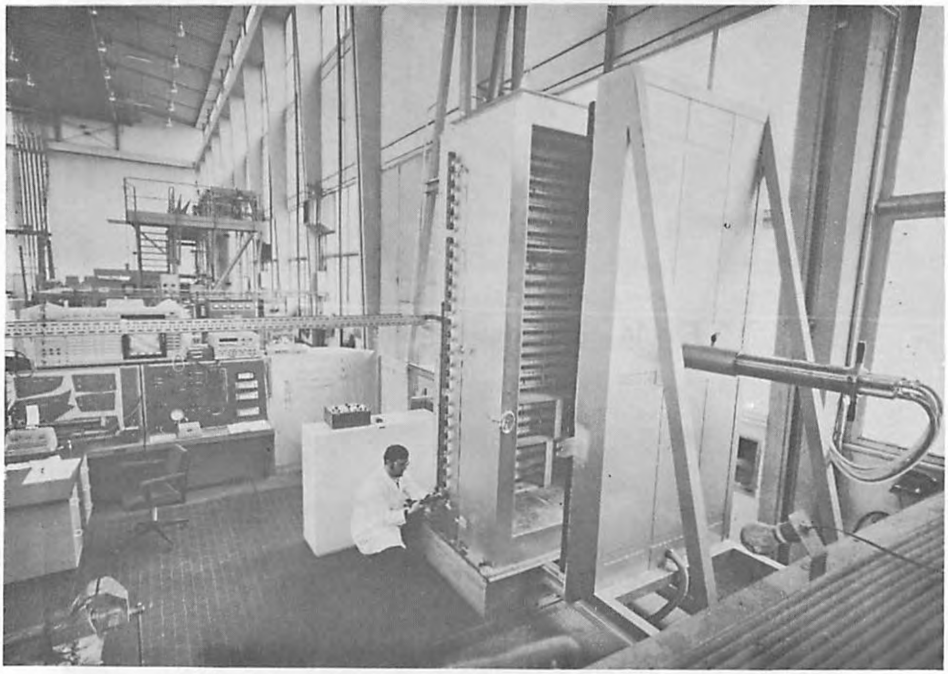


Fig.15 - Thermal test facility

DISCUSSION

Q A. J. WILLIAMS, U. K.

1. The analysis in this paper concerns cells which are sealed from one another, whereas in practice a degree of interconnection exists. Does this interconnection affect the conclusions drawn in this paper ?

2. I agree that ideally it would be beneficial to decrease the spacing towards the cold face. I would hesitate to use this principle however because of the difficulty of ensuring that the layers of insulation are erected in the correct order.

A E. ARANOVICH, JRC Ispra, Italy

1. For safety reasons, because of possible rapid depressurization phenomena, such as pump failures, communications must exist between the cells and between the cells and the outside of the insulation. Because of these communications, natural convection phenomena, which we call "macroconvection", involving the whole structure, can take place. These phenomena can be quite important in the case of existing pressure gradients, and should be added to the other heat transmissions. In optimization calculations the level of permissible heat losses should take this effect into consideration.

2. Optimization calculations optimize two points: the optimum spacing between layers and the optimum number of layers. As we are in the region of optima, the values of the spacing distances need not to be observed with an extreme accuracy. I think the optimization of the number of layers is more important as it has a direct influence on the price of the insulation and on the mounting time.