STATISTICAL TREATMENT OF SEISMIC LOADING ON REACTOR BUILDINGS AND EQUIPMENTS

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ABSTRACT

A statistical approach for the seismic analysis and design of nuclear reactor buildings and facilities is proposed. The concepts are derived through random vibration theory; a comparison of specific results obtained on this basis with those obtained for a normalized set of recorded accelerograms attests to the validity of the proposed statistical approach.
1. INTRODUCTION

Random vibration concepts have been used in seismic analysis since the early fifties; see for example Goodman, Rosenblueth, and Newmark [1]. The procedure of taking the square-root of the sum of squares of the maximum modal responses, for example, is based on certain approximations derived from the theory of random vibration. This leads to reasonable approximations for regular and symmetrical buildings for which only the lateral deflections and the story drifts are the quantities of primary interest. However, in the seismic analysis of nuclear reactor facilities, the systems are quite complex, and response quantities other than displacements are also of significance. Moreover, in certain soil dynamic evaluations, the number of exceedances beyond a high stress level is also of interest. Because of the complexity of the systems involved in a reactor facility, a direct random vibration analysis, therefore, appears to be potentially useful and desirable.

In this paper the available random vibration concepts that are of practical significance for reactor facilities are summarized, and specific numerical results obtained therefrom are compared with those from a normalized set of recorded accelerograms. This comparison demonstrates the validity of using a direct random vibration approach in seismic analysis.

The implementation of the procedure requires a description of the spectral density for the design earthquake. At present this information can be analytically derived from the postulated average response spectrum, which is presumably specified for purposes of design. The dispersion of the maximum response is taken into account by specifying a response level corresponding to a probability of exceedance during the earthquake. This probability serves as a consistent basis for specifying the response level for design; this provides an alternative to the current practice of using an amplified design spectrum.

2. PREDICTION OF EXTREME STRUCTURAL RESPONSE

2.1 Approximate Distribution

It is well-known that the exact distribution of the extremes of the random system response $X(t)$ within an earthquake duration $t_d$ is not available. However, a number of studies, including the works of Amin, Tsao, and Ang [2] and Goto and Kameda [3] have shown that when high response levels of design interest are considered, it is reasonable to ignore the statistical dependence of the up and down-crossings of the response levels $x = \pm b$, as shown in Fig. 1. If it is assumed that the response process has zero mean and the up and down-crossings of level $x = \pm b$ occur in accordance with a Poisson process, the distribution function of the random variable

$$ X_m = \max_{0 \leq t \leq t_d} |X(t)| $$

(1)

can be described as,

$$ F_{X_m}(b, t_d) = \Pr (X_m < b) = 1 - p_e(t_d) = \exp \left[ -2 \int_0^{t_d} v_b^{+}(t) dt \right] $$

(2)
in which \( v^+_b(t) \) is the rate of up-crossing at level \( x = b \) and is obtained from the joint density function of \( X \) and \( \dot{X} \) at time \( t \) as follows

\[
v^+_b(t) = \int_0^\infty \dot{X} \phi_{X,\dot{X}}(b,\dot{x},t)d\dot{x}
\]

Assuming that the earthquake response is a Gaussian Process, the design response level \( b \) corresponding to an exceedance probability \( p_e(t_d) \) can be evaluated numerically from the above equations. Also if the response is assumed to be a stationary process, the evaluation of \( b \) can be obtained in closed form as follows:

\[
b = \alpha \sigma_X
\]

where,

\[
\alpha = \left[ 2 \ln \left( \frac{t_d \sigma_X}{\pi q_e \sigma_X} \right) \right]^{1/2}
\]

\[
q_e = -\ln(1 - p_e)
\]

In these equations \( \sigma_X \) and \( \sigma_{\dot{X}} \) are the standard deviations of \( X(t) \) and \( \dot{X}(t) \). The evaluation of these quantities is described subsequently.

2.2 Range of Validity of Stationary Response Assumption

Recorded earthquake accelerograms are clearly not stationary except perhaps in the strong-phase duration. The duration of strong shaking, in turn, increases with the earthquake magnitude. In addition to accelerometer nonstationarity, additional evolutionary trends are also introduced into the response process because of zero initial conditions.

A quantitative study of the influence of nonstationarity was recently reported by Amin and Gungor [4]. The findings of this study can be stated as follows: For a single-degree-of-freedom (SDF) system having 2 percent damping and a period less than 2 seconds, no more than 15 percent error is introduced if the response process is treated as a stationary process. This conclusion applies to a magnitude 7 earthquake which is comparable to the El Centro record of May 18, 1940. For larger magnitude earthquakes, for which the duration of strong shaking is longer, smaller error will be introduced by ignoring nonstationarity. The same is true for systems with lower periods and/or larger values of damping. Since the fundamental periods of most reactor facilities are less than 1 second and most structural dampings are taken to be larger than 2 percent, the above discussion indicates that treating the earthquake response of power plants as a stationary random process is reasonable. The error involved is, of course, on the conservative side.

2.3 Response Statistics

The use of eqs. (4) and (5) requires the variances of \( X(t) \) and \( \dot{X}(t) \). Evaluation of these statistics for linearly elastic system is treated in many textbooks on random
vibration. Pertinent equations of relevance to the subsequent presentation can be summarized as follows:

\[ G_x(\omega) = \omega^2 G_y(\omega) \]  
\[ \sigma_x^2 = \int_{-\infty}^{\infty} G_x(\omega) d\omega \]  
\[ \sigma_x^2 = \int_{-\infty}^{\infty} G_x(\omega) d\omega \]

in which \( G_x(\omega) \) is the power spectral density of the response \( X(t) \) which is related to the input earthquake spectral density \( G_y(\omega) \) according to eq. (10)

\[ G_x(\omega) = H_x(\omega) \cdot H_y(\omega) \cdot G_y(\omega) \]  

where \( H_x \) is the complex frequency response function and \( \cdot \) denotes the complex conjugate.

In seismic analysis, the method of modal analysis is usually used and a small constant modal damping is assumed. On this assumption, an arbitrary response \( X(t) \) can be written as

\[ X(t) = \sum_{k=1}^{n} B_{xk} q_k(t) \]  

where \( n \) = number of active modes, \( B_{xk} \) = time invariant quantity depending on the shape of mode \( k \) and the definition of the response \( X \), and

\[ \ddot{q}_k + 2B \omega_k \dot{q}_k + \omega_k^2 q_k = -\ddot{y}(t) \]  

in which \( \omega_k \) = modal frequency (sec\(^{-1}\)); and \( \ddot{y} \) = ground acceleration. A steady-state response analysis of eqs. (11) and (12) (\( \ddot{y} = e^{i\omega t} \)), yields

\[ H_x(\omega) = -B_{xk} \frac{1}{\omega_k^2} \]  

\[ z_k(\omega) = 1 - (\frac{\omega}{\omega_k})^2 + i \cdot 2B \frac{\omega}{\omega_k} \]  

Using this information in eq. (8) we obtain

\[ \sigma_x^2 = \int_{-\infty}^{\infty} G_x(\omega) d\omega = \sum_{k} \sum_{k'} B_{xk} B_{xk'} (\omega_{k'}^2 \omega_k^2)^{-1} \sigma_{k'k} \]  

Similarly, eq. (9) yields
\[ \sigma_{x}^{2} = \sum_{k} \sum_{k'} B_{xk} B_{xk'} \left( \omega_{k} \omega_{k'} \right)^{-1} \delta_{kk'} \]  

(16)

in which

\[ \sigma_{kk} = \int_{-\infty}^{\infty} \frac{G_{y}(\omega)}{z_{k}(\omega)z_{k}^{*}(\omega)} \, d\omega \]  

(17)

and,

\[ \delta_{kk} = \int_{-\infty}^{\infty} \frac{\omega^{2} G_{y}(\omega)}{z_{k}(\omega)z_{k}^{*}(\omega)} \, d\omega \]  

(18)

For most power spectral density functions, \( G_{y}(\omega) \), developed to describe earthquakes, the integrals in eqs. (17) and (18) can be readily evaluated by the method of residues and using complex arithmetic features available in most computers. Finally, the double summation in eqs. (15) and (16) may sometimes be approximated, respectively, by

\[ \sigma_{x}^{2} = \sum_{k=1}^{n} \left( \frac{B_{xk}}{\omega_{k}} \right)^{2} \sigma_{kk} \]  

(19)

and

\[ \sigma_{x}^{2} = \sum_{k=1}^{n} \left( \frac{B_{xk}}{\omega_{k}} \right)^{2} \delta_{kk} \]  

(20)

When the frequencies of the active modes of a particular response are well-separated, eqs. (19) and (20) are reasonable approximations. This is usually true, for example, for the lateral deflections of a cantilevered structure with regular distribution of mass and stiffness. However, eqs. (19) and (20) are poor approximations when floor accelerations in a beam-like structure is considered. For this reason, eqs. (15) and (16) ought to be used when treating the response of a complex system.

In the following results, when eqs. (15) and (16) are used in eqs. (4) and (5), the resulting value of \( b \) will be identified as \( b^{\Sigma} \), whereas the corresponding value of \( b \) obtained with eqs. (19) and (20) will be referred to as \( b^{\Sigma} \).

### 3. COMPARISONS WITH NORMALIZED ACCELEROMETERS

#### 3.1 Description of Excitations

Two horizontal components of the following four records obtained in the West coast of the U.S. were used in this study: El Centro (12/30/34 and 5/18/40),

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*When \( k = n \) and if most of the contribution to the integral in eq. (17) comes from the values of \( \omega \) near \( \omega_{n} \), a well-known approximation is \( \sigma_{kk} = \pi G_{y}(\omega_{n})/2B \omega_{n}^{2} \). However, in earthquake response analysis, frequency conditions exist for which this approximation produces noticeable inaccuracy.*
Olympia (4/13/49), and Taft (7/21/52). The records were normalized to have the same area under the undamped pseudo-velocity curve \( V = \omega_m \) for \( T = 0.1 \) to 2.5 second.

The following expression is used for the spectral density of the ground acceleration,

\[
G_y(\omega) = G_1(\omega)G_2(\omega)
\]

\[
G_1(\omega) = \frac{1 + 4B_1^2(\frac{\omega}{\omega_1})^2}{[1 - (\frac{\omega}{\omega_1})^2]^2 + 4B_1^2(\frac{\omega}{\omega_1})^2} \quad S_0
\]

\[
G_2(\omega) = \frac{0.650 + 2.24 (\frac{\omega}{\omega_2})^2 + 1.63 (\frac{\omega}{\omega_2})^4}{[1 - (\frac{\omega}{\omega_2})^2]^2 + 2.24 (\frac{\omega}{\omega_2})^2}
\]

In which \( \omega_1 = 15.5 \, \text{sec}^{-1} \), \( B_1 = 0.642 \), \( \omega_2 = 15.7 \, \text{sec}^{-1} \), \( S_0 = 0.0052 \, \text{ft}^2 \, \text{sec}^{-3} \). Figure 2 shows the median spectra, \( P_0(30) = .5 \), obtained using eq. (21) in eq. (4) and its comparison with the corresponding average spectra of the 8 normalized records.

### 3.2 Systems Considered

Three systems, shown in Fig. 3 as structures 1, 2, and 3 are used in the comparative studies. Structure 1 is a 10 degree-of-freedom shear beam. Structure 2 is a single-story shear structure with eccentricity \( e \) between the centers of mass and stiffness in the direction perpendicular to the ground motion; there are, therefore, two degrees of freedom. Structure 3 is composed of 5 identical stories, each resembling structure 2; this structure, therefore, involves 10 degrees of freedom.

The parameters of structures 2 and 3 are selected such that the periods of the active modes are close to each other: for structure 2 the ratio \( T_2/T_1 = 1.105 \); for structure 3 this ratio is 1.05. More details pertaining to these structures are described in Ref. [4].

### 3.3 Comparison of Results

Columns 2, 3, and 4 of Table 1 summarize, respectively, the range of the response values, the average, and the second highest value obtained from a step-by-step integration of the equations of motion of structures 1, 2 and 3 using the 8 normalized accelerograms. Generally, \( \gamma \) is the displacement of a mass center relative to the ground, \( \gamma \) is the absolute displacement of the mass center, and \( \theta \) denotes the floor rotation. The second highest value reported would have an exceedance probability of about 1/8. A 5-percent damping was used for all the modes of vibration; therefore, for smaller damping, a larger range of response values than those reported in Table 1 would be expected.

The above results are compared with the response quantities obtained with the random vibration method. The latter results are \( b_{.5} \), \( b_{.5} \), \( b_1 \), and \( b_2 \), which are
respectively, the median and the response level corresponding to a 10% exceedance probability. These are presented in Columns 5 through 8 in Table 1, and are given in terms of the average and second highest values of Columns 3 and 4, respectively.

The results summarized in Table 1 indicate that the approximation of eq. (19) may be poor even for cantilevered beam-like structures; see for example the response \( \tilde{y}_1 \) in Column 5, versus the corresponding result in Column 6 obtained using eq. (15) for the variance. The errors caused by the approximation of eq. (19) become more pronounced for structures 2 and 3, as evidenced in Table 1; these are structures having modal frequencies close to each other. Since this property is not uncommon in the higher mode responses of complex systems, the evaluation of the variances through eqs. (15) and (16) appears necessary when considering complex structures.

Columns 6 and 8 of Table 1 demonstrate the validity of the proposed random vibration approach. Specifically, these show that the assumptions of a Gaussian response and Poisson occurrence of level-crossings produce a reliable means for estimating maximum earthquake responses in MDF-systems.

4. RESPONSE OF SECONDARY SYSTEMS

4.1 Generation of Floor Response Spectra

In current practice it is often necessary to specify floor spectra for the design of light equipment items mounted on the primary structure. When a deterministic analysis is used, this information may be generated by making a time-history analysis of the primary system using several accelerograms. For each input accelerogram, the floor motions are determined and used to evaluate floor spectra. Final floor spectra are specified by smoothing the floor spectra which are generated from various accelerograms. Clearly this is an expensive and time-consuming process; for this reason, it is impractical to study the effects of possible changes in the primary system frequencies on the floor spectra. Since the primary system is a sharply resonant system, the effects of changes in its frequencies on the floor spectra is far more important than the significance to the deformational response of the primary system itself; minor changes in the frequencies of the primary system can produce a significant effect on the response of equipment mounted on the primary system.

A random vibration approach can be used to generate floor spectra rapidly and, therefore, the influence of changes in the primary system frequencies on floor spectra can be evaluated by making a sensitivity analysis of the results. This involves the use of eqs. (4) and (5), in which \( X(t) \) and \( \hat{X}(t) \), in this case, refer to the response of the equipment as shown in Fig. 4. The required equations for evaluating \( \sigma_x \) is given below.

If the inertia and damping coupling between the equipment and the primary system in Fig. 4 is ignored, and damped normal coordinates are used to consider, approximately, the effects of damping, the equipment response \( X(t) \) in Fig. 4 can be written as

\[
X(t) = \sum_{k=1}^{n} \gamma_k \psi_k(j) x_k(t)
\]

(24)
\begin{align}
\dddot{x}_k + 2\beta_2 \omega_2^2 x_k + \omega_2^4 x_k &= \omega_{1k}^2 q_k + 2\beta_1 \omega_1^4 \dddot{q}_k \\
\dddot{q}_k + 2\beta_1 \omega_1^4 \dddot{q}_k + \omega_{1k}^2 q_k &= -\dddot{y}(t)
\end{align}

where \( \omega_{1k} \), \( \gamma_k \) = modal frequency and participation factor, respectively, for the primary mode \( k \); \( \psi_k(j) \) = amplitude of mode \( k \) at floor \( j \); and \( \beta_1, \beta_2 \) = damping values for the primary and secondary systems, respectively, and \( \omega_2 \) = frequency of the secondary system.

The complex frequency response of the equipment, determined from eqs. (24) through (26), is

\[ H_x(\omega) = -\sum_{k=1}^{n} \frac{\gamma_k \psi_k(j)}{\omega_1^4} a_k(\omega) \left[ \omega_2^2 z_2(\omega) z_{1k}(\omega) \right]^{-1} \]

in which \( z_{1k} \) is given by eq. (14) and

\[ a_k(\omega) = 1 + 2\beta_1 \frac{\omega_k}{\omega_{1k}} \]

\[ z_2(\omega) = 1 - \frac{\omega_2}{\omega_2} + i 2\beta_2 \frac{\omega_2}{\omega_2} \]

Using eq. (27) in eq. (10) and using eq. (8) yields the variance of \( X(t) \) as

\[ \sigma_X^2 = \sum_k \sum_k \frac{\gamma_k \gamma_k \psi_k(j) \psi_k(j)}{\omega_2^4} \int \frac{a_k(\omega) a_k^*(\omega) c_\gamma(\omega)}{\left| z_2(\omega) \right|^2 z_{1k}(\omega) z_{1k}^*(\omega)} d\omega \]

The integral in eq. (30) can be readily evaluated by residues; the tedious algebra can be avoided by using complex arithmetic routines available in most computers.

When the primary system consists of a single mass

\[ \sigma_X^2 = \frac{1}{\omega_2^4} \int \frac{[1 + 4\beta_1^2 \left( \frac{\omega_2}{\omega_{11}} \right)^2] c_\gamma(\omega)}{\left| z_2(\omega) \right|^2 \left| z_{11}(\omega) \right|^2} d\omega \]

It is discussed in Ref. [5] that because of the tail behavior of \( \left| z_2 \right|^2 \) and \( \left| z_{11} \right|^2 \) for lightly damped systems, certain bounds exist for the floor response spectrum. Specifically, when \( \omega_2/\omega_{11} \leq 1/3 \), the spectrum approaches that of a SDF-system with damping \( \beta_2 \) and directly subjected to the ground motion \( \dddot{y} \); whereas when \( \omega_2/\omega_{11} \geq 3 \), the pseudo-acceleration of the floor spectrum approaches the pseudo-acceleration of the primary system. This behavior is illustrated in Fig. 5 for the NS component of the May 18, 1940 El Centro accelerogram, with \( f_{11} = \omega_{11}/2\pi = 5.8 \) cps.

4.2 Random Vibration of MDF-Equipment

The floor response spectra may be used with currently available rules for combining modal maxima to analyze MDF equipments mounted on a single floor of a reactor
building. However, the floor-spectra cannot be directly used for equipments, such as piping systems, that are connected to several floors. A response spectrum approach for piping systems is described in Ref. [5]; it is mentioned that a random vibration approach to the analysis of piping systems would be preferable. Such an approach should also improve the accuracy of results for MDF systems resting on a single floor. No conceptual difficulty exists for the application of random vibration principles to MDF secondary systems along the lines proposed herein. However, additional studies are needed to simplify the four-fold summation involved in calculating all the coupled terms in the expression for $\sigma^2_\text{x}$ (two summations for the building modes and two for the equipment). This item is currently under study.

5. CONCLUSIONS

The fundamental periods and damping of most reactor facilities are such that the seismic responses of these structures can be treated as a stationary random process. Practically feasible procedures for the analysis of reactor facilities and equipments are available from random vibration theory. The use of random vibration leads to results that are in agreement with those obtained from the direct integration of a normalized set of recorded accelerograms. This shows that the proposed statistical approach can be used to obtain reliable estimates of seismic loading on the main structure as well as for the analysis of equipment responses. This avoids the use of specific accelerograms and lengthy step-by-step integration; also, the allowance for the dispersion in the maximum response can be made more systematically than by means of a simple amplification of the average response spectra.

6. ACKNOWLEDGMENT

The study described herein is part of a continuing research program on the probabilistic aspects of structural safety and design, being carried out in the Department of Civil Engineering, University of Illinois at Urbana, Illinois, USA. The program is currently supported by the National Science Foundation through research grant GK-1812X. The numerical results reported in the comparative studies were obtained by Mr. I. Gungor as a part of his doctoral dissertation. His contributions are gratefully acknowledged.

REFERENCES


TABLE 1 - COMPARISON OF RESULTS: RANDOM VIBRATION VS. INTEGRATION OF ACCELEROMETERS ($\delta = .05$ in all modes)

<table>
<thead>
<tr>
<th>Response Quantities</th>
<th>Values from Records</th>
<th>b$_{50}$ (30)</th>
<th>b$_{10}$ (30)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Range (2)</td>
<td>Ave. Highest (3)</td>
<td>2nd Highest (4)</td>
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<tr>
<td>(a) Structure 1, $T_1 = 2$ Sec</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$u_1 \times 10^4$</td>
<td>291-477</td>
<td>383</td>
<td>467</td>
</tr>
<tr>
<td>$(u_7 - u_6) \times 10^4$</td>
<td>208-409</td>
<td>283</td>
<td>322</td>
</tr>
<tr>
<td>$(u_{10} - u_9) \times 10^4$</td>
<td>98-158</td>
<td>122</td>
<td>144</td>
</tr>
<tr>
<td>$\overline{y}_1$</td>
<td>2.19-4.08</td>
<td>3.07</td>
<td>3.49</td>
</tr>
<tr>
<td>$\overline{y}_{10}$</td>
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<td>(b) Structure 2, $T_1 = 0.66$ Sec</td>
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<td>565</td>
<td>659</td>
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<tr>
<td>$\theta \times 10^6$</td>
<td>469-935</td>
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<td>906</td>
</tr>
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<td>$\overline{y}$</td>
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<td>6.68</td>
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<td>$\overline{\theta} \times 10^5$</td>
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<td>901</td>
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<td>(c) Structure 3, $T_1 = 2$ Sec</td>
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FIG. 1 FIRST PASSAGE PROBLEM

FIG. 2 COMPARISON OF MEDIAN PSEUDO VELOCITY SPECTRA TO AVERAGE SPECTRA OF NORMALIZED ACCELEROMETERS \( \beta = 0.05 \)
FIG. 3  STRUCTURES USED IN THE COMPARATIVE STUDIES
FIG. 4  SDF EQUIPMENT MOUNTED ON A PRIMARY SYSTEM

FIG. 5  RESPONSE SPECTRUM FOR STRUCTURE 2 AND ITS LIMITING VALUES—$\beta_1 = 0.02$, $\beta_2 = 0.005$, EL CENTRO
DISCUSSION

N. N. KULKARNI, India

Q
In case of pressure tube type of reactors fuelling machines also operate during operation for on-loading fuelling. If an earthquake occurs during operation "MCA" can result. It is necessary to establish a criterion for the design of fuelling machines. Can the authors present some statistical data for such a case?

A
J. M. DOYLE, U. S. A.

It seems that if you consider the fuelling machines as an equipment item, the methods outlined in our paper could be used to obtain the floor motion at the location of the machine. Therefore, the information you want would need to be calculated for each individual case. It would depend, of course, on the design basis earthquake, and the dynamic properties of the primary structure.