SEISMIC DESIGN COEFFICIENTS OF EQUIPMENT IN NUCLEAR POWER PLANTS

C.-W. LIN,

Westinghouse Electric Corporation,
Power Systems, Pittsburgh, Pennsylvania, U.S.A.

ABSTRACT

Seismic coefficients for the design of equipment in nuclear power plants have been shown to be proportional to the single degree response spectrum. Using the two degree response spectra, constructed according to a numerical time integration study for the El-Centro 1940 N-S earthquake obtained by previous investigators, equipment seismic design coefficients for other earthquakes can be easily computed using present theory.

1. INTRODUCTION

Although seismic analysis of a multi-story building has been much simplified since the response spectra technique was introduced by Alford et al. [1], and the root-mean-square combination of different modes was recommended by Clough [2], the design of a piece of equipment to resist an earthquake is still a difficult task for which a universal approach has yet to be found. There are, to the author's knowledge, at least two different approaches being adopted in the United States. These are briefly stated as follows:

a) When time history accelerogram of one or more actual earthquake records normalized to the proper density or maximum ground acceleration is available, a direct integration will generate the response spectra of equipment (see Blume, Keith [3], [4]).

b) If the N-S component of the El-Centro 1940 earthquake can be used as the design earthquake, the results of the two degree of freedom response spectra constructed by Penzien and Chopra [5], [6], may be readily used.

When design criteria is such that response spectra different from those of the El-Centro site have to be used, one would be likely to choose a time history representative of the past earthquake or an earthquake generated artificially by using a stationary, Gaussian distributed input function having a power spectral density which is filtered to represent the site characteristics, and either go through a time integration study or repeat the same analysis outlined by Penzien and Chopra [5], to obtain the required two degree of freedom spectra. These analyses are not only costly, but also time consuming. Very often, when proper specification has to be made even before preliminary design of a building is completed, the sophistication of these analyses does not seem to justify the loss of accuracy due to the use of preliminary design data and the delay of schedule.
Using the same model, this paper illustrates how the results presented by Penzien and Chopra [5], in obtaining two degree of freedom response spectra, can be easily converted to useful information for different site conditions when either the local ground power spectral density or the single degree response spectrum is known.

2. DYNAMIC RESPONSE OF A TWO DEGREE OF FREEDOM SYSTEM

Using the fact that the mass of equipment is usually small compared with the supporting structure, the interaction effect will be more from the structure to the equipment than vice versa. Consequently, Penzien and Chopra [5] suggest a separate two degree of freedom system for each of the \( N \) normal modes of the building without the equipment in analyzing this interaction effect. Figure 1 shows this system, in which subscripts \( n \) and \( a \) indicate the quantities derived from the \( n \)th building mode without the equipment and from the equipment, respectively. \( M \) is the mass, \( K \) the spring constant, \( C \) the damping factor, \( X \) the displacement relative to the support, and \( \bar{U}_n(t) \) is the support motion, with

\[
\bar{U}_n(t) = \left( \sum_{i=1}^{N} \frac{m_i}{m} \gamma_{in}^2 \right) \sum_{i=1}^{N} \frac{m_i}{m} \gamma_{in}^2 \bar{U}_i(t) = a_n \bar{U}_n(t)
\]

where \( \gamma_{in} \) is the dimensionless \( n \)th building mode shape quantity for \( i \)th floor, with \( n \) representing the floor level mass, \( \omega_n \) and \( \xi_n \) the corresponding frequency and damping ratio, respectively, \( \omega_a \) is the frequency of the equipment by itself, and \( \xi_a \) is the corresponding damping ratio. The use of the ground motion time history multiplied by the \( n \)th participation factor \( a_n \) to represent the support motion indicates that when quantities from the equipment are taken equal to zero, and the remaining system is equivalent to the generalized building model with the \( n \)th mode being excited alone.

The equations of motion can be derived from the mathematical model shown in Figure 1. However, instead of working directly with those equations, a combined single equation can be obtained as the following:

\[
\ddot{X}_a - \ddot{X}_n = \frac{2\omega^2_a (\xi_a + 1)}{\omega_n^2} (\dot{X}_a - \dot{X}_n) + \omega_n^2 (\dot{X}_a - \dot{X}_n) + 2\omega^2_a (\xi_a + 1) (X_a - X_n)
\]

where

\[
\delta_a = M_a / M_n
\]

Recall at this point that one of the basic assumptions made by Penzien and Chopra [5], in using the mathematical model of Figure 1 is that the building entirely controls the response of the equipment, while the effect from the equipment is little felt by the building. With this approximation in mind, one has:

\[
\ddot{X}_n + 2\omega_n \xi_n \dot{X}_n + \omega_n^2 X_n = -a_n \bar{U}_n
\]

Eq. (4) can be solved in terms of a convolution integral. By differentiating the solution twice with respect to \( t \) and neglecting the integration terms containing damping ratio, one arrives at:
\[
\ddot{X}_n + a_n \ddot{U}_n = a_n \omega_n \int \sin \omega_n (t - \zeta) e^{-\omega_n \zeta} (t - \zeta) d\zeta
\]

After rearranging eq. (4) by grouping \(\ddot{X}_n + a_n \ddot{U}_n \) to one side of the equation and then eliminating this using eq. (5), the resulting equation can be substituted into eq. (2) and then solved for \(X_a - X_n\). The net result is:

\[
\langle X_a - X_n \rangle = -\frac{a_n \omega_n}{\omega_a^a} \int_0^\infty \sin \omega_n (t - \lambda) e^{-\omega_n \lambda} (t - \lambda) d\lambda
\]

where:

\[
\omega_n^a = \omega_a (\beta_a + 1)^{1/2}, \quad \xi_n^a = \xi_a (\beta_a + 1)^{1/2}
\]

3. VARIANCE OF \(X_a - X_n\)

Since design earthquakes are usually very strong and have long duration, it can be deduced from Caughey, Stumpf, and Bycroft [7], [8], that each of such earthquakes forms a process which is ergodic, Gaussian distributed with zero mean. From these observations, one can take the temporal auto-correlation function of \(U_g\) as:

\[
\langle U(t) U(t') \rangle = \int_0^\infty G(\omega) \cos \omega(t - t') d\omega
\]

where \(G(\omega)\) is the power spectral density. Very often this is assumed to be a constant representing a white noise process. However, physically, such a process can never exist. For a large majority of strong earthquakes, a semi-empirical formula suggested by Kanai [9] can be used.

With the help of eq. (7), the variance of \(X_a - X_n\) may be derived from eq. (6) as:

\[
\sigma^2(t) = \langle X_a(t) - X_n(t) \rangle^2 = \int_0^\infty G(\omega) \cos \omega(t - t') d\omega
\]

Knowing the integrals involved in eq. (8) are convergent, one may reverse the order of integration. When \(G(\omega)\) is known, the integration can be carried out either explicitly or numerically, depending on how \(G(\omega)\) is given. Since \(\xi_n\) is considered to be small in most of the structural analysis, when integrations with respect to \(\lambda\) and \(\lambda'\) have been carried out by reversing the order of integration for \(\lambda, \lambda'\), and \(\omega\), the quantity \(|Z(\omega)|^2\) is given by:

\[
|Z(\omega)|^2 = \left[ \xi_n^2 a_n \omega_n^2 + 2 \xi_n \omega_n (\omega_n^2 + \omega_n^2)^2 + (\omega_n^2 - \omega_n^2)^2 \right]
\]
as obtained in the process will have a sharp peak at \( \omega = \omega_n \). Hence, when making contour integration with respect to \( \omega \), the main contribution to the integral will come from the region around \( \omega = \omega_n \). Using the same analogy as Caughey and Stumpf [7], which originated from the Laplace's method of evaluating integrals, eq. (8) may be very closely approximated by:

\[
\sigma^2(t) = G(\omega_n) \left( \frac{\omega_n}{\omega} \right)^2 \omega_n^2 \int \left( \int \frac{1}{Z(\omega)} \right) \frac{\cos(\lambda - \lambda')}{\cos(\lambda + \lambda')} \sin^2(\lambda' - \lambda - \lambda') e^{\omega_n \xi_n \lambda'} [\cos(\omega \sin(\omega_n \lambda + \xi_n \omega_n \lambda') \cos(\omega_n \lambda') + \omega_n \sin(\omega_n \lambda + \xi_n \omega_n \lambda')] + e^{-\omega_n \xi_n \lambda'} [\cos(\omega \sin(\omega_n \lambda + \xi_n \omega_n \lambda') \cos(\omega_n \lambda') + \omega_n \sin(\omega_n \lambda + \xi_n \omega_n \lambda')] d\omega d\lambda'
\]

or, one may write:

\[
\sigma^2(t) = G(\omega_n) f(\omega_n, \omega, \xi_n, \xi_n, \xi_n, \xi_n, \beta_n, \alpha_n)
\]

in which \( f(\omega_n, \omega_n, \xi_n, \xi_n, \xi_n, \beta_n, \alpha_n) \) represents the rest of the terms of the right hand side of eq. (10) multiplied by \( G(\omega_n) \). Thus, the maximum of the root-mean-squared relative displacement of \( X_a - X_n \) is given by:

\[
\sigma_{\text{max}} = [G(\omega_n)]^{1/2} [f_{\text{max}}(\omega_n, \omega_n, \xi_n, \xi_n, \xi_n, \beta_n, \alpha_n)]^{1/2}
\]

For any given \( \omega_n, \xi_n, \xi_n, \omega_n \) and \( \beta_n \), eq. (12) shows that the root-mean-squared relative displacement of \( X_a - X_n \) is proportional to the square root value of the ground power spectral density evaluated at the frequency \( \omega_n \). This implies that when analysis of the two degree of freedom system of the present model has been carried out for any particular ground power spectral density, results obtained can be converted immediately to yield root-mean-squared relative displacement of \( X_a - X_n \) for any other ground power spectral density. This is:

\[
\sigma_{\text{max}}^*(\omega_n, \omega_n, \xi_n, \xi_n, \xi_n, \beta_n, \alpha_n) = \frac{G^*(\omega_n)}{G(\omega_n)}^{1/2} \sigma_{\text{max}}(\omega_n, \omega_n, \xi_n, \xi_n, \xi_n, \beta_n, \alpha_n)
\]

where \( \sigma_{\text{max}}^* \) represents the known value of the root-mean-squared relative displacement calculated based on the particular ground power spectral density \( G(\omega_n) \), and \( \sigma_{\text{max}}^* \) represents the unknown root-mean-squared relative displacement corresponding to the given ground power spectral density \( G^*(\omega_n) \). In the case that the spectral acceleration response \( S_a^*(\omega_n, \xi_n) \) is given rather than \( G^*(\omega_n) \) a similar result as eq. (13) can be derived.

Using eq. (7) and the same analogy in obtaining eq. (12), it is possible to show that the maximum probable response of a single degree of freedom system, which approximates the
single degree response spectrum \( S_a(\omega_n, \xi_n) \), is in proportion to the square root of its power spectrum density. Therefore,

\[
\sigma_{\text{max}}(\omega, \omega_a, \xi_a, \xi_n, \beta_a, \beta_n) = \frac{S_a^*(\omega, \xi_n)}{S_a^*(\omega, \xi_n)} \sigma_{\text{max}}(\omega, \omega_a, \xi_n, \xi_a, \beta_a, \beta_n) \tag{14}
\]

Eq. (14) has a direct application when the two degree of freedom response spectra obtained by Penzien and Chopra [5] for the El-Centro 1940 earthquake are used. Using their notation with:

\[
C_{\text{an}}(\omega_n, \xi_n, \xi_a, \beta_a, \beta_n) \equiv |(X_n - X_a)/\omega_a g|_{\text{max}} \tag{15}
\]

representing the seismic coefficient based on El-Centro earthquake, eq. (14) can be written as:

\[
C_{\text{an}}^*(\omega_n, \omega_a, \xi_n, \xi_a, \beta_a, \beta_n) = \frac{S_a^*(\omega_n, \xi_n)}{S_a^*(\omega_n, \xi_n)} C_{\text{an}}(\omega_n, \omega_a, \xi_n, \xi_a, \beta_a, \beta_n) \tag{16}
\]

where

\[
C_{\text{an}}^*(\omega_n, \omega_a, \xi_n, \xi_a, \beta_a, \beta_n) = \sigma_{\text{max}}(\omega, \omega_a, \xi_n, \xi_a, \beta_a, \beta_n) \frac{\omega^2}{g} \tag{17}
\]

4. NUMERICAL EXAMPLES

To illustrate how eq. (16) can be used to convert the results obtained by Penzien and Chopra [5], corresponding to the El-Centro earthquake to results required for equipment design based on other design earthquakes, two different sets of single degree spectral accelerations, \( S_a(\omega_n, \xi_n) \) and \( S_a^*(\omega_n, \xi_n) \) representing the El-Centro earthquake and a typical earthquake, respectively, are plotted in Figure 2. Figure 3 shows the spectral ratio which is defined as \( S_a^*(\omega_n, \xi_n) / S_a(\omega_n, \xi_n) \).

Knowing the spectral ratio, seismic coefficient \( C_{\text{an}}^* \) for the typical earthquake can then be computed using eq. (16). Figures 4 to 12 show these results. Table 1 shows the parameters involved in identifying each curve in these figures.

The following procedure for seismic design of equipment located at the i-th floor of a building is outlined briefly in order to present a self-contain treatment.

a) Plot spectral ratio \( S_a^*(\omega_n, \xi_n) / S_a(\omega_n, \xi_n) \) against \( T_n \) for each \( \xi_n \) in consideration.

b) Plot seismic coefficients \( C_{\text{an}}^*(\omega_n, \xi_n, \omega_a, \xi_a, \beta_a, \beta_n) \) for each combination of \( \xi_n \) and \( \xi_a \) for different mass ratio \( \beta_a \) and equipment frequency \( \omega_a \) against the building period \( T_n \).

c) Compute dynamic characteristics of the building and the equipment such as \( M_n, \omega_n, M_a, \omega_a, \beta_a, \beta_n, \) etc.

d) From the figure showing the pair of damping ratios \( \xi_n \) and \( \xi_a \) used for the building and equipment, read in the seismic coefficients \( C_{\text{an}}^* \) for each mode.
e) Multiply each $C_{an}$ by its proper building participation factor $\alpha_n$ and the normalized mode shape $\theta_{in}$ and take the root-mean-squared sum, i.e.,

$$C_{ia}^* = \left[ \sum_{n} (\alpha_n \theta_{in} C_{an}^*)^2 \right]^{1/2}$$

f) Design the equipment to resist a maximum horizontal seismic force of:

$$F_a = C_{ia}^* W_a$$

where $W_a$ is the weight of the equipment.

5. DISCUSSION AND CONCLUDING REMARKS

For a given set of building and equipment parameters, such as $\omega_n$, $\omega_a$, $\zeta_n$, $\zeta_a$, $\beta_a$ and $\alpha_n$, the present study shows that the ratio of the seismic coefficients for two different design earthquakes is in proportion to the ratio of the spectral accelerations. Using this conclusion, together with the two degree response spectra obtained by Penzien and Chopra [5], based on a numerical time integration study for El-Centro 1940 N-S earthquake, equipment seismic design coefficients for other earthquakes can be easily obtained.

In contrast to the time consuming and high cost process of the numerical integration scheme, which in general requires the help of a large capacity electronic computer, the semi-graphic procedure reported in the present analysis is simple to apply and easy to construct.

When results by Penzien and Chopra [5] are not available, the relationship established within the text allows one to use the present typical spectral accelerations and seismic coefficients in constructing the necessary seismic coefficient curves. For damping ratios $\zeta_n$ and $\zeta_a$, and the building frequency $\omega_n$ not appearing in the prepared figures, extrapolation will yield a close estimate to the true value.

The response spectra of the El-Centro 1940 earthquake shown in Figure 2 are actually the design spectra averaged over four different earthquakes. The use of these averaged spectra in conjunction with Penzien and Chopra's [5] results undoubtedly provides some safety margin over the design since these spectral values are lower than the true spectra of the El-Centro 1940 earthquake.

Finally, it may be pointed out that the shaded area in Figures 4-15 indicates where one would be had the peak of the ground response spectra been used in the design.
REFERENCES


APPENDIX II - NOTATION

The following symbols are used in this paper:

- \( C_a \) = damping factor of the equipment.
- \( C_n \) = damping factor of the \( n \)th building mode.
- \( K_a \) = spring constant of the equipment support.
- \( K_n \) = spring constant of the generalized \( n \)th building mass.
- \( M_a \) = mass of the equipment.
- \( M_n \) = generalized mass of the \( n \)th building mode.
- \( W_a \) = equipment weight.
- \( S_{a^*} \), \( S_a \) = spectral accelerations.
- \( m_i \) = mass of the building at \( i \)th floor.
- \( G \) = power spectral density.
- \( C_{an} \), \( C_{an^*} \) = seismic coefficients.
- \( C_{a^*} \) = resultant seismic coefficient.
- \( X_a \) = displacement of equipment relative to the support.
- \( X_n \) = displacement of generalized \( n \)th building mode relative to the support.
- \( U_g \) = ground movement.
- \( U_s \) = generalized \( n \)th building mode support movement.
- \( \omega_a \) = equipment frequency.
- \( \omega_n \) = \( n \)th mode building frequency.
- \( \zeta_a \) = equipment damping ratio.
- \( \zeta_n \) = \( n \)th building mode damping ratio.
- \( \theta_{in} \) = dimensionless \( n \)th building mode shape quantity for \( i \)th floor.
- \( \beta_a \) = mass ratio of equipment to building.
- \( \omega \) = frequency variable.
- \( B \) = spectral density at bedrock.
- \( h_g \), \( v_g \) = parameters depending on local geology.
- \( g \) = gravity acceleration.
- \( T_a \) = \( 2\pi/\omega_a \), period of the equipment.
- \( T_n \) = \( 2\pi/\omega_n \), period of the building.
- \( \alpha_n \) = participation factor of \( n \)th building mode.
- \( F_a \) = design seismic force for the equipment.
### TABLE 1
PARAMETERS FOR TWO DEGREE RESPONSE SPECTRA FIGS. 4–12

<table>
<thead>
<tr>
<th>CURVE NO.</th>
<th>$T_a = \frac{2\pi}{\omega_a}$</th>
<th>$\beta_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.20</td>
<td>0.002</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.010</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0.025</td>
</tr>
<tr>
<td>4</td>
<td>0.40</td>
<td>0.002</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0.010</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>0.025</td>
</tr>
<tr>
<td>7</td>
<td>0.60</td>
<td>0.002</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>0.010</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>0.025</td>
</tr>
<tr>
<td>10</td>
<td>0.80</td>
<td>0.002</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>0.010</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>0.025</td>
</tr>
<tr>
<td>13</td>
<td>1.00</td>
<td>0.002</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>0.010</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>0.025</td>
</tr>
</tbody>
</table>
FIG. 1 TWO DEGREE OF FREEDOM SYSTEM

LEGEND

\(- S_0 (\omega_0, f_n), \text{EL-CENTRO} \) -- \(- \tilde{S}_0 (\omega_0, f_n), \text{A 7.1 TIMES} \)

1940 EARTHQUAKE  EMPIRED TYPICAL EARTHQUAKE

FIG. 2 ACCELERATION SPECTRUM
**FIG. 3** SPECTRAL RATIOS \( \frac{S_{a}^{*}(\omega_n, \xi_n)}{S_{a}(\omega_n, \xi_n)} \) VS. \( T_n \)

**FIG. 4** SEISMIC COEFFICIENTS
FIG. 5 SEISMIC COEFFICIENTS

FIG. 6 SEISMIC COEFFICIENTS
FIG. 7 SEISMIC COEFFICIENTS

FIG. 8 SEISMIC COEFFICIENTS
FIG. 9 SEISMIC COEFFICIENTS

FIG. 10 SEISMIC COEFFICIENTS
FIG. 11 SEISMIC COEFFICIENTS

FIG. 12 SEISMIC COEFFICIENTS
DISCUSSION

P. MITTERBACHER, Switzerland

Q  Do you consider the case of a horizontal tube or tank with end-closure and not completely filled with water under the influence of an earthquake?

A  J. D. STEVENSON, U. S. A.

For design purposes the simplified method presented in TID 7024, chapter 6, earthquake design of nuclear facilities is normally used. This procedure does consider slosh effect.

H. WÖLFEL, Germany

Q  I think your method is very conservative. Did you compare your results with a time history analysis and can you give us an estimation of the failure?

A  J. D. STEVENSON, U. S. A.

No comparison with time history was performed. The method does tend to be conservative in so far as it applies to single degree of freedom systems. However, in systems which might not respond as a single degree of freedom system some additional conservatism should be required. In addition the technique is meant to be used as simple procedure to develop design criteria to the manufacturer. Avoiding to evaluate a design in detail requires to be conservative to insure design adequacy.

H. SHIBATA, Japan

Q  How do you define the mass ratio of equipment to a building? In most cases, the mass of a building might be less than the total mass of it, especially in a coupling condition of higher modes, if you use a set of two-degrees-of-freedom systems.

A  J. D. STEVENSON, U. S. A.

Generally the mass of that portion of the structure that will be excited by a particular piece of equipment is much less than the total mass of the structure. Actual points of support have to be evaluated to estimate affected building mass. An over-estimation of affected building mass is preferred since this will lead to more conservative results.