DYNAMIC TEMPERATURE DISTRIBUTION OF A POWER REACTOR FUEL ELEMENT

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ABSTRACT

An accurate model for the dynamic heat transfer of a power reactor with a cylindrical type fuel element is presented.

We consider a cylindrical fuel element composed of a hollow fuel core and its cladding, separated by a vacuum gap. We suppose these two regions homogeneous and isotropic with thermal properties independent from the temperature.

The mathematical model is based on heat diffusion equations in fuel and cladding with radiative heat transfer in the vacuum gap between them and on the convection equation for the coolant fluid.

The integration of the equations with respect to the variables involved in the problem is performed by the following steps: first, we operate the separation of the radial variable by means of modal expansion via a finite integral transformation specially constructed for a multiregion system; then, we arrive to an integral-differential equation system for time and the axial variable. Lastly, the Laplace transform with respect to time reduces the problem to a system of ordinary differential equations in the axial variable. An analytical integration performed for this variable gives axial temperature and power transfer functions particularly suited for analog simulation.

The long term solution is then obtained by an iterative numerical procedure whose steps reproduce on the digital computer the cascading blocks of the analog simulation.

The short term solution, particularly interesting for the thermoelastic and safety analysis of power plants, needs a different handling because the iterative procedure converges too slowly. Therefore we construct this solution by numerical inversion of the Laplace transform of the indicial response and by convolution operations.

1. INTRODUCTION

In the present paper we shall consider the problem of determining the transient temperature distribution in a long cylindrical reactor fuel element.

To determine the radial temperature field we perform a modal expansion of radial variable via a finite integral transformation particularly suitable for a multiregion system.

To determine the axial temperature distribution we use a numerical procedure.

We develop formally the calculation for a system constituted by two solid regions, the core and the cladding, the latter cooled by a fluid. Conduction in the solid and convection in the fluid are considered. Radiation is taken into account in the gap between core and cladding.

2. THE PROBLEM

Let us consider a cylindrical fuel element composed of a hollow fuel core and its cladding, separated by a gap. We suppose that these two regions are homogeneous isotropic bodies with thermal properties independent from the temperature.

In this paper we shall evaluate the radial and axial temperature fields in both fuel element and coolant fluid.

The following assumption are made:

a) axial heat diffusion is negligible in both fuel and fluid
b) coolant temperature has been radially averaged
c) coolant heat transfer occurs only by convection
d) symmetry in cylindrical geometry subsists
e) radiation phenomena in the gap between the core and cladding are taken into account in a linearized form.
f) thermal properties are independent from the temperature

The fuel equations with dimensionless variables are therefore:

$$\frac{\partial T_1(\tau,R,X)}{\partial \tau} = [\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R}]\, T_1(\tau,R,X) + \frac{r_0^2}{\lambda_1} W_1(\tau,X) \qquad R_0 \le R \le R_1$$

$$(\frac{a_1}{a_2})^2 \frac{\partial T_2(\tau,R,X)}{\partial \tau} = \frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R}\, T_2(\tau,R,X) + \frac{r_0^2}{\lambda_2} W_2(\tau,X) \qquad R_2 \le R \le R_3 \tag{1}$$

The boundary conditions are:

$$|\frac{\partial T_1}{\partial R}|_{R_0} = 0 \; ; \; \frac{\lambda_2}{r_0} |\frac{\partial T_2}{\partial R}|_{R_3} + H|T_2(\tau,R_3,X) - t(\tau,X)| = 0 \tag{2}$$

while the interface conditions are:

$$\frac{\lambda_1}{r_0}\left|\frac{\partial T_1}{\partial R}\right|_{R_1} + h_{12}(T_1(\tau,R_1,X) - T_2(\tau,R_2,X)) = 0 \tag{3}$$

$$R_1\lambda_1\left|\frac{\partial T_1}{\partial R}\right|_{R_1} = R_2\lambda_2\left|\frac{\partial T_2}{\partial R}\right|_{R_2}$$

The fluid equation is

$$T_2(\tau,R_3,X) - t(\tau,X) = \mu\frac{\partial t(\tau,X)}{\partial\tau} + \frac{\partial t(\tau,X)}{\partial X} \tag{4}$$

where

$$\mu = \frac{\gamma_r C_r S_r a_1^2}{2\pi r_3 r_0^2 H}$$

3. CONSTRUCTION OF FINITE INTEGRAL TRANSFORMS FOR THE FUEL AND CLADDING.

The finite integral transform method is used to eliminate the radial variable from the system of equations (1). In this way the problem is reduced to integrating with respect to τ and X variables.

When geometry takes simple forms, such as a bare cylinder, the transform's kernel is already found on the literature; for more complicated geometries the kernel must be constructed in accordance with the geometry itself and the boundary and interface conditions.

To this aim, let us define the integral transforms:

$$\bar{T}_1(\tau,X;p) = \int_{R_0}^{R_1} RT_1(\tau,R,X)V_1(pR)dR \quad \text{for the fuel} \tag{5}$$

$$\bar{T}_2(\tau,X;p) = \int_{R_2}^{R_3} RT_2(\tau,R,X)V_2(pR)dR \quad \text{for the cladding}$$

Now, integrating by parts, we can write the following identity:

$$\int_\alpha^\beta \frac{\partial}{\partial R}\left(R\frac{\partial W}{\partial R}\right)V(pR)dR = \int_\alpha^\beta RW\left(\frac{d^2}{dR^2} + \frac{1}{R}\frac{d}{dR}\right)V(pR)dR + \left|RV\frac{dW}{dR}\right|_\alpha^\beta - \left|RW\frac{dV}{dR}\right|_\alpha^\beta$$

Therefore, for the fuel we have:

$$\int_{R_0}^{R_1} \frac{\partial}{\partial R}\left(R\frac{\partial T_1}{\partial R}\right)V_1(pR)dR = \int_{R_0}^{R_1} T_1(\tau,R,X)R\left(\frac{d^2}{dR^2} + \frac{1}{R}\frac{d}{dR}\right)V_1(pR)dR + H_1(\tau,X;p) \qquad (6)$$

If we add and subtract $p^2\bar{T}(\tau,X;p)$ on the right-hand side of eq. (6), we find:

$$\int_{R_0}^{R_1} \frac{\partial}{\partial R}\left(R\frac{\partial T_1}{\partial R}\right)V_1(pR)dR = \int_{R_0}^{R_1} T_1(\tau,R,X)R\left(\frac{d^2}{dR^2} + \frac{1}{R}\frac{d}{dR} + p^2\right)V_1(pR)dR - p^2\bar{T}(\tau,X;p) + H_1(\tau,X;p) \qquad (7)$$

where

$$H_1(\tau,X;p) = \left|RV_1\frac{\partial T_1}{\partial R}\right|_{R_0}^{R_1} - \left|RT_1\frac{dV_1}{dR}\right|_{R_0}^{R_1} \qquad (8)$$

Now let us impose to the function $V_1(pR)$ to satisfy the Bessel differential equation

$$\frac{d^2V_1}{dR^2} + \frac{1}{R}\frac{dV_1}{dR} + p^2V_1 = 0 \qquad (9)$$

that is

$$V_1(pR) = A_1J_0(pR) + B_1Y_0(pR) \qquad (10)$$

Then eq. (7) becomes:

$$\int_{R_0}^{R_1} \frac{\partial}{\partial R}\left(R\frac{\partial T_1}{\partial R}\right)V_1(pR)dR = -p^2\bar{T}_1(\tau,X;p) + H_1(\tau,X;p) \qquad (11)$$

For the cladding material, with analogous procedure, we obtain

$$\frac{dV_2}{dR} + \frac{1}{R}\frac{dV2}{dR} + \left(\frac{a_1}{a_2}p\right)^2 V_2 = 0 \qquad (12)$$

$$V_2\left(p\frac{a_1}{a_2}R\right) = A_2J_0\left(p\frac{a_1}{a_2}R\right) + B_2Y_0\left(p\frac{a_1}{a_2}R\right) \qquad (13)$$

and

$$H_2(\tau,X;p) = \left|R\frac{\partial T_2}{\partial R}V_2\right|_{R_2}^{R_3} - \left|RT_2\frac{dV_2}{dR}\right|_{R_2}^{R_3} \qquad (14)$$

The formal steps which have brought to the eq. (9) and eq.(12) are suggested and justified as extension of simpler cases. By the same formal steps, in fact,classic integral transforms are obtained, such as Hankel and Laplace transforms.

In the case of multiregion domains it is impossible to define an unique transform kernel for the complete domain, as occurs, e.g., for the Laplace transform, but it is necessary to define a transform kernel for every region.

For a more complete description of this theory see S. Kaplan and G. Sonneman |1| ; for its applications to multiregion domains see G. Papa |2| . The purpose of the following calculations is to define uniquely the kernels V_1 and V_2 (that is to determine the costants A_1 , B_1 and A_2 , B_2) as functions of the boundary conditions of the problem.

Let us then multiply both sides of eq.(1) by $RV_1(pR)$ and take into account eq.(11). We obtain:

$$\frac{d\bar{T}_1}{d\tau}(\tau,X;p) = - p^2\bar{T}_1(\tau,X;p) + \frac{r_0^2}{\lambda_1}\bar{W}_1(\tau,X;p) + H_1(\tau,X,p) \quad \text{for the fuel} \qquad (15)$$

and

$$\frac{d\bar{T}_2}{d\tau}(\tau,X;p) = - p^2\bar{T}_2(\tau,X;p)+\left(\frac{a_2}{a_1}\right)^2\frac{r_0^2}{\lambda_2}\bar{W}_2(\tau,X;p)+\left(\frac{a_2}{a_1}\right)^2H_2(\tau,X;p) \quad \text{for the cladding} \qquad (16)$$

where

$$\bar{W}_1(\tau,X;p)=W_1(\tau,X)\int_{R_0}^{R_1}RV_1(pR)dR;\ \bar{W}_2(\tau,X;p)=W_2(\tau,X)\int_{R_2}^{R_3}RV_2\left(p\,\frac{a_1}{a_2}\,R\right)dR$$

Adding side by side eq.(15) and eq.(16) we have:

$$\frac{d}{d\tau}\,\bar{\Theta}(\tau,X;p)+p^2\bar{\Theta}(\tau,X;p)=H_1(\tau,X;p)+\left(\frac{a_2}{a_1}\right)^2H_2(\tau,X;p)+\frac{r_0^2}{\lambda_1}\bar{W}_1(\tau,X;p)+\left(\frac{a_2}{a_1}\right)^2\frac{r_0^2}{\lambda_2}\bar{W}_2(\tau,X;p) \qquad (17)$$

where

$$\bar{\Theta}(\tau,X;p) = \bar{T}_1(\tau,X;p) + \bar{T}_2(\tau,X;p) \qquad (18)$$

Accounting for eq. (2) and eq. (3) we can write:

$$H_1(\tau,X;p)+\left(\frac{a_2}{a_1}\right)^2H_2(\tau,X;p)=\left(\frac{a_2}{a_1}\right)^2\frac{r_1H}{\lambda_2}V_2\left(p\,\frac{a_1}{a_2}R_3\right)t(X,\tau)+T_1(\tau,R_1,X)\left\{\left(\frac{a_2}{a_1}\right)^2\frac{r_1h_{12}}{\lambda_2}V_2\left(p\,\frac{a_1}{a_2}\,R_2\right)\right.$$

$$-R_1\left[\frac{dV_1}{dR}\Big|_{R_1}-\frac{r_1h_{12}}{\lambda_1}V_1(pR_1)\right\} + R_0\frac{dV_1}{dR}\Big|_{R_0}T_1(\tau,R_0,X)+T_2(\tau,R_2,X)\left\{\frac{r_1h_{12}}{\lambda_1}V_1(pR_1)-\left(\frac{a_2}{a_1}\right)^2\frac{r_1h_{12}}{\lambda_2}V_2\left(p\frac{a_1}{a_2}R_2\right)\right.$$

$$\left.+\left(\frac{a_2}{a_1}\right)^2R_2\frac{dV_2}{dR}\Big|_{R_2}\right\}-T_2(\tau,R_3,X)\left\{\left(\frac{a_2}{a_1}\right)^2\frac{r_1H}{\lambda_2}V_2\left(p\,\frac{a_1}{a_2}R_3\right)+\left(\frac{a_2}{a_1}\right)^2R_3\frac{dV_2}{dR}\Big|_{R_3}\right\}. \qquad (19)$$

Let us note that in eq. (19) appear the unknown functions $T_1(\tau,X,R)$ and $T_2(\tau,X,R)$ evaluated respectively in R_0 , R_1 and R_2 , R_3 . These functions can be eliminated from eq. (19) by setting:

$$
\begin{aligned}
&\left|\frac{dV_1}{dR}\right|_{R_0} = 0 \\
&\left|\frac{dV_1}{dR}\right|_{R_1} = \frac{r_0 h_{12}}{\lambda_1}\left[\left(\frac{a_2}{a_1}\right)^2 \frac{\lambda_1}{\lambda_2} V_2\left(p\,\frac{a_1}{a_2}R_2\right) - V_1(pR_1)\right] \\
&\left|\frac{dV_1}{dR}\right|_{R_1} - \left(\frac{a_2}{a_1}\right)\frac{r_2}{r_1}\left|\frac{dV_2}{dR}\right|_{R_2} = 0 \\
&\left|\frac{dV_2}{dR}\right|_{R_3} + \frac{r_0 H}{\lambda_2} V_2\left(p\,\frac{a_1}{a_2}R_3\right) = 0
\end{aligned}
\tag{20}
$$

It is found that:

$$
H_1(\tau,X;p)+\left(\frac{a_2}{a_1}\right)^2 H_2(\tau,X;p)=\left(\frac{a_2}{a_1}\right)^2 \frac{r_3 H}{\lambda_2} V_2\left(p\,\frac{a_1}{a_2}R_3\right)t(\tau,X) \tag{21}
$$

Before integrating eq. (15) and eq. (16), we rewrite the system of eq.(20), using eq. (10) and eq. (13):

$$
\begin{aligned}
&A_1J_1(pR_0) + B_1Y_1(pR_0) = 0 \\
&A_1\left[\frac{r_0h_{12}}{\lambda_1} J_0(pR_1)-pJ_1(pR_1)\right]+B_1\left[\frac{r_0h_{12}}{\lambda_1} Y_0(pR_1)-pY_1(pR_1)\right]-A_2\left[\frac{r_0h_{12}}{\lambda_2}\left(\frac{a_2}{a_1}\right)^2 J_0\left(p\frac{a_1}{a_2}R_2\right)\right] \\
&\quad - B_2\left[\frac{r_0h_{12}}{\lambda_2}\left(\frac{a_2}{a_1}\right)^2 Y_0\left(p\,\frac{a_1}{a_2}R_2\right)\right] = 0 \\
&A_1J_1(pR_1)+B_1Y_1(pR_1)-A_2\left[\frac{a_2}{a_1}\frac{r_2}{r_1} J_1\left(p\frac{a_1}{a_2}R_2\right)\right] -B_2\left[\frac{a_2}{a_1}\frac{r_2}{r_1} Y_1\left(p\,\frac{a_1}{a_2}R_2\right)\right] = 0 \\
&A_2\left[\frac{r_0H}{\lambda_2} J_0\left(p\,\frac{a_1}{a_2}R_3\right)-p\,\frac{a_1}{a_2}J_1\left(p\frac{a_1}{a_2}R_3\right)\right]+B\left[\frac{r_0H}{\lambda_2} Y_0\left(p\frac{a_1}{a_2}R_3\right)-p\,\frac{a_1}{a_2}Y_1\left(p\,\frac{a_1}{a_2}R_3\right)\right] = 0
\end{aligned}
\tag{22}
$$

This is a homogeneous system with respect to the arbitrary constants A_1, B_1, A_2, B_2 ; therefore it admits solutions only if the determinant of the coefficients matrix vanishes; this occurs for a set of values of p which constitutes the spectrum of an eigenvalue problem consisting of the differential equations (9) and (12) with the boundary and the interface conditions (20).

For the k-th eigenvalue p_k of the set the constants A_{1k} , B_{1k}, A_{2k}, B_{2k} are therefore uniquely determined. Setting, e.g.

$$
A_{1k} = Y_1(p_kR_0)
$$

we must still write the inversion formula, that is the formula which expresses the temperature functions $T_1(\tau,R,X)$ and $T_2(\tau,R,X)$ in terms of $\bar{\Theta}(\tau,X;p_k)$ solution of eq. (17).

The functions $V_1(p_kR)$, $V_2(p_kR)$ have the orthogonality property for $i \neq k$

$$\int_{R_0}^{R_1} RV_1(p_kR)V_1(p_iR)dR + \int_{R_2}^{R_3} RV_2(p_kR)V_2(p_iR)dR = 0 \tag{23}$$

This property permits us to write the inversion formula for the function $\bar{\Theta}(\tau,X;p_k)$ as a series of eigenfunctions $V_1(p_kR)$, $V_2(p_kR)$ on the p_k eigenvalues.
In fact, let us write the inversion formulae in this way:

$$\begin{aligned}\bar{T}_1(\tau,X;p_i)&=\int_{R_0}^{R_1}\Big[\sum_{k=1}^{\infty} A(p_k)\bar{\Theta}(\tau,X,p_k)V_1(p_kR)\Big]V_1(p_iR)dR\\ \bar{T}_2(\tau,X;p_i)&=\int_{R_2}^{R_3}\Big[\sum_{k=1}^{\infty} A(p_k)\bar{\Theta}(\tau,X,p_k)V_2(p_kR)\Big]V_2(p_iR)dR\end{aligned} \tag{24}$$

Substituting in eq. (18) we obtain:

$$\bar{\Theta}(p_i,X,\tau)=\sum_{k=1}^{\infty} A(p_k)\bar{\Theta}(p_k,X,\tau)\Big[\int_{R_0}^{R_1} RV_1(p_kR)V_1(p_iR)dR+\int_{R_2}^{R_3} RV_2(p_kR)V_2(p_iR)dR\Big]$$

Owing to eq. (23), this expression reduces to:

$$\bar{\Theta}(p_i,X,\tau)=A(p_i)\bar{\Theta}(p_i,X,\tau)\Big[\int_{R_0}^{R_1} RV_1^2(p_iR)dR+\int_{R_2}^{R_3} RV_2^2(p_iR)dR\Big]$$

and lastly to:

$$A(p_i) = \Big[\int_{R_0}^{R_1} RV_1^2(p_iR)dR+\int_{R_2}^{R_3} RV_2^2(p_iR)dR\Big]^{-1}$$

The inversion formula may be written:

$$T_i(\tau,R,X) = \sum_{k=1}^{\infty} \frac{\bar{\Theta}(\tau,X;p_k)V_i(p_k\alpha_iR)}{\sum_i^2 \int_{R_{i-1}}^{R_i} RV_i^2(p_k\alpha_iR)dR} \qquad i = 1,2. \tag{25}$$

where $\alpha_1=1$ for $R_0 \leq R<R_1$ and $\alpha_2= \frac{a_1}{a_2}$ for $R_2 \leq R \leq R_3$.

The function $\bar{\Theta}(\tau,X;p_k)$ appearing in eq. (25) is a particular solution of eq. (17) and takes the form

$$\bar{\theta}(\tau,X;p_k)=\bar{\theta}(0,X;p_k)e^{-p_k^2\tau}+e^{-p_k^2\tau}\int_0^\tau e^{p_k^2\zeta}\Big[\left(\frac{a_2}{a_1}\right)^2\frac{r_3H}{\lambda_2}V_2\left(p_k\frac{a_1}{a_2}R_1\right)t(\zeta,X)+\frac{r_0}{\lambda_1}\bar{W}_1(\zeta,X,p_k)$$

$$+\left(\frac{a_2}{a_1}\right)^2\frac{r_0^2}{\lambda_2}\bar{W}_2(\zeta,X,p_k)\Big]d\zeta$$

where $\bar{\theta}(0,X;p_k)$ is the initial value of temperature eq. (18).

4. AXIAL TEMPERATURE AND POWER TRANSFER FUNCTION

The eq. (25) for $R=R_3$ gives $T_2(\tau,R_3,X)$; this one, substituted in eq.(4), gives us an integral-differential equation in the unique unknown $t(\tau,X)$.

If now we execute the L-transform with respect to τ , we obtain the following ordinary differential equation in the independent variable X :

$$\frac{dt(s,X)}{dX}+q(s)t(s,X)=t(0,X)+\sum_{k=1}^{\infty}\frac{\bar{\theta}(0,X;p_k)}{V_2(p_k\frac{a_1}{a_2}R_3)}\frac{p_k^2}{s+p_k^2}C(p_k)+\frac{r_0^2}{\lambda_1}W_1(s,X)\sum_{k=1}^{\infty}\frac{p_k^2}{s+p_k^2}g_1(p_k)$$

$$+\left(\frac{a_2}{a_1}\right)^2\frac{r_0^2}{\lambda_2}W_2(s,X)\sum_{k=1}^{\infty}\frac{p_k^2}{s+p_k^2}g_2(p_k) \qquad (26)$$

where $t(0,X)$ is the initial value of coolant temperature, and

$$q(s)=s\left(\mu+\left(\frac{a_2}{a_1}\right)^2\frac{r_3H}{\lambda_2}\sum_{k=1}^{\infty}\frac{C(p_k)}{s+p_k^2}\right)$$

$$C(p_k)=\frac{1}{p_k^2}\frac{V_2^2(p_k\frac{a_1}{a_2}R_3)}{\int_{R_0}^{R_1}RV_1^2(p_kR)dR+\int_{R_2}^{R_3}RV_2^2(p_k\frac{a_1}{a_2}R)dR}$$

$$g_1(p_k)=C(p_k)\frac{\int_{R_0}^{R_1}RV_1^2(p_kR)dR}{V_2(p_k\frac{a_1}{a_2}R_3)} \qquad (27)$$

$$g_2(p_k)=C(p_k)\frac{\int_{R_2}^{R_3}RV_2^2(p_k\frac{a_1}{a_2}R)dR}{V_2(p_k\frac{a_1}{a_2}R_3)}$$

The right-hand side of eq. (26) is a known function, say $F(s,X)$, of the power in fuel element and cladding.

As we integrate eq. (26) with respect to X we have

$$t(s,X) = e^{-q(s)X}\{t(s,0) + \int_0^X e^{q(s)\zeta} F(s,\zeta)d\zeta\} \qquad (28)$$

When the power generated within the fuel remains constant and the fluid temperature entering the channel is given a unitary step variation, eq. (28) becomes:

$$t(s,X) = \frac{1}{s} e^{-q(s)X} \qquad (29)$$

On the other hand,as the power distribution in the fuel is length-independent and it is given a unitary step variation, while the fluid temperature entering the channel is kept at zero, eq. (28) becomes:

$$\frac{\lambda_1}{r_0^2} t(s,X) = \frac{1}{s^2} G(s) \{1-\exp(-\mu Xs)\exp(X(\frac{a_2}{a_1})^2 \frac{Hr_3}{\lambda_2} \sum_{k=1}^{\infty} \frac{p_k^2}{s+p_k^2} C(p_k))\} \qquad (30)$$

where:

$$G(s) = \frac{\sum_{k=1}^{\infty} \frac{p_k^2}{s+p_k^2} g_1(p_k)}{\mu+(\frac{a_2}{a_1})^2 \frac{Hr_3}{\lambda_2} \sum_{k=1}^{\infty} \frac{C(P_k)}{s+p_k^2}}$$

5. NUMERICAL RESULTS

The inverse Laplace transform of eq.(28) cannot be expressed in analitic form. The operator $\exp(-q(s)X)$ in eq.(28) has no poles in the complex plane and therefore the residues techniques cannot be used.

However, this operator can be written as a product of an infinite number of operators, the k-th of them being:

$$\Phi_k(s,X) = \exp\{X(\frac{a_2}{a_1})^2 \frac{Hr_3}{\lambda_2} \frac{p_k^2}{s+p_k^2} C(p_k)\} \qquad (31)$$

Then we can write:

$$e^{-q(s)X} = e^{-X(1+\mu s)} \prod_{k=1}^{\infty} \Phi_k(s,X) \qquad (32)$$

The operator in eq.(31) can be simulated on an analog computer to obtain a system of infinite cascading blocks to represent eq.(32) , or on a digital computer to obtain an iterative procedure (see R. Forghieri and G.Papa |3|,|4|).

Obviously, the infinite process must be truncated on the machine. The approximation so obtained is satisfactory for the long term solution.

On the other hand the short term solution, for the problem considered, is not obtainable with a good approximation. Then we construct the solution valid on the whole time axis by a numerical inversion method of the Laplace transform using a digital computer. This method approximates the transfer function by means of a piece-wise linear function, which is then inverted analytically (this method is presented by Solodovnikov |5| pp.40-46).

The method of analysis applied in the present paper can be easily extended to handle systems with more than two regions separated by vacuum gaps.

The Authors have applied these techniques to the analysis of the transient temperature distribution in a fuel element like the one of the PEC reactor (see Table 1).

In figures 1 and 2 there are displayed the transients for a coolant temperature step at the inlet of the channel and in figures 3 and 4 the transients due to an uniform power step in the fuel element.

6. NOMENCLATURE

θ	time, sec
x	axial distance, m
r_0, r_1	inner and outer radius of fuel, m
r_2, r_3	inner and outer radius of cladding, m
T_1	fuel temperature, °C
T_2	cladding temperature, °C
t	coolant temperature, °C
γ_1	fuel density, Kg m^{-3}
γ_2	cladding density, Kg m^{-3}
γ_r	coolant density, Kg m^{-3}
C_1	specific heat of fuel, joule $°C^{-1}$ Kg^{-1}
C_2	specific heat of cladding, joule $°C^{-1}$ Kg^{-1}
C_r	specific heat of coolant, joule $°C^{-1}$ Kg^{-1}
λ_1	heat conduction coefficient in fuel, joule $°C^{-1} sec^{-1} m^{-1}$
λ_2	heat conduction coefficient in cladding, joule $°C^{-1} sec^{-1} m^{-1}$
$C_1\gamma_1$	specific heat per unit volume of fuel, joule $°C^{-1} m^{-3}$
$C_2\gamma_2$	specific heat per unit volume of cladding, joule $°C^{-1} m^{-3}$
$a_1^2 = \frac{\lambda_1}{C_1\gamma_1}$	, $m^2 sec^{-1}$
$a_2^2 = \frac{\lambda_2}{C_2\gamma_2}$	, $m^2 sec^{-1}$
H	heat transfer coefficient from cladding to coolant, joule $°C^{-1} m^{-2} sec^{-1}$
h_{12}	heat transfer coefficient from fuel to cladding, joule $°C^{-1} m^{-2} sec^{-1}$
S_r	coolant area per element, m^2
u	coolant velocity, m sec^{-1}
W_1	power density in fuel, joule $sec^{-1} m^{-3}$
$R = \frac{r}{r_0}$	dimensionless radius
$\tau = \frac{a_1^2}{r_0^2}\theta$	dimensionless time
$X = \frac{2\pi r_3 H}{\gamma_r C_r S_r u}$	dimensionless axial distance

7. REFERENCES

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TABLE 1

Data for numerical sample:

r_0 = $7{,}75\ 10^{-2}$ cm

r_1 = $2{,}5\ 10^{-1}$ cm

r_2 = $2{,}92\ 10^{-1}$ cm

r_3 = $3{,}29\ 10^{-1}$ cm

h_{12} = 2,84 joule $°C^{-1} sec^{-1} cm^{-2}$

H = 12 joule $°C^{-1} sec^{-1} cm^{-2}$

γ_1 = 10,5 gr cm^{-3}

C_1 = $3{,}386\ 10^{-1}$ joule $°C^{-1} gr^{-1}$

λ_1 = $4{,}68\ -0^{-2}$ joule $°C^{-1} sec^{-1} cm^{-1}$

γ_2 = 7,92 gr cm^{-3}

C_2 = $5{,}02\ 10^{-1}$ joule $°C^{-1} gr^{-1}$

λ_2 = $2{,}195\ 10^{-1}$ joule $°C^{-1} sec^{-1} cm^{-1}$

γ_r = 0,844 gr cm^{-3}

C_r = 1,27 joule $°C^{-1} gr^{-1}$

S_r = 0,1887 cm^2

x = 90 cm

u = 732 cm sec^{-1}

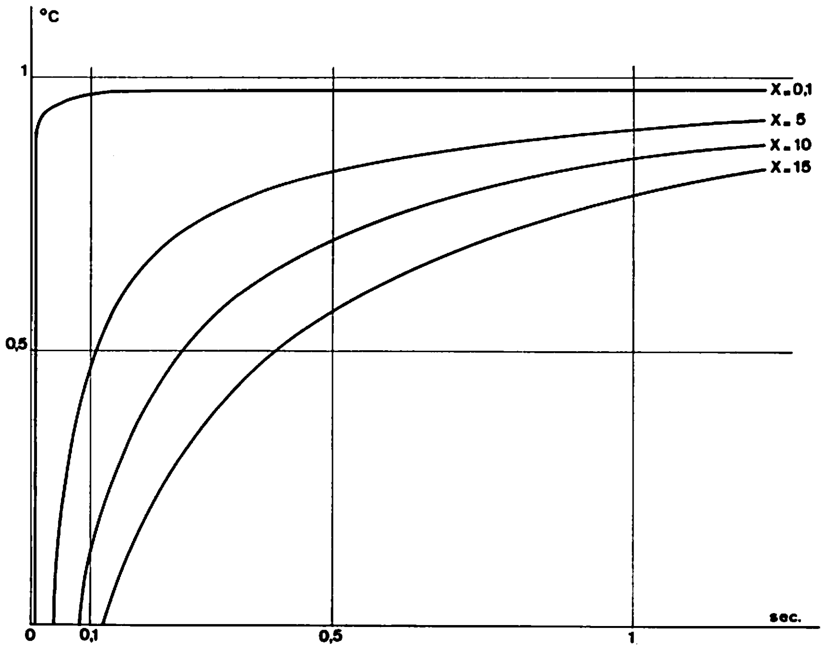

Fig. 1 - Coolant transient temperature response to a step variation of inlet coolant temperature.

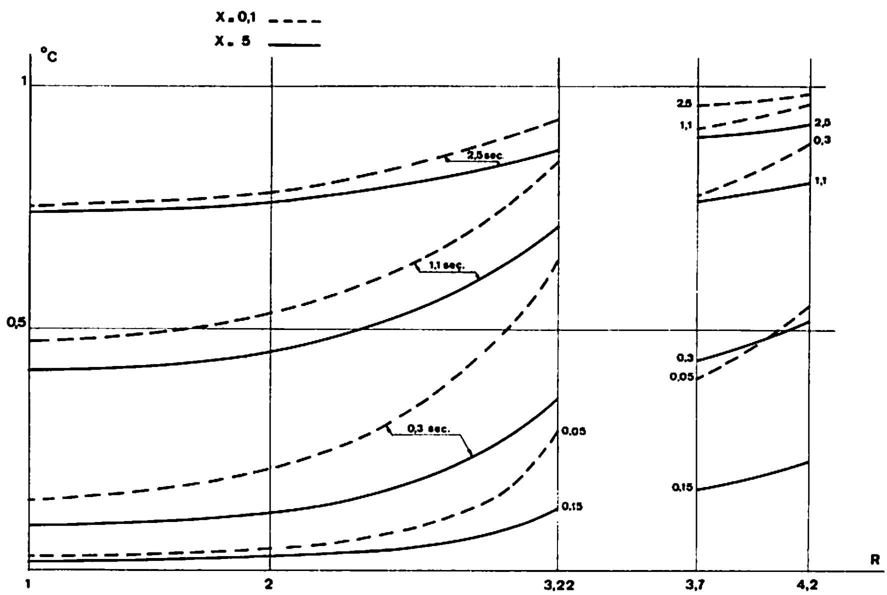

Fig. 2 - Transient temperature distribution in fuel element and cladding to a step variation of inlet coolant temperature.

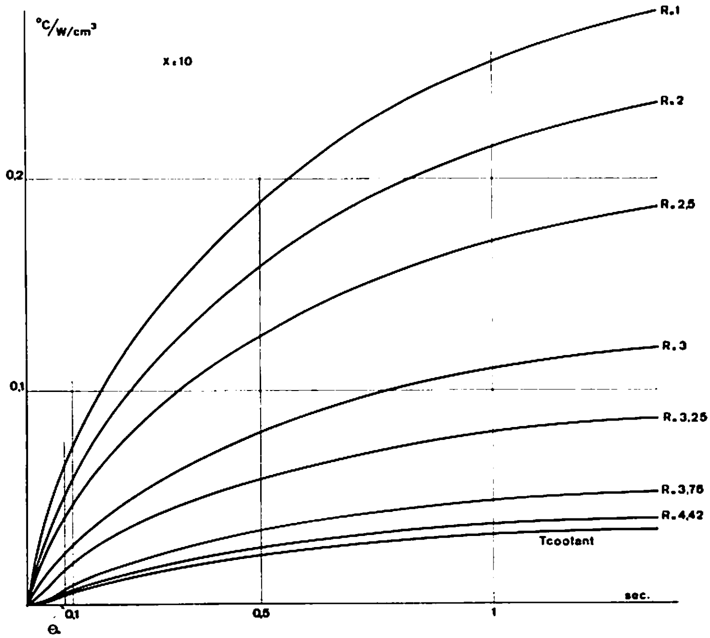

Fig. 3 - Transient temperature response to a step variation of power density.

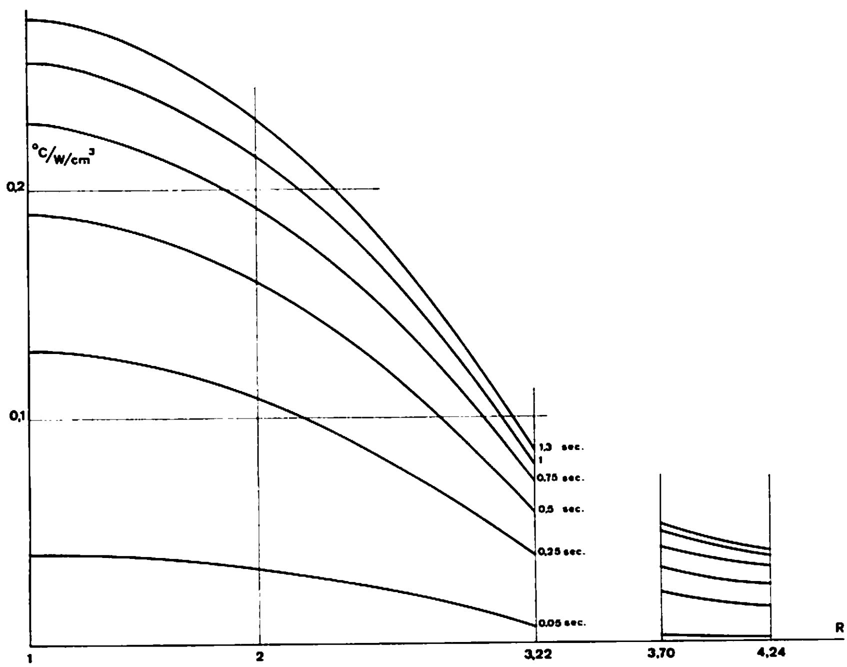

Fig. 4 - Radial transient temperature distribution to a step variation of power