TWO-DIMENSIONAL, MULTIREGION ANALYSIS OF TEMPERATURE FIELDS IN FINITE ROD BUNDLES COOLED BY LIQUID METALS

L. WOLF, K. JOHANNSEN,
Technische Universität Berlin, Berlin, Germany

ABSTRACT

This paper presents the results of a single-rod-three-region (fuel, clad, coolant) analysis of steady-state fully developed heat transfer to slug flow along symmetrically posed corner and lateral rods of un baffled hexagonal rod bundles with equilateral triangular spacing. Account is taken for thermal contact resistance at fuel-clad interface and clad surface. The effect of round dispersers inserted into the peripheral flow channel is investigated. From the explicit temperature solutions obtained in terms of Fourier series, the effect of variations in geometrical parameters like pitch-to-diameter ratio, relative rod distance from the channel wall, and relative clad thickness as well as in thermal parameters like relative conductivities of fuel and clad on heat transfer and temperature distributions is studied.

1. INTRODUCTION

Present designs for liquid-metal-cooled fast breeder reactors involve parallel flow past tightly packed rods arranged in equilateral triangular array within hexagonal channels (Fig. 1). In any of these subassemblies one can distinguish two types of fuel rods which operate under nonequivalent conditions from the point of view of cooling by the liquid metal: the fuel rods of the central and peripheral regions. The fuel rods of the central region are surrounded by twelve identical cells through each of which the same amount of liquid flows; therefore the temperature distributions in the central fuel rods are symmetric with respect to 0, 30, and 60° etc. As shown in Figure 1, a completely different type of coolant flow area is attached to the peripheral fuel rods at the plane walls of the subassembly which is only symmetric with respect to 0, and 180°. As a result of this higher degree of nonuniformity of cooling, the circumferential variations in wall and coolant temperatures as well as heat fluxes are considerably more pronounced for these corner and edge rods than for central...
rods. The same effect occurs if a central rod is displaced from its symmetrical position as recently demonstrated by both experimental and theoretical investigations [1, 2, 3]. The peripheral variations in wall temperatures and heat fluxes have an important consequence in determining the average convective heat transfer coefficient, since these depend on the internal thermal properties and dimensions of the fuel rod. As a result, a reliable prediction of the thermal performance of the rod bundle can only be achieved if the heat transfer within the coolant and fuel rod is determined simultaneously (multiregion analysis). Single-region analyses which consider only the heat transfer within the coolant region by assuming constant azimuthal heat flux at the rod surface become more and more questionable if the irregularity of fluid flow area increases. For liquid-metal cooling, this has been recently confirmed by Nijsing and Elfier [4] and Marchese [5] who performed thorough analyses for axial turbulent flow through closely spaced rods in an infinite array \(1.00 \leq P/D \leq 1.15\) neglecting turbulent heat transport in the coolant, and by the present authors [3, 6] who investigated the effect of displacing a central rod from its symmetrical position assuming slug flow.

In predicting the heat transfer behavior of central rods of a rod bundle with equilateral triangular spacing one has to distinguish (1) the symmetric case where the geometry of the bundle is characterized by only one geometrical parameter, the pitch-to-diameter ratio, \(P/D\), and (2) the asymmetric case where a rod is displaced from its symmetrical position which is characterized by two additional parameters, e.g., by direction, \(\omega\), and relative magnitude, \(e/e_{\text{max}}\), of displacement. To describe the geometry of corner and lateral rods in both symmetric and asymmetric position, accordingly, an additional dimensionless geometrical parameter, the relative distance from the channel wall, \(W/R_2\), has to be introduced. Thus corner and lateral rods in their proper lattice position as shown in Figure 1 are characterized by \(P/D\) and \(W/R_2\).

In calculating velocity and eddy diffusivity fields as well as temperature distributions in rod bundles with in-line flow, the procedure generally applied is to divide the bundle into a number of unit cells more or less by estimation of which the smallest characteristic area then is analyzed (see Fig. 1, the crosshatched areas). The cell selected is considered to be isolated from the other cells, i.e., it is assumed that this cell is neither influenced by the neighboring cells nor itself influences them. In physical terms, this means that there is neither momentum nor heat transfer across the boundaries of the cells. This procedure of analysis may be designated as "single-rod analysis". In case of a regular and infinite array of rods each with axisymmetric and same mean power generation, this method is certainly correct. However, if the bundle is limited by a fixed wall, irregularities of rod position exist, and/or heat generation of rods varies crosswise in the bundle, the single-rod analysis may be considered only as an approximation that will give conservative heat transfer results for the rod of maximum irregularity of coolant area or heat generation.
compared to neighbouring rods.

Obviously, the validity of a single-rod analysis decreases as the inequality of neighbouring cells increases. Thus one would expect that its application has been restricted to symmetrically posed central rods with equal heat release. However, due to the mathematical difficulties that primarily arise in describing a characteristic bundle section, e.g., that of Figure 1, "multi-rod analyses" have only been presented hitherto for some very special cases. For a finite bundle containing \( m + 1 \) rods in a cylindrical channel, Chen and Fulford [7, 8] performed a two-region heat-flow analysis for both slug and laminar flow to determine the effects of displaced rods, unequal rod heat generation, or eccentric bundle location on fully developed heat transfer using an extension of the method of Axford [9] to evaluate the laminar velocity field. Buleev et al. [10] carried out a three-dimensional multi-rod heat transfer analysis for a hexagonal stack of 19 fuel rods in an infinite triangular array to determine the effect of unequal heat release on certain design temperatures applying an approach which is based on the superposition principle as previously formulated by Sutherland and Kays [11]. For finite hexagonal rod bundles as considered for application in fast breeder reactors, multi-rod analyses have not been published in the available literature hitherto, and considerable analytical investigations seem to be necessary to find methods of solution which are fully satisfactory with respect to consumption of digital computer time. Thus single-rod multi-region analyses at present prevail for estimating fuel element thermal performance and furnishing the input information for thermal stress calculations.

This paper presents the results of a single-rod three-region (fuel, clad, coolant) analysis of steady-state fully developed heat transfer to slug flow around symmetrically posed corner and lateral rods of hexagonal rod bundles with equilateral triangular spacing. Account is taken for thermal contact resistance at fuel-clad interface and clad surface. The effect of rod displacers inserted into the peripheral flow channel is investigated. From the explicit temperature solutions obtained in terms of Fourier series, the effect of variations in geometrical parameters like pitch-to-diameter ratio, relative rod distance from the channel wall, and relative clad thickness as well as in thermal parameters like relative conductivities of fuel and clad on heat transfer and temperature distribution is studied.

2. PREVIOUS INVESTIGATIONS

Published results of theoretical and experimental investigations of liquid metal heat transfer in the peripheral region of closely packed finite rod bundles are very scarce. A common characteristic of the few investigations reported hitherto is that all have been performed in
course of the thermal design of a specific fast breeder reactor core and, for that reason, are very limited in view of the geometric and thermal parameter considered. This section will review these with respect to the present analysis.

The only experimental data for liquid metal heat transfer in the peripheral region of finite hexagonal rod bundles are those by the workers of the Energy-Physics Institute (FEI) at Obninsk, USSR, who investigated heat transfer models of the fuel elements of the BN-350 and BOR-60 fast reactors. The mockups of the BN-350 core consisted of 37 tubes (which could be heated) placed in hexagonal shells at the vertices of regular triangles with pitch-to-diameter ratios equal to 1.24, 1.15, and 1.04 whereof only the models with P/D equal to 1.24 and 1.15 had unflinned tubes [12, 13, 14]. The relative wall distance, 2 W/D in the assemblies of unflinned tubes was equal to pitch-to-diameter ratio. Results related to the present study are those obtained with liquid NaK as the coolant and for P/D = 1.15. Measurements of temperature distributions of central, lateral, and corner rods have been performed in the range of Peclet numbers of 6 to 4000 and at several distances of heated length of which only those, obtained for a length of l/d_e ≥ 100 where stabilization of maximum temperature variation begins [13], may be used for comparison with the present investigation. Unfortunately, results of fully developed heat transfer are only reported for the lateral rod in terms of maximum variation in rod surface temperature as function of Pe number in graphical form for 6 ≤ Pe ≤ 4000 [12, 14] and by an empirical formula for 100 ≤ Pe ≤ 700 [13]. It further may be noted that Subbotin et al. [13] also present a single peripheral wall temperature profile of an unflinned lateral rod for Pe = 350 and l/d_e = 60 which has been calculated on an analogue computer using the velocity profile and the distribution of eddy viscosity in the fluid region computed by the method of Buleev and Ibragimov et al. [15], however, they do not furnish further details with regard to the method of solution and calculation procedure.

More recently, Subbotin et al. [16, 17, 18] published some results of the experimental investigations on thermal models of the BOR-60 reactor that essentially differ from the BN-350 experiments by closer rod spacing (P/D = 1.1), relative clearance between the peripheral rods and the shell, (2 W - D) / (P - D) = 2, which is twice that in BN-350, and lower effective angular thermal conductivity, k_2/k_3 = 0.22, compared to 0.69 in the BN-350 models. The coolant was liquid sodium. Models were used with smooth and flinned rods with and without displacers in the peripheral cells. It was found that, with Pe > 100, temperature nonuniformity of side rods increases practically linearly because no stabilization occurred along the heated length of the assembly (l/d_e = 173 for the smooth rods). To determine the temperature nonuniformities to the lateral fuel rods of the BOR-60 reactor cassettes, the experimental data had to be extrapolated to the working length of l/d_e = 202. Again, results are given in terms of maximum surface temperature variations for lateral and corner rods as function of Pe number in graphical form for 20 ≤ Pe ≤ 600.
and of lateral rods also by empirical formula for $100 \leq \text{Pe} \leq 600$ [16, 17, 18].

Numerical calculations of local heat transfer in peripheral rods have been performed on EBR-II subassemblies [19, 20, 21] and on the SEFOR fuel channel [22]. A nodalized three-dimensional model of an edge fuel rod in fuel bundles of $1, 2$ P/D was set up in [19] to study the effect of the rod-to-channel wall spacing and the wire spacer, on the circumferential clad temperature distribution. Incorporated in the model, which was set up on the THTD heat transfer code, was the assumption of slug flow neglecting any mixing [19]. The effects of coolant mixing from flow channel to adjacent flow channel were included in a subsequent analysis [20], but the calculational method used did not predict circumferential temperature variations. Calculations of the central and peripheral coolant subassembly with different spacer designs are reported in [21]. A series of heat transfer calculations on the SEFOR fuel channel using an improved version of the TIGER II heat transfer code is reported by Meler et al. [22]. In these calculations constant heat flux at inner clad surface was assumed and clad and coolant regions were coupled by an "effective" circumferentially varying clad-to-coolant heat transfer coefficient which was calculated assuming molecular conduction only, with no contribution from eddy heat transport. Thus, this analysis may be considered as a single-region analysis of the clad. A comparison of the very few results given in all of these reports with the present ones may only be accomplished qualitatively, since, unfortunately, in not a single case the complete set of both geometrical and thermal input data used in the calculations is specified.

3. THEORETICAL ANALYSIS

3.1 Geometry and Assumptions

For reasons of symmetry, the single-rod analysis in the peripheral region of an equilateral triangular rod array within a hexagonal channel may be confined to the domains in the interval $0 \leq \phi \leq \pi$ as illustrated in Fig. 1 by the cross-hatched areas. The regions considered are that of the fuel for $0 \leq r \leq R_1$, that of the clad for $R_1 \leq r \leq R_2$, and that of the coolant for $R_2 \leq r \leq a_j / \cos \phi$, $j = 1, 2, 3$ (Fig. 2).

A uniform spatial distribution of heat generation in the fuel is specified. To reduce the analysis to two dimensions, the temperature profiles were assumed to be fully developed; that is, the temperature gradient in the flow direction is independent of radial and circumferential position, and is determined by an energy balance on a control volume consisting of the unit of symmetry and a unit distance in the flow direction.

Slug-flow conditions of the fluid are considered. This idealization is unquestionably the assumption most subject to debate when the relevance of present heat transfer results to practical reactor situations where conditions of turbulent coolant flow prevail should be judged. The two simplifications implied by assuming slug flow, uniform velocity every-
where in the coolant area and negligible turbulent diffusivity, tend to have counter balanc-
ing effects on heat transfer, and the results of slug-flow analyses have thus often proved
to be rather reasonable approximations for liquid-metal heat transfer in the "practical"
Peclet range, though there is considerable doubt, particularly in case of asymmetric flow
channels where large circumferential velocity and temperature gradients exist, to relate
results more specifically to certain Re and Pr numbers. This would require either reliable
experimental results over a large range of relevant parameters or a detailed knowledge of
two-dimensional velocity distributions as well as eddy diffusivity fields for heat in both
radial and tangential direction (ε_Hr , ε_Hφ ) for the geometrical configuration to enable
an appropriate theoretical analysis. To the present authors's knowledge, this information
is not available in sufficient completeness, either experimentally or theoretically. There-
fore, the present slug-flow solutions should be taken as semi-quantitative indicators of the
probable effects of the different situations examined with the single-rod model.

Further assumptions underlying the analysis are: Constant physical properties of the ma-
terials, negligible viscous dissipation, negligible axial conduction, negligible heat gen-
eration in clad and coolant, and circumferentially uniform contact resistance between fuel
and clad and between clad and coolant.

3.2 Differential Equations and Boundary Conditions.

In view of the above mentioned assumptions, the governing differential energy equations for
heat transfer in the fuel, clad and elemental coolant flow area shown in Fig. 2 can be
written as:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial t_1}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 t_1}{\partial \phi^2} = - \frac{q''}{k_1} \]  \hspace{1cm} (1)

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial t_2}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 t_2}{\partial \phi^2} = 0 \]  \hspace{1cm} (2)

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial t_3}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 t_3}{\partial \phi^2} = \frac{C''}{k_3} \left( -\frac{R_1}{R_2} \right)^2 \]  \hspace{1cm} (3)

where the subscripts 1, 2, and 3 pertain to the fuel, clad, and coolant region, respectively.
I represents the cross sectional area of flow (cross-hatched area shown in Fig. 2) divided
by the cross sectional area of the fuel rod, and is given by

\[ I = \frac{1}{\pi} \int_0^\pi \left( \frac{a_1}{R_2} \right)^2 \cos^2(\phi - \phi_0) d\phi - 1. \]  \hspace{1cm} (4)

Symmetry requirements applied to the boundaries \( \phi = 0 \) and \( \phi = \pi \) of the domain shown in
Fig. 2 result in the following boundary conditions for each of the three regions:

\[ \frac{\partial t_1}{\partial \phi} = 0 \quad \text{for} \quad \phi = 0 \quad \text{and} \quad \phi = \pi. \]  \hspace{1cm} (5)
The fuel temperature nowhere assumes an infinite value, hence:
\[ t_1 = \text{finite} \quad \text{at} \quad r = 0. \]  
(6)

The joining conditions at the interfaces between the adjacent regions may be expressed by:
\[ -k_i \gamma_i \frac{\partial t_i}{\partial r} = t_i - t_2 \]  
\[ k_i \frac{\partial t_i}{\partial r} = k_2 \frac{\partial t_2}{\partial r} \]  
\[ \text{at} \quad r = R_1 \]  
(7)

\[ -k_2 \frac{\partial t_2}{\partial r} = t_2 - t_3 \]  
\[ k_2 \frac{\partial t_2}{\partial r} = k_3 \frac{\partial t_3}{\partial r} \]  
\[ \text{at} \quad r = R_2 \]  
(9)

where \( \gamma_i \) in the boundary conditions of the third kind (7) and (9) denote circumferentially uniform contact resistances existing between the different regions.

At the outer contour 1234 of the characteristic flow area (Fig. 2) the coolant temperature field must satisfy the Neumann boundary condition:
\[ \mathbf{\bar{n}} \cdot \nabla t_j = \cos(\varphi - \varphi_j) \frac{\partial t_j}{\partial r} \sin(\varphi - \varphi_j) \left( \frac{\cos(\varphi - \varphi_j)}{a_j} \right) \frac{\partial t_j}{\partial \varphi} = 0 \]  
(11)

at \( r = a_j / \cos(\varphi - \varphi_j) \), and for \( 0 \leq \varphi \leq \pi \) \( (j = 1, 2, 3) \).

Applying the conditions of symmetry and finiteness (5) and (6) to the general series solutions of eqs. (1) - (3) results in the well known cosine Fourier series representations for the temperature fields in the fuel, clad, and coolant region (cf. e.g. [23]). Using the boundary conditions (7) - (10) the yet undetermined integration constants in these series solutions can be expressed in terms of one type of coefficients that are determined by applying eq. (11) to the complete solution for the coolant temperature field and satisfying the resulting equation only at a finite number of preselected points along the outer boundary of coolant region \( 0 \leq \varphi \leq \pi \) (point-matching). For \( N \) points this results in a set of \( N \) linear algebraic equations,
\[ \sum_{m=1}^{N} M_{mn} Y_n = P_m \quad m = 1, 2, \ldots, N \]  
(12)

which can be used to evaluate the coefficients \( Y_1, Y_2, \ldots, Y_N \) if the series in the equations are truncated after \( N \) terms. This method of collocation has been formerly used by the authors to obtain approximate solutions of boundary value problems involving second-order elliptic equations when the boundary did not correspond to the coordinate surfaces of any orthogonal coordinate system [24, 25, 6].

For the present problem it is found that the number of points has to be increased as the P/D ratio decreases to attain the same order of accuracy. However, as this is done, the
equations became more and more singular, and the error in the coefficients is increased resulting in an oscillatory behavior. This could be controlled to a limited extent by varying the number of points as well as their distribution along the irregular and distinct boundary lines. Thus this point-matching technique results in a compromise choosing sufficient, properly placed matching points to keep the truncation error of the series small enough and yet not have so many that the error in the coefficients, which arises from fixed length solution on the digital computer, becomes too great. From this point of view, point-matching seems to be ineffective. To overcome this difficulty of point spacing we met the boundary condition over a larger set of points than coefficients were wanted. This led to the following system of overdetermined linear equations:

$$\sum_{n=1}^{N} M_{mn} \psi_n = p_m \quad m=1,2,\ldots,M, \quad M > N \quad (13)$$

where

$$M_{mn} = f_n \left( \frac{R_2}{a_j} \cos^n(\phi - \phi_j) \cos((n+1) \psi - \psi_j) \right)$$

$$- g_n \left( \frac{R_2}{a_j} \cos^{n-1}(\phi - \phi_j) \cos((n-1) \psi - \psi_j) \right), \quad j=1,2,3 \quad (14)$$

$$f_n = \frac{1}{4} \left[ (1+\kappa_1 + \frac{n}{B_{i_2}})(1+\kappa_2 + \frac{n}{B_{i_2}}) \right.$$ \n
$$\left. + (1-\kappa_1 - \frac{n}{B_{i_2}})(1-\kappa_2 + \frac{n}{B_{i_2}})(\frac{R_L}{R_2})^{2n} \right] \quad (15)$$

$$g_n = \frac{1}{4} \left[ (1+\kappa_1 + \frac{n}{B_{i_2}})(1+\kappa_2 + \frac{n}{B_{i_2}}) \right.$$ \n
$$\left. + (1-\kappa_1 - \frac{n}{B_{i_2}})(1-\kappa_2 + \frac{n}{B_{i_2}})(\frac{R_L}{R_2})^{2n} \right] \quad (16)$$

$$p_n = (\frac{a_j}{R_2})^2 \frac{1}{1 - \left(1 + \frac{1}{I_2} \right) \cos(\phi_m - \phi_j)} \quad (17)$$

$$\psi_n = \frac{n R_2}{B_n} \frac{R_n}{q^m R_1^2/2 k_3} \quad (18)$$

The unknown coefficients \( \psi_n \) are determined in the least square sense using the so-called "Gauss transformation" [26]. Then as \( M \to \infty \) this process becomes that of minimizing the integral of the squared error (cf. [27]). The solution is obtained from the matrix equation:
where $A^T$ denotes the transpose of $A$. $A$ is the $M \times N$ matrix, $x$ is the $N$-fold solution vector and $B$ is a $M$-fold right hand side vector containing all nonhomogeneous terms. Several investigators [28, 29, 30] concluded from numerical examinations that it is sufficient to take $M \approx 2N$, which corresponds to a doubly-redundant system, however, this experience cannot be applied without further computational evidence to the present case of crude boundary [27]. Solving eq. (19) with standard matrix routines and substituting the coefficients in the solutions of the eq. (1) - (3) one finds the temperature fields for the fuel region:

$$\theta_1(\rho, \psi) = \frac{1}{2x_1x_2} \left[ 1 - \left( \frac{\rho}{R_1/R_2} \right)^2 \right] + \sum_{n=1}^{N} \frac{V_n}{n} \rho^n \cos(n\psi)$$

(20)

for the clad region:

$$\theta_2(\rho, \psi) = -\frac{1}{x_2} \ln\left( \frac{\rho}{R_1/R_2} \right) - \frac{1}{x_1x_2} \frac{1}{Bi_1}$$

$$+ \sum_{n=1}^{N} \frac{V_n}{2n} \left[ \rho^n (1 + x_1 + \frac{n}{Bi_1}) + (\frac{R_1}{R_2})^{2n} \rho^{2n} (1 - x_1 + \frac{n}{Bi_1}) \right] \cos(n\psi)$$

(21)

for the coolant region:

$$\theta_3(\rho, \psi) = \frac{1}{2l} (\rho^2 - 1) + \frac{1}{x_2} \ln\left( \frac{R_1}{R_2} \right) - \left( 1 + \frac{1}{1} \right) \ln(\rho) - \frac{1}{x_1} \left( \frac{1}{Bi_1} + \frac{1}{Bi_2} \right)$$

$$+ \sum_{n=1}^{N} \frac{V_n}{4n} \left[ \rho^n (1 + x_2 + \frac{n}{Bi_2})(1 + x_1 + \frac{n}{Bi_1}) + (\frac{R_1}{R_2})^{2n} (1 - x_2 - \frac{n}{Bi_2})(1 - x_1 - \frac{n}{Bi_1}) \right]$$

$$+ \rho^{2n} \left[ (1 - x_2 + \frac{n}{Bi_2})(1 + x_1 + \frac{n}{Bi_1}) + (\frac{R_1}{R_2})^{2n} (1 - x_2 - \frac{n}{Bi_2})(1 - x_1 + \frac{n}{Bi_1}) \right] \cos(n\psi)$$

(22)

The average Nusselt number in terms of the dimensionless fluid temperature $\theta_3$ is given by:

$$\overline{Nu} = \frac{\overline{Hd}}{k_3} = \frac{2l}{x_2} \left( -\frac{1}{x_2} \ln\left( \frac{R_1}{R_2} \right) - \theta_3 \right)$$

(23)
where \( d \) is the equivalent hydraulic diameter of coolant area associated to a single rod and the average heat transfer coefficient is defined as \( \bar{h}(R_2) / (\bar{m}(R_2) - \bar{m}) \).

4. DISCUSSION OF NUMERICAL RESULTS.

The analysis was programmed and used to predict the effect of variations in geometrical parameters \( P/D, W/R_2, R_1/R_2 \) as well as in thermal parameters \( k_1/k_2, k_2/k_3, \beta_1, \beta_2 \) on temperatures and Nusselt numbers. In view of the large number of parameters that must be specified to obtain a solution only results are presented for a limited number of illustrative examples. Geometrical and thermal parameters were chosen such that they coincide approximately with present parameter values under consideration for LMFBR subassemblies, e.g., the value selected for the relative thermal conductivity of clad, \( k_2/k_3 = 0.3 \), is almost characteristic for a sodium-cooled fuel rod with stainless steel clad whereas the values chosen for relative thermal conductivity of fuel, \( k_1/k_2 = 0.1 \) and \( 1.0 \), represent oxide and carbide fueled rods, respectively, with stainless steel clad.

In special view of the accuracy of temperature solutions, the numerical method applied for solving eq. (19) seems to be of considerable importance and will be discussed by the first author in [27]. The results presented herein were obtained solving eq. (19) by means of Gauss-elimination with pivoting in main diagonal to preserve symmetry in remaining coefficient matrices. This method seems to be a rather good compromise between accuracy and computing time.

An illustrative example of the temperature fields in the three regions of both lateral and corner cells with fuels of different thermal conductivities is presented in Fig. 3. It may be noted that the "liquid metal" is at its coolest in the region near the cassette and considerable circumferential temperature variations exist that are more pronounced for the "oxide fueled" rod. The maximum temperatures occur near the "infinite" lattice zone. The temperature variations strongly depend on the relative rod distance from the channel wall as shown in Fig. 4 for the temperature around the outer clad surface of lateral and corner rods. From this figure, it may be concluded that for each pitch-to-diameter ratio a specific relative wall distance exists where the circumferential temperature variation is minimum. Plotting the maximum rod surface temperature above bulk coolant temperature, \( \delta_2(R_2)_{\text{max}} \), normalized by the same difference of a central rod, \( \delta_2(R_2)_{\text{max}} \), for constant \( P/D \) versus relative wall distance results in v-shaped curves with distinct minima that specify the optimum \( W/R_2 \) ratio with respect to design temperatures (Fig. 5). The minimum temperature ratio occurs when the circumferential temperature distribution has two absolute maxima of equal level as may be seen from Fig. 4. The values of minimum temperature ratio decrease with increasing \( P/D \) ratio and shift to higher relative wall distances; the temperature ratio is the more sensitive to changes in relative wall distance the lower the \( P/D \) ratio and the relative thermal conductivity of fuel. Since for a given \( P/D \)
ratio the minima occur at different relative wall distance for lateral and corner rods, a compromise will have to be made in choosing the design wall distance of the bundle. For estimating absolute values of maximum clad surface temperatures of the peripheral rods, Fig. 6 may be used where \( \left[ \delta_2(R_2) \right]^{\text{max}} \) of the central rod (infinite array) is plotted vs. P/D ratio. Apparently, there also exist an optimum P/D ratio of about 1.18 to 1.19 in view of maximum clad surface temperature which varies only slightly with relative thermal conductivity of fuel.

The preceding results have been computed assuming negligible thermal contact resistances at fuel-clad interface and clad surface. The effect of contact resistances, characterized by the modified Blot numbers \( \text{Bl}_1 \) and \( \text{Bl}_2 \), on maximum (inner) clad temperature and circumferential temperature difference of a lateral rod is illustrated in Figs. 8 and 9. As the thermal contact resistance between fuel and clad increases (\( \text{Bl}_1 \rightarrow 0 \)) the greater is the maximum temperature as well as the circumferential temperature variation around inner clad surface because of the reduction in circumferential heat flow in the fuel (Fig. 8). Fig. 9 deals with the effect of a thermal contact resistance at the outer clad surface taking the same geometrical and thermal parameters as of Fig. 8. In line with the expectations it is again observed that the maximum temperature at inner clad surface above coolant average temperature increases as contact resistance becomes greater. On the other hand, the circumferential temperature variation decreases as \( \text{Bl}_2 \rightarrow 0 \) since the rod regions become more and more decoupled from coolant region.

Besides of temperature distributions, information on heat transfer coefficient and Nusselt number is of prime interest, because the latter are commonly used to predict wall temperatures and estimate the effectiveness of specific heat transfer configurations. For a lateral rod of P/D = 1.2, Fig. 7 shows the circumferential variation of the normalized local heat transfer coefficient with relative wall distance as a parameter. In the interval \( 0 \leq \varphi < 70 \text{ deg} \), \( \frac{h_L}{h} \) is negative because in that region the outer wall temperatures fall below the coolant bulk temperatures. It is apparent that the circumferential location where \( \delta_2(R_2, \varphi) = 0 \) becomes infinite ( \( \varphi = 70 \text{ deg} \)). For the lowest relative wall distance shown, \( W/R_2 = 1.1 \) two of those poles exist demonstrating that \( \delta_2(R_2, \varphi) \) changes sign twice in the interval \( 0 \leq \varphi \leq \pi \). Because of this behavior, local heat transfer coefficients are obviously of very limited significance and the common convention defining the mean heat transfer coefficient as its circumferential average becomes inappropriate. Thus, in the present study, the average Nusselt number is defined by eq. (23), where \( \bar{h} \) is, in turn, defined as \( \frac{q^{\text{in}}(R_2)}{\left| \bar{t}_2(R_2) - \bar{t}_1 \right|} \). Fig. 10 gives, for reference, values of \( \text{Nu}_1 \) and \( \text{Nu}_2 \) for lateral and corner fuel rod cells, normalized by the corresponding \( \text{Nu} \) values of the central rod, for the two limiting thermal boundary conditions:

1. Uniform wall-heat flux in axial direction and uniform wall temperature in circumferential direction.
2. Uniform wall-heat flux in all directions (single-region analysis).

First, it may be noticed that average Nusselt numbers of peripheral rod cells are always lower than \( \text{Nu}_1 \) of the geometrically more uniform central rod cell as expected. For a given P/D
ratio, it is seen that the curves for boundary condition (2) fall below that for boundary condition (1). The curves show a maximum that in case of uniform wall-heat flux occurs at lower $W/R_2$ ratio than in case of uniform wall temperature and at lower values for the lateral than for the corner rod cell. The maxima approach unity as the $P/D$ ratio increases. From results of multiregion analysis for the normalized average Nusselt number, $\frac{\bar{Nu}_{C1L}}{\bar{Nu}_1}$, as shown in Fig. 11, it can be observed that for a given $P/D$ ratio and constant thermal parameters the dependence of the $Nu$ ratio on relative wall distance is essentially the same for both peripheral rods except that the maxima in case of the lateral rod occur at slightly lower $W/R_2$ ratios. $\frac{\bar{Nu}_{C1L}}{\bar{Nu}_1}$ values pertaining to a carbide fuel, which is characterized by a high thermal conductivity, always exceed those related to oxide fueled rods.

Comparing the results of single and multiregion analysis (Fig. 12), it seems worth noting that the results of the single-region analysis using boundary conditions of first and second kind, respectively establish the upper and lower bounds of the multiregion analysis. The effect of variations in relative clad thickness, $R_1/R_2$, in the range of 0.8 to 0.95 is of minor importance.

To improve the heat transfer behavior of the peripheral cells, the insertion of displacers in the wall region of coolant area has been studied recently both experimentally [16,18] and theoretically [21]. The effect of a round displacer of different relative radius on the normalized peripheral temperature on outer clad surface is demonstrated in Fig. 13. For both lateral and corner cells it is observed that the temperature is reduced with an increase in relative displacer radius, $R_d/R_{d_{max}}$. However, the maximum circumferential temperature variation remains almost unaffected. Apparently, the displacer practically influences only the temperature drop between outer clad surface and coolant which is equivalent to an increase of the average dimensionless heat transfer coefficient, $\frac{\bar{Nu}_d'}{\bar{Nu}_{d'}}$ as illustrated by Fig. 14 where $\bar{Nu}_{d'}$ normalized by the Nusselt number for a cell without displacer, $\bar{Nu}_d$, is plotted versus dimensionless displacer radius. It is interesting to note that for $P/D \leq 1.25$ a displacer of given radius has a greater effect on the average slug-flow Nusselt number of the carbide fueled rod whereas the opposite holds for $P/D \geq 1.25$.

A comparison of the analytical results presented herein to the very few theoretical ones of other authors [13, 19-22] could only be performed qualitatively for the reasons stated in Section 2 and showed agreement on principal.

A comparison to the experimental results given in [13, 14, 18] is possible to the most extent with respect to the maximum circumferential temperature variation on the outer clad surface of a lateral rod. As mentioned in Section 2, these results were correlated in dependence on the Peclet number by empirical equations. For an assembly of $P/D = W/R_2 = 1.15$ without displacers the maximum temperature nonuniformity is described by

$$\frac{t_{2_{max}}(R_2) - t_{2_{min}}(R_2)}{q'' R_2^2 / 2k} = 0.21 + 2.75 \exp(-0.0088 \text{Pe})$$  (24)
for 100 \leq \text{Pe} \leq 700 [13] , and for an assembly of $P/D = 1.1$ and $W/R_2 = 1.2$ without and with round displacers, respectively, by

$$\frac{t^{\text{max}}_2 - t^{\text{min}}_2 (R_2)}{q'' R_1^2/2 k_2} \approx 0.4 + 5.9 \exp(-0.00902 \text{Pe}), \quad (25)$$

$$\frac{t^{\text{max}}_2 - t^{\text{min}}_2 (R_2)}{q'' R_1^2/2 k_2} \approx 0.2 + 3.08 \exp(-0.00730 \text{Pe}) \quad (26)$$

for 100 \leq \text{Pe} \leq 600 [18]. Evaluating these equations for various Peclet numbers, the curves 1 to 3 in Fig. 15 are obtained. In addition, experimental values of the assembly with $P/D = W/R_2 = 1.15$ of Zhukov et al. [14] are shown.

Evaluating the present analytical solution for the geometrical and thermal parameters associated to the models used in the experiments, numerical values for the temperature variation are obtained that match the experimental curves at distinct Peclet numbers. It may be noted that in the experiments the boundary condition of constant heat flux at inner clad surface was simulated. The slug flow results for this boundary condition correspond to Peclet numbers in the range of 40 to 100. Obviously, slug flow results are conservative for higher Peclet numbers. When the fuel region is taken into account, the temperature variation is reduced and the Peclet numbers at which the analytical results match the empirical curves are shifted to higher values what is in line with the expectations. The most interesting finding of this comparison is that the adequacy of the slug flow assumptions for calculations of clad temperature variations in case of turbulent liquid metal cooling is appreciably diminished as the coolant area becomes asymmetric.

ACKNOWLEDGEMENTS

The authors would like to express their appreciation to Mrs. Marianne Hein for typing the manuscript and to K.-H. Rüster for his invaluable help in preparing the graphs.

Gratitude is also due to the German Research Foundation for its financial support of portions of this work.
NOMENCLATURE

\( A_o \) = integration constant, or average fuel surface temperature
\( B_n \) = integration constant
\( Bl_1 = \frac{R_1}{\beta_1 k_1} \) modified Blot number
\( D \) = outer diameter of rods
\( I \) = relative cross sectional area of coolant flow, def. by eq. (4)
\( Nu = \frac{h d}{k_2} \) average Nusselt number
\( P \) = pitch, or distance between rod centers
\( Pe \) = Peclet number
\( R_1 \) = inner radius of clad
\( R_2 \) = \( D/2 \), outer radius of fuel rod
\( W \) = distance of rod center from channel wall
\( Y_{\eta n} \) = coefficients, def. by eq. (18)
\( z_j \) = distances, def. in Fig. 2
\( d \) = equivalent hydraulic diameter of coolant area associated to a single rod for in-line flow
\( d_{\eta e} \) = equivalent hydraulic diameter of rod bundle for in-line flow
\( e \) = displacement of a rod from its symmetrical position
\( h = \frac{q''(R_2, \varphi)}{\delta_2(R_2, \varphi)} \), local heat transfer coefficient
\( \bar{h} = \frac{q''(R_2)}{\delta_2(R_2)} \), average heat transfer coefficient
\( k \) = thermal conductivity
\( l \) = heated length of rod bundle
\( m \) = number of tubes in outer ring
\( q'' \) = heat flux
\( q^{in} \) = heat source density in fuel
\( r \) = radial distance from rod center
\( t \) = temperature
\( \Theta = \frac{(t - A_o)}{(q^{in} R_1^2 / 2 k_3)} \), dimensionless temperature
\( \beta_1 \) = thermal contact resistance at interface between region 1 and region \((1 + 1)\)
\( \varepsilon_{H} \) = eddy diffusivity for heat
\( \delta = t - \bar{t}, \) temperature above bulk coolant temperature
\( \kappa_1 = \frac{k_1}{k_1 + 1} \), relative thermal conductivity of region 1
\( \phi \) = angle
\( \psi_j = \) angles shown in Fig. 2
\( \omega \) = angle of rod displacement from symmetrical position
\( \rho = \frac{(r/R_2)} \), dimensionless radial distance from rod center

Subscripts
\( C \) = corner fuel rod
\[ \begin{align*}
I &= \text{fuel rod in infinite array} \\
L &= \text{lateral fuel rod} \\
d &= \text{displacer} \\
max &= \text{maximum} \\
min &= \text{minimum} \\
q &= \text{uniform wall heat flux in all directions} \\
r &= \text{radial direction} \\
t &= \text{uniform wall heat flux in axial direction and uniform wall temperature in circumferential direction} \\
\varphi &= \text{tangential direction} \\
1 &= \text{fuel} \\
2 &= \text{clad} \\
3 &= \text{coolant} \\
- &= \text{average}
\end{align*} \]

REFERENCES


[27] WOLF, L., "Application of boundary points least squares method to heat transfer analysis of finite rod bundles", to be published.


Fig. 1. Characteristic section of equilateral triangular rod array within hexagonal channel.

Fig. 2. General symmetric cross section of symmetrically posed, peripheral fuel rods and associated coolant channel.
Fig. 3. Temperature fields of fully developed heat transfer to slug flow along a symmetrically posed lateral (a) and corner rod (b) of an unbaffled hexagonal rod bundle with equilateral triangular spacing. (P/D = 1.2, W/R₂ = 1.4, R₁/R₂ = 0.9, k₁/k₂ = 0.3).
Fig. 4. Typical curves showing the effects of variations in wall distance and thermal conductivity of fuel on peripheral variation of the temperature on the outer wall of clad of lateral (a) and corner rod (b).

Fig. 5. Effects of variations in relative wall distance, rod spacing, and thermal conductivity on maximum clad surface temperature of lateral (a) and corner rod (b).
Fig. 6. Effects of variations in relative rod spacing and thermal conductivity of fuel on maximum temperature on the outer clad wall of a rod within infinite triangular array.

Fig. 7. Typical curves showing the effect of variations in relative wall distance on normalized circumferential heat transfer coefficient of lateral rod.
Fig. 8. Effect of thermal contact resistance at fuel-clad interface on maximum temperature and maximum temperature variation on inner clad surface of lateral rod.

Fig. 9. Effect of thermal contact resistance at outer clad surface on maximum temperature and maximum temperature variation on inner clad surface of lateral rod.
Fig. 10. Effects of variations in relative rod spacing and wall distance on the average, normalized, slug-flow Nusselt number of lateral (a) and corner rod (b) for the two limiting cases of boundary condition. 

- [$\frac{Nu_L}{Nu_1}$]$_t$; $\frac{Nu_L}{Nu_1}$]$_i$; $\frac{Nu_L}{Nu_1}$]$_q$; $\frac{Nu_L}{Nu_1}$]$_q$;
Fig. 11. Typical curves showing the effects of relative wall distance, rod spacing, and thermal conductivity of fuel on average, normalized, slug-flow Nusselt number of lateral (a) and corner rod (b).
Fig. 12. Typical curves showing the effects of variations in relative wall distance, thermal conductivity of the fuel, and relative clad thickness on the average, normalized, slug-flow Nusselt number of lateral rod.

Fig. 13. Typical curves showing the effects of variations in relative radius of a displacer inserted in lateral (a) and corner (b) rod cells on the peripheral variation of the temperature on the outer wall of clad.
Fig. 14. Effects of variations in relative radius of a round displacer inserted in a lateral rod cell, pitch-to-diameter ratio, relative wall distance, and relative thermal conductivity of fuel on normalized slug-flow Nusselt number \( \frac{N_{\text{d}}}{N_{\text{L}}} \). (\( R_1/R_2 = 0.9, k_2/k_3 = 0.3 \)).

Fig. 15. Experimental results of maximum circumferential temperature variation on the outer clad surface of lateral rod for liquid metal in-line flow in dependence on Peclet number and theoretical predictions of present study.
DISCUSSION

Q

C. F. BONILLA, U. S. A.

Could you describe differences in treatment briefly, and in results, if available, between this work and others that have been published on the same topic (as I recall, there have been by Dwyer, by Friedland, by Nijsing, and possibly by others)?

A

K. JOHANNSEN, Germany

To the best of our knowledge, published results of theoretical and experimental investigations of liquid metal heat transfer in the peripheral region, have been summarized in Section 2 of this paper. The present contribution and those cited by Prof. Bonilla differ in the geometry studied (Dwyer, Friedland and Nijsing treated rods within in infinite array) and in the number of regions taken into account (except Nijsing's work). The method of solution applied is the boundary points least squares method whereas Dwyer used finite differences and Nijsing the point-collocation method.

Q

G. MELESE-d'HOSPITAL, U. S. A.

In cases where filler pieces are used in the corners to shape the coolant cross section, the slides show very little effect on the circumferential rod temperature distributions. Were changes in flow distribution in subchannels and changes in momentum and energy exchanges between subchannels considered in your calculations?

A

K. JOHANNSEN, Germany

The relative small effect of inserting a filler piece in the peripheral coolant area on the circumferential temperature variation even for $R_d/R_d$ max ratios greater than 0.5 is due to the assumption of slug flow and the implications of the single-rod analysis. Therefore, the filler piece has no effect on the velocity (and eddy diffusivity) distributions and on momentum and heat transport across the cell boundaries as it does in practice.

Q

F. HOFMANN, Germany

Why do you think that the assumption of fully developed thermal profile is conservative?

A

K. JOHANNSEN, Germany

For channel flow with axially uniform heat flux from the wall, the wall-to-bulk temperature difference is greatest, when the temperature gradient in flow direction is independent of radial and circumferential direction, or, in other words, the temperature profile is fully developed. Theoretically, this is the case as the channel length approaches infinity.
Thus, for finite channels, wall-to-bulk temperature differences are always less, since the temperature profile is not fully developed, and, therefore the assumption of a fully developed profile will give conservative results with respect to the actual case. This fact was demonstrated first by Graetz in 1883 (Ann. Phys. (N. F.) 18(1883)79) and is confirmed by numerous experimental and theoretical investigations. The conclusion that it also holds for channels of noncircular cross section and asymmetric heating may, for example, be drawn from the experimental findings of Tan and Charters (Solar Energy, 12 (1969) 513 and 13 (1970) 121).