A MATHEMATICAL MODEL FOR ELEVATED TEMPERATURE DEFORMATION BEHAVIOR OF STRUCTURAL METALS

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ABSTRACT

The salient features of the isothermal deformation behavior of structural metals subjected to monotonic, static, and slowly varying loads are documented. These features include the total time-dependent picture (strain-rate sensitivity, creep, relaxation, and cyclic hardening or softening), the effect of prior deformation history and the response to hydrostatic loading. The materials considered are metallurgically stable in the sense that exposure to temperature alone will not change their mechanical properties.

The concept of a discrete and fading memory of the past history is introduced and formalized. The memory is for the stress tensor at the transition from loading to unloading provided that certain specified conditions are fulfilled. The proposed stress-strain law contains the stress as well as the small strain tensor and their respective time derivatives. The coefficients of the strain tensor are the usual elastic constants whereas coefficient functions multiply the time derivatives of the stress and the strain tensor. The coefficient functions contain as arguments the present stress and strain tensor as well as the time-dependent memory parameter representing the discrete fading memory. Isotropic and anisotropic relations are discussed.

For the isotropic case the equations are specialized for two two-step creep tests where the order of stress application is high stress followed by low stress and vice versa. The equations exhibit the necessary properties to model the influence of prior history shown by the experimental results. Finally, an approach is outlined how specific coefficient functions can be obtained.

Introduction

The trend in nuclear power generation is towards increase of operating temperatures. Breeder reactors are scheduled to generate power in the 1980's. The operating temperatures of these reactors can exceed those of the present fossil fired plants. There is a much greater concern for the safe and reliable operation of the nuclear plants than for the conventional units. It follows that very accurate reliability analysis procedures have to be developed and utilized in the design of these plants. At the present time this objective cannot be completely realized since great uncertainties exist about the deformation and fracture behavior of structural materials at elevated temperature. In this temperature regime, time and history dependent effects such as creep, relaxation, strain-rate sensitivity, hold-time as well as cyclic hardening (softening) and ratchetting are important. Their influence on mechanical failure cannot yet be determined accurately.

Reliability analysis consists of two main elements, the determination of stresses (strains) as a function of location and time, the stress analysis part, and a decision whether the calculated stresses (strains) will permit an operation without mechanical failure of the component, the failure analysis part. In this step, a failure criterion is necessary.

In the stress analysis part, two major problems are encountered in the analysis of structures, one is geometrical in nature (boundary conditions), the other is in the proper description of the material behavior, the stress-strain relation. Complex geometries can now be handled by finite-element computer codes [1, 2]. The proper characterization of the deformation behavior of structural materials for monotonic loading, static and cyclically varying loads, however, is not at an equal footing with the capability of handling complex geometries. Indeed, it has been stated that "progress in analytical methods has far outstripped our knowledge of the material to be analyzed" [3]. In a survey article on the finite-element method, Zienkiewicz [4] has noted "Perhaps the most justifiable work is now in the study and introduction of realistic material parameters and constitutive description, and as this work progresses, many novel applications will be made".

In this paper, the isothermal deformation behavior of structural materials as observed in laboratory tests is documented. The test results quoted are from experiments which establish failure criteria to be used in the reliability analysis of components operating at elevated temperature. Especially the recent studies on hold-time and frequency effects in elevated temperature low-cycle fatigue [5, 6] provided much of the background.

From these papers important and rather general characteristics of deformation behavior are extracted. They provide necessary conditions which a realistic constitutive equation has to fulfill. None of the conventional stress-strain laws are capable of reproducing all these characteristics and consequently work is started on a new stress-strain relation. It is believed to have the capability of reproducing the important characteristics of the deformation behavior. A detailed evaluation is in progress.

Characteristics of Isothermal Elevated Temperature Deformation Behavior

A phenomenological approach is adopted. Unless otherwise specified, the information is derived from cylindrical specimens loaded in the direction of their axis. The relations of interest are between the components of the stress and strain tensor in the direction of the cylinder axis. Loadings are such that the components of the displacement gradient are below ± 2% and therefore the small strain tensor will be used in describing the deformation. The motions considered are slow, acceleration terms are insignificant [7]. The material is considered a black box [8]. A forcing function (ramp, step, cyclic) is imposed either in terms of load or displacement (input) and the response in terms of displacement or load is observed (output). The relation of the input-output pairs as a function of time contain information about the black box, the material, which we want to characterize. In this study only metallurgically stable materials are considered. These materials do not change their mechanical properties if exposed to constant temperature alone.

Total Time-Dependent Picture

For annealed materials without any prior mechanical history, the following general deformation characteristics are observed. Under a ramp input (monotonic loading) of either displacement or force the usual stress-strain diagram results. As the rate of loading increases the deviation from linearity occurs at higher and higher stresses, the material shows positive strain-rate sensitivity.

If the force is kept constant at a certain level, the deformation may continue to increase with time, the material creeps. A general characteristic of the creep curve is the never negative creep rate $\left(\frac{d\varepsilon}{d\tau} \geq 0\right.$, ε designates the strain component of interest, τ is the time coordinate$\left.\right)$.

A constant displacement of a certain magnitude can cause a time-dependent change of the load (stress), the material relaxes. The relaxation curve is characterized by a never positive stress rate $\left(\frac{d\sigma}{d\tau} \leq 0\right.$, σ is the stress component of interest$\left.\right)$.

For a slowly varying cyclic forcing function the annealed material usually cyclic strain hardens. A periodic forcing function causes a non-periodic response until a so-called steady or shakedown condition is reached. It is possible that shakedown will not occur. In the steady state the response function is periodic. The transient period from the start of the test untill the steady state condition is reached is characterized by a varying amplitude response function. The mean value of the response function follows the pattern of the creep or the relaxation curve. Details of these variations are described in [7, 9].

For sufficiently small stresses the response of the material will be time-independent and linear. The above-described phenomena are very pronounced and of significant engineering importance at high stresses [7].

Effect of Prior Deformation

The deformation behavior of a structural material at a given instant in time is influenced by its prior deformation history. An influence of prior deformation history can only be expected if the loading was such that the linear, elastic range was exceeded during that history. Only loading histories beyond that range can have an influence on the subsequent behavior.

Experiments quoted in [7] indicate that the influence of prior deformation history is such that the basic, general characteristics mentioned before are not changed in the case of monotonic and static loading. For example, a creep curve has still the characteristic $\frac{d\varepsilon}{d\tau} \geq 0$, although the actual variation of ε vs. time may be quite different for the virgin and the prestrained material. Also, positive strain-rate sensitivity can still be expected even if the departure from linearity of the stress-strain curve may occur at a different stress level for the deformed than for the virgin material.

In the cyclic case prior deformation in the form of cold work (monotonic pull and subsequent unloading to zero) causes cyclic softening. In the displacement controlled case, the stress amplitude is decreasing from cycle to cycle, whereas an increase is observed for annealed materials [9]. All other general characteristics remain unchanged.

Prior deformation beyond the linear range apparently causes a change in the subsequent stress-strain behavior in degree but not in kind. Significant, however, is the fact that these observed changes in the stress-strain behavior are observed after unloading and subsequent reloading. The change from loading to unloading must have a decisive influence on the subsequent deformation behavior [9].

Permanent Deformation, Aftereffect, Anelasticity

A specimen is subjected to an arbitrary homogeneous deformation history and then the load is reduced to zero. Since the deformation was homogeneous, no residual stresses in the continuum mechanics sense are remaining, they are everywhere zero.

If the material reaches its initial position again at the instant when zero external load is reached, then the deformation was elastic.

In general, this behavior cannot be expected. Let $\tau = t_o$ denote the time when zero load is reached. At this time, a certain strain is associated with this zero load. Let the strain be represented by the small strain tensor $\underset{\sim}{\varepsilon}$ with components ε_{ij} taken with respect to a suitable Cartesian coordinate system. The magnitude of the strain at this point is given by $+(\varepsilon_{ij}\varepsilon_{ij})^{\frac{1}{2}}$ for $\tau = t_o$. For $\tau > t_o$, the magnitude of the strain decreases further, we observe anelasticity or an aftereffect. While it may be possible that anelasticity reduces the magnitude of the strain to zero for a long time rest at elevated temperature, the usual observations are limited to comparatively short times (hours, days instead of years). They show that the rate of decrease of the magnitude of the strain decreases rapidly with time so that it is permissible to talk about a permanent set (strain) which is reached for long times of interest ($\tau \to \infty$). The magnitude of the permanent strain (zero magnitude is not excluded) is always less than the magnitude of the strain at $\tau = t_o$ so that we have

$$(\varepsilon_{ij}\varepsilon_{ij})^{\frac{1}{2}}_{\tau=t_o} \geq (\varepsilon_{ij}\varepsilon_{ij})^{\frac{1}{2}}_{\tau\to\infty} \qquad (1)$$

where $+(\varepsilon_{ij}\varepsilon_{ij})^{\frac{1}{2}} = \|\underset{\sim}{\varepsilon}\|$ is the magnitude of the small strain tensor. The difference between the strain at $\tau = t_o$ and $\tau \to \infty$ is called the anelastic strain or the strain due to the aftereffect [14].

Response to a Spherical Stress Tensor (Hydrostatic Pressure)

Specific experiments investigating the influence of a spherical stress tensor on the low-cycle fatigue and creep behavior of structural metals at elevated temperature are rare, if not completely absent. Following the observations of [10] it is not unreasonable to assume that the resistance to a spherical stress tensor is such that the original shape is retained after the loading has been removed. Therefore, it is assumed that the resistance of the material to a spherical stress tensor is linear elastic at pressures which are of interest in stress analysis.

Isothermal Aging

In the temperate range of interest the structural materials considered in this study do not change their mechanical properties if exposed to constant temperature in the absence of mechanical loading. Mechanical loading, however, can change the stress-strain behavior so that the properties after unloading are different from those of the virgin material. An extended exposure to constant temperature of a deformed metal without any subsequent loading will cause a change in the mechanical properties, some deformation induced aging will occur. Insufficient experimental evidence is available to decide whether the virgin mechanical properties can be restored through a prolonged past deformation exposure [12]. This question is intimately related to the question whether a zero permanent set is reached for $\tau \to \infty$. However, it is established that the stress-strain behavior will change during no load exposure after prior deformation at elevated temperature. No such change is expected at room temperature.

The Proposed Isothermal Stress-Strain Law

The features of material behavior discussed in the previous section have to be incorporated in a mathematical relation, the stress-strain law or constitutive equation, which provides the link between the kinematic and kinetic variables of a continuum. To the author's knowledge, such a realistic stress-strain law is not in existence. Therefore, an attempt will be made to develop such an equation which incorporates the previously mentioned deformation characteristics.

The Memory of Structural Materials

It was shown [7] that prior deformation has an effect on the subsequent deformation behavior; the present response is not sufficiently determined by the present values of the relevant parameters alone. Experimental evidence exists that the present response depends also on the deformation history of the material. It follows that the material must have a memory of events in its past deformation history.

The concept of fading memory [11] is widely used and states that events in the distant past have lost their influence on the present behavior. A material possessing fading memory will restore its initial properties after sufficiently long rest periods.

Fading memory is not a property of structural metallic materials at room temperature. Permanent changes in the stress-strain behavior are observed, they must be caused by a type of memory which is different from the fading one. Experimental evidence from studies of the hysteresis loop of steels at room temperature suggested [9] that the memory of structural steels was discrete and nonfading, and was for the stress at the transition from loading to unloading. The findings of [9] will now be extended to elevated temperature. In doing so it is assumed that the basic features of metallic deformation are not altered by the increase in temperature. Thermal activation, however, is increased and consequently redistributions of microstructural stresses can occur, so that time-dependent changes in the material may occur under no external load, changes which are insignificant at room temperature.

> It is postulated that at elevated temperature a part of the memory of a structural metal is still discrete but partially or completely fading. The discrete memory is for the stress tensor at the transition from loading to unloading. A necessary condition for a change in the contents of the memory is the change from loading to unloading. A formal definition of these concepts will now be given.

The components of the stress tensor $\underset{\sim}{\sigma}$ with respect to a Cartesian coordinate system are σ_{ij}. Its magnitude $\|\underset{\sim}{\sigma}\|$ is defined by

$$\|\underset{\sim}{\sigma}\| = +(\sigma_{ij}\sigma_{ij})^{\frac{1}{2}} . \tag{2}$$

Loading is defined as an increase of $\|\underset{\sim}{\sigma}\|$ such that

$$d\|\underset{\sim}{\sigma}^2\| = 2\sigma_{ij}d\sigma_{ij} > 0. \tag{3}$$

For unloading the magnitude of the stress tensor decreases and we have

$$d\|\underset{\sim}{\sigma}^2\| = 2\sigma_{ij}d\sigma_{ij} < 0. \tag{4}$$

The transition from loading to unloading is marked by

$$d\|\underset{\sim}{\sigma}^2\| = 0. \tag{5}$$

At this point the stress tensor has a characteristic value σ^*_{ij}. It is this value of the stress tensor which can enter into the "memory of the material" provided that other conditions are met. A necessary condition for the change in the content of the memory of the material is therefore $d\|\underset{\sim}{\sigma}^2\| = 0$.

One obvious condition for which a change in the content of the memory cannot take place is immediately identified. If the change from loading to unloading occurs in the linear elastic range, then no change in the memory is possible, i.e., if

$$\frac{d\sigma_{ij}}{d\epsilon_{k\ell}} = \beta_{ijk\ell} \tag{6}$$

where $\beta_{ijk\ell}$ is a constant (independent of $\underset{\sim}{\sigma}$ or $\underset{\sim}{\epsilon}$). A change in the content of the memory is possible if

$$\frac{d\sigma_{ij}}{d\epsilon_{k\ell}} \neq \beta_{ijk\ell}. \tag{7}$$

In this way two necessary conditions [Eqs.(5) and (7)], for a change in the content of the memory are obtained.

Let $\tau = t$ designate the present time. At time t the memory of a structural material can now be represented in the stress-strain law (which contains other variables of course) by a stress tensor, the memory parameter. The memory parameter can only change if Eqs.(5) and (7) are fulfilled. It is assumed that the equation representing a mechanically virgin material contains an initial parameter σ^o_{ij}. (The possibility of σ^o_{ij} being equal to the zero tensor is not excluded.) If Eqs.(5) and (7) are fulfilled at an instant t, then the content of the memory can change. In view of experimental evidence (upon loading the yield surface expands, any state of stress within the yield surface will only cause elastic deformations subsequently) we assume that the contents of the memory will change if

$$\sigma^*_{ij}\sigma^*_{ij} > \sigma^o_{ij}\sigma^o_{ij} \tag{8}$$

where σ^*_{ij} is the stress tensor for $d\|\underset{\sim}{\sigma}^2\| = 0$ at $\tau = t$. If this condition is met then the new memory parameter will be σ^*_{ij} which replaces σ^o_{ij}. The mathematical equation which represents the material has now irreversibly changed which corresponds to the experimentally observed evidence. Future changes in the memory can occur if Eqs.(5) and (7) are met if

$$\sigma^{**}_{ij}\sigma^{**}_{ij} > \sigma^*_{ij}\sigma^*_{ij} \tag{9}$$

where σ^{**}_{ij} is the stress tensor for the present $d\|\underset{\sim}{\sigma}^2\| = 0$.

In some cases, e.g., creep, the condition $d\|\underset{\sim}{\sigma}^2\| = 0$ will prevail for a long time before unloading takes place. For definiteness we assume that the change in the parameter takes place at the time when $d\|\underset{\sim}{\sigma}^2\| = 0$ for the first time, e.g., immediately after the cessation of loading ($\sigma_{ij}d\sigma_{ij} > 0$).

To introduce the concept of fading memory, we assume that the last change in the content of the memory has taken place s units of time before the present time t. At this time, the tensor σ^*_{ij} became the memory parameter. Now the memory is assumed to be fading. An obliviator [11] is defined which is a monotonically decreasing, dimensionless function of the time lapse (t-s).

Although experimental evidence is insufficient, it is postulated that no load exposure after prior loading and unloading can restore the virgin material properties. (It provides no difficulty to formulate a partially fading memory in the sense that the original parameter σ^o_{ij} will never be obtained again; this formulation will not be pursued here.) Therefore the memory parameter σ^*_{ij} at the present time t, has to return to the virgin parameter σ^o_{ij} for $\tau \to \infty$ provided that the future deformation for $\tau > t$ is such that no further change in the memory parameter takes place.

Let h(t-s) be the dimensionless scalar-valued obliviator with the property

and
$$\left.\begin{aligned} & h(0) = 1 \\ & \lim_{(t-s)\to\infty} h(t-s) = \alpha, \quad 0 \le \alpha \le 1 \end{aligned}\right\} \tag{11}$$

The content of the memory at time t is then

$$\sigma^*_{ij}\, h(t-s). \tag{12}$$

In view of the fact that the original properties can be restored after a prolonged exposure, we have to require that

$$\lim_{(t-s)\to\infty} \sigma^*_{ij}\, h(t-s) = \sigma^o_{ij} \tag{13}$$

and therefore

$$\sigma^*_{ij}\alpha = \sigma^o_{ij} \tag{14}$$

since σ^*_{ij} does not change if the material is at rest. It follows that the initial parameter σ^o_{ij} does not fade, it is a constant in time within our assumptions.

Equation (14) can only be true if the loading is proportional all the time. In general, σ^*_{ij} and σ^o_{ij} are not related by a scalar-valued factor, they are not necessarily proportional. In the general case a tensor-valued obliviator $h_{ijk\ell}(t-s)$ has to be defined such that

$$\sigma^*_{ij}\, h_{ijk\ell}(0) = \sigma^*_{k\ell} \tag{15}$$

and

$$\lim_{(t-s)\to\infty} h_{ijk\ell}(t-s) = \alpha_{ijk\ell}. \tag{16}$$

We then require

$$\lim_{(t-s)\to\infty} \sigma^*_{ij}\, h_{ijk\ell}(t-s) = \sigma^*_{ij}\alpha_{ijk\ell} = \sigma^o_{k\ell}. \tag{17}$$

To model a permanent, nonfading memory, it is sufficient to require that the defined obliviators are constants (independent of time) and equal to unity. This type of memory corresponds to the behavior of structural materials at room temperature.

The Proposed Constitutive Equation

The type of memory proposed in the previous section is decisively important when load changes are involved, especially for cyclic loading. Creep, relaxation and strain-rate sensitivity occur during monotonic loading and do not involve unloading; these phenomena must be reproduced without a change in the memory parameter.

In the linear case, the standard linear solid exhibits creep, relaxation and strain-rate sensitivity. It is the simplest viscoelastic material which exhibits these phenomena and contains stress, strain and their respective time derivatives [13]. The presence of time derivatives of kinematic and kinetic variables is indicative of a fading memory. On the other hand, the modeling of strain-rate sensitivity, creep and relaxation requires these time derivatives also. It appears that fading memory and creep, relaxation and strain-rate sensitivity are complementary, the presence of one necessitates the other. The fading memory represented by the time derivatives is different from the discrete memory postulated previously.

In a recent paper, Appleby [15] has considered equations of the function type and finite deformations. He concludes that at least one time derivative of a kinetic and a kinematic variable has to be present in a constitutive equation to model creep and relaxation behavior.

For monotonic loading it is customary to separate the strains additively into an elastic, plastic and creep part, a convention which is dictated by the constitutive equations used in the analysis [16-19]. In this paper, the total time-dependent deformation picture is considered and, following the conventional approach, would require the additional use of a relaxation, low-cycle and high-cycle fatigue strain. This method is not practical.

Following the experimental evidence, the approach taken here says that the "nature of the deformation" is revealed after unloading from a homogeneous deformation history. The permanent or plastic strain is obtained after the anelastic strain has "died out". Under load no possibility exists to discern between the anelastic and permanent components of the strain. A similar approach was taken in [20].

With these concepts in mind, equations are proposed which are believed to be the simplest expressions which can model the phenomena described previously.

$$f_{ijk\ell}(\sigma_{ij}, \epsilon_{ij}, \sigma^o_{ij}) \frac{d\sigma_{k\ell}}{d\tau} + \sigma_{ij} = c_{ijk\ell}\epsilon_{k\ell} + g_{ijk\ell}(\sigma_{ij}, \epsilon_{ij}, \sigma^o_{ij}) \frac{d\epsilon_{k\ell}}{d\tau} . \tag{18}$$

In this equation the symbols denote

$c_{ijk\ell}$ - the elastic constants of the material.

$f_{ijk\ell}(\sigma_{ij}, \epsilon_{ij}, \sigma^o_{ij})$ and $g_{ijk\ell}(\sigma_{ij}, \epsilon_{ij}, \sigma^o_{ij})$ - fourth-order tensor functions of the indicated tensor arguments.

$\frac{d\sigma_{k\ell}}{d\tau}$ - a proper, objective time derivative of the stress tensor.

σ^o_{ij} - any possible memory parameter which may occur during a deformation history.

Equation (18) represents anisotropic characteristics and shows time-dependent response for a spherical stress tensor

$$\sigma_{ij} = p\delta_{ij}. \tag{19}$$

To insure purely elastic response to spherical stress tensor, one has to require

and
$$\left.\begin{aligned} f_{ij\ell\ell}(p\delta_{ij},\ \epsilon_{ij},\ \sigma^{o}_{ij})\ \frac{dp}{d\tau} &= 0 \\ g_{ijk\ell}(p\delta_{ij},\ \epsilon_{ij},\ \sigma^{o}_{ij})\ \frac{de_{k\ell}}{d\tau} &= 0\,. \end{aligned}\right\} \tag{20}$$

Anisotropy will not be considered further in this paper although it offers the possibility of modeling the experimentally observed phenomenon that an initially isotropic material becomes anisotropic with respect to the onset of permanent deformation.

In the isotropic case Eq.(18) reduces to

$$f(\sigma_{ij},\ \epsilon_{ij},\ \sigma^{o}_{ij})\ \frac{d\sigma_{ij}}{d\tau} + \sigma_{ij} = \lambda\epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij} + g(\sigma_{ij},\ \epsilon_{ij},\ \sigma^{o}_{ij})\ \frac{d\epsilon_{ij}}{d\tau} \tag{21}$$

where

λ, μ - Lamé constants

$f(\sigma_{ij},\ \epsilon_{ij},\ \sigma^{o}_{ij})$, $g(\sigma_{ij},\ \epsilon_{ij},\ \sigma^{o}_{ij})$ - isotropic, scalar-valued tensor functions of the indicated arguments.

In the isotropic case stress and strain tensors are coaxial so that for a spherical stress tensor (19) we have

$$\epsilon_{ij} = q\delta_{ij}. \tag{22}$$

To insure elastic response for (19), two choices are open, viz

and
$$\left.\begin{aligned} f(p\delta_{ij},\ q\delta_{ij},\ \sigma^{o}_{ij}) &= 0 \\ g(p\delta_{ij},\ q\delta_{ij},\ \sigma^{o}_{ij}) &= 0 \end{aligned}\right\} \tag{23}$$

or

$$\frac{dp}{d\tau}\ \delta_{ij} = \frac{dq}{d\tau}\ \delta_{ij} = 0. \tag{24}$$

The latter possibility is chosen by introducing

$$\begin{aligned} s_{ij} &= \sigma_{ij} - \frac{\sigma_{kk}}{3}\ \delta_{ij} \\ \frac{ds_{ij}}{d\tau} &= \frac{d\sigma_{ij}}{d\tau} - \frac{1}{3}\ \frac{d\sigma_{kk}}{d\tau}\ \delta_{ij} \end{aligned} \tag{25}$$

and

$$\begin{aligned} e_{ij} &= \epsilon_{ij} - \frac{\epsilon_{kk}}{3}\ \delta_{ij} \\ \frac{de_{ij}}{d\tau} &= \frac{d\epsilon_{ij}}{d\tau} - \frac{1}{3}\ \frac{d\epsilon_{kk}}{d\tau}\ \delta_{ij}. \end{aligned} \tag{26}$$

With this choice, Eq.(21) becomes

$$f(\sigma_{ij},\ \epsilon_{ij},\ \sigma^{o}_{ij})\ \frac{ds_{ij}}{d\tau} + \sigma_{ij} = \lambda\epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij} + g(\sigma_{ij},\ \epsilon_{ij},\ \sigma^{o}_{ij})\ \frac{de_{ij}}{d\tau} \tag{27}$$

This equation must be invariant if σ_{ij} and ϵ_{ij} is replaced by $-\sigma_{ij}$ and $-\epsilon_{ij}$, respectively [9]. It follows that f( ) and g( ) have to be even in the tensor variables σ_{ij} and ϵ_{ij}.

Equation (27) can model the Bauschinger effect; it is not invariant if σ_{ij} is replaced by $-\sigma_{ij}$, moreover during the simulation of the Bauschinger experiment using Eq.(27), the parameter σ^o_{ij} will have been replaced by a new one at the change from loading to unloading.

[For real materials, the Bauschinger effect is observed after loading in one direction (say tension), unloading and loading in the opposite direction (say compression. Experience then shows that the stress-strain curve on first loading is different from that for the subsequent loading in the opposite direction.]

A further general requirement is that the solutions to the differential equations obtained by setting either $\sigma_{ij} = \alpha_{ij}$ or $\epsilon_{ij} = \beta_{ij}$, where α_{ij} and β_{ij} are not dependent on time, are non-oscillatory.

Equation (27) represents the material in the virgin state. If loading is followed by unloading and if the conditions (5) and (7) are met at the transition from loading to unloading, then the memory parameter σ^o_{ij} will be replaced by a new one, $\sigma^*_{k\ell}h_{k\ell ij}(t-s)$, [Eqs.(15)-(17)], and therefore the material characteristic has also changed. The substitution of a new memory parameter can take place very frequently during cyclic loading, so that a new material can emerge after each cycle. Indeed, in design calculations involving low-cycle fatigue conditions, this fact is already recognized, the "cyclic stress-strain diagram" is frequently used instead of the regular monotonic one [5].

Application to a Two Step Creep Loading

To illustrate the capability of the proposed equation a two level creep loading is considered, the results of which are reproduced in [16], p.240, Fig.78. The following two creep tests were performed on the same material. In the first test the specimen was loaded to a constant uniaxial stress $\underset{\sim}{\sigma}_1$ for a period of time, say s_1, then the stress was increased to $\underset{\sim}{\sigma}_2$ and kept for the same time period s_1. In the second test the order of stress application was reversed, $\underset{\sim}{\sigma}_2$ first followed by $\underset{\sim}{\sigma}_1$. The same time steps were employed. The curves of creep strain versus time reported show that the total accumulated creep strain was different in the two cases at the end of the loading sequence and that the two creep curves for $\underset{\sim}{\sigma}_1$ differed markedly with very little difference in the two curves for $\underset{\sim}{\sigma}_2$.

In plotting Fig.78 of [16] only creep strains are considered (any strains obtained during loading and change of load are not shown) and this method will be followed in the duplication of these tests with the use of Eq.(27).

For the purpose of this analysis it is assumed that $||\underset{\sim}{\sigma}_2|| > ||\underset{\sim}{\sigma}_1|| > ||\underset{\sim}{\sigma}^o||$ and that the loading is proportional. The sequence $\underset{\sim}{\sigma}_1$, $\underset{\sim}{\sigma}_2$ is modeled first.

Upon first loading $\underset{\sim}{\sigma}_1$ is reached, loading ceases and $d||\underset{\sim}{\sigma}^2|| = 0$ at $\tau = s$. For $\tau = t > s$ we have $\sigma_{ij} = \sigma_{ij(1)}$ which is independent of time. Equation (27) together with (5), (7), (12), (13) yields for $0 \le t-s \le s_1$

$$\sigma_{ij(1)} = \lambda\epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij} + g\left(\sigma_{ij(1)}, \epsilon_{ij}, \sigma_{ij(1)}h(t-s)\right)\frac{de_{ij}}{d\tau} \tag{28}$$

since $\|\underset{\sim}{\sigma}_1\| > \|\underset{\sim}{\sigma}^o\|$. The solution of Eq.(28) subject to the initial condition $\epsilon_{ij} = \epsilon^o_{ij}$ for t-s = 0 will give the variation of the creep strain as a function of time. At the end of the loading period (t-s) = s_1 and a certain creep strain is obtained which corresponds to the total strain accumulated during $0 \le (t-s) \le s_1$.

Now the stress is changed from $\underset{\sim}{\sigma}_1$ to $\underset{\sim}{\sigma}_2$ and from (27) and conditions (5), (7), (12), (13) we have for $0 \le (t-s) \le s_1$

$$\sigma_{ij(2)} = \lambda\epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij} + g\left(\sigma_{ij(2)}, \epsilon_{ij}, \sigma_{ij(2)}h(t-s)\right)\frac{de_{ij}}{d\tau} \tag{29}$$

since $\|\underset{\sim}{\sigma}_2\| > \|\underset{\sim}{\sigma}_1\|$. The solution of this differential equation will give the creep strain accumulated for the second portion of the loading. The strains obtained from the solution of (28) and (29) are generally different.

For the second sequence we have $\underset{\sim}{\sigma}_2$ first and get

$$\sigma_{ij(2)} = \lambda\epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij} + g\left(\sigma_{ij(2)}, \epsilon_{ij}, \sigma_{ij(2)}h(t-s)\right)\frac{de_{ij}}{d\tau} \tag{30}$$

for $0 \le (t-s) \le s_1$. We note that Eqs.(29) and (30) are identical; they predict the same creep curve.

For a change to $\underset{\sim}{\sigma}_1$ we get for $s_1 \le (t-s) \le 2s_1$

$$\sigma_{ij(1)} = \lambda\epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij} + g\left(\sigma_{ij(1)}, \epsilon_{ij}, \sigma_{ij(2)}h(t-s)\right)\frac{de_{ij}}{d\tau} \tag{31}$$

The difference between Eq.(28) and (31) is twofold. In the second case $\underset{\sim}{\sigma}_2$ is the memory parameter compared to $\underset{\sim}{\sigma}_1$ in the first case, and $0 \le (t-s) \le s_1$ in Eq.(28) but $s_1 \le (t-s) \le 2s_1$ in Eq.(31). It follows that the ideas introduced previously lead to equations which have the necessary theoretical foundation so that this particular example of deformation behavior in creep can be reproduced.

The approach outlined has parallels to the equation of state concept, it is, however, decisively different in its outlook as it definitely says what the dependence on past history is; the state parameter is not left unspecified, it is determined through the concept of discrete, fading memory.

The presence of the first time derivatives in the stress-strain laws (18) and (27) requires in each possible loading case the solution of a first order, nonlinear, nonhomogeneous differential equation. In general the loading histories are piecewise continuous (e.g., loading at a constant rate up to a certain load level, then continuing with constant load), the discontinuities being in the slopes of the corresponding stress or strain vs. time curves. This behavior can be reproduced by requiring that the arbitrary constant entering into the solution of the first order differential equation is such that the stress (strain) values at the end of one particular loading are equal to those at the beginning of the next loading segment. Indeed, Eq.(27) could have been used to model the entire sequence of the loading and unloading of the experiment in [16] if the experimental conditions and results would have been known.

So far the coefficient functions of the time derivatives were left unspecified. From a theoretical point of view every isotropic tensor function qualifies. For the modeling of material behavior the proper choice is all important. To arrive at a solution, the following steps suggest themselves. A specific choice of the coefficient functions has to be made, then the stress-strain law has to be specialized for the uniaxial case for the particular loading situations quoted under total time-dependent picture and [7]. The solution of the corresponding differential equations has to reproduce the experimentally observed behavior. If such a correspondence fails to emerge, the coefficient functions or even the stress-strain law have to be modified and the procedure has to be repeated. A trial and error process emerges which will finally converge on the desired realistic stress-strain law.

Conclusion

In this paper salient features of the elevated temperature deformation behavior of structural metals are documented and a stress-strain law of minimum complexity is proposed which is believed to have the capability of reproducing these salient features. Details of the proposed approach are still to be evaluated in a trial and error process, so that a specific stress-strain law can emerge.

In view of the general availability of finite-element computer codes with nonlinear and time following capabilities in the design office such a realistic stress-strain law is urgently needed. In the absence of such a stress-strain law, existing ones will be used beyond the limits for which they were developed. The computer, instead of helping to solve a difficult problem, makes it then easier to get the wrong answers. Aside from the economic loss due to computations which are based on irrelevant input, the possibility cannot be discounted that the outputs of these computations form the basis on which components operating in the future high temperature reactors are sized and built. If the basis for such decisions is wrong, the safe operation of these components during the lifetime of a reactor cannot be assessed with the certainty which is required of nuclear units. It is therefore mandatory to develop and use as accurate information about material deformation behavior as is possible.

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DISCUSSION

Q Z. ZUDANS, U. S. A.

In the last equation shown in the slide the tensor M_o cannot take value equal to zero. How is the equation changed if M_o is equal to zero and what is the meaning of M_o in the formula shown ?

A E. KREMPL, U. S. A.

The last equation shown is not printed in the paper. It will appear in a forthcoming article in the International Journal of Fracture Mechanics. If $M_o = 0$ a new solution has to be found which is given in the above mentioned article.

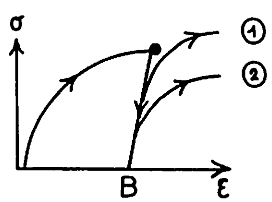
Q A. PHILLIPS, U. S. A.

I have three comments to make:

1. At point B the Baushinger Effect predicts experimentally that there is already plasticity and creep. Hence, it is not surprising that there is an effect depending on the time spent at B.
2. The constitutive equation used seems to me similar although without memory effects to a paper written by Hohenemser and Prager in ZAMM about 40 years ago.
3. We know very little experimentally, which is safely reproducible, and the theoretical work has far outstripped the experimental advances.

A E. KREMPL, U. S. A.

1.



Curve (1) - Room temperature behaviour.

Curve (2) - Elevated temperature behaviour after rest at B.

I am glad to learn that Prof. Phillips finds the behaviour as shown in curve 2 not surprising. To model this behaviour in the constitutive equation the concept of the fading, discrete memory was introduced.

2. I will consult the ZAMM. It is, however, necessary to have the memory parameter σ^o_{ij} in the constitutive equation if plasticity has to be reproduced.

3. The approach proposed in the paper started from careful examination of existing experimental information which showed that the existing stress-strain laws are inadequate. All the information used are qualitative. For a further development of the proposed approach accurate experimental information will become necessary.

Q T. MALMBERG, Germany

I have one question with respect to your constitutive equation. Does this relation represent steady state creep and if so is it true that then the creep rate depends on the elastic constants, i. e. Young's modulus and Poisson's ratio ?

A E. KREMPL, U. S. A.

The theory I developed is based on general principles and requires that experimental data will be used to determine the unknown elements. A point in case is that the coefficient function $g(\sigma_{ij}, \varepsilon_{ij}, \sigma_{ij}^{o})$ has to be chosen in such a way that steady state creep can be reproduced by the solution of equations (29), (28). The solution will depend on the elastic constants. In my approach only total strains are considered. The experimental results plot creep strains and elastic strains are omitted. It is also not clear at this point how significant the influence of the elastic constants on the solution of equations (28), (29) will be.

Q W. L. GREENSTREET, U. S. A.

Did I understand correctly that you can develop a theory, as you propose, mainly from uniaxial results ? At what point in the development would you introduce combined stress tests ?

A E. KREMPL, U. S. A.

I did not say that it will be possible to develop a theory from uniaxial test results alone. I made the statement that uniaxial results show that plasticity and creep theories are inadequate to describe material behaviour at elevated temperature.

Tests by Conway (GEMP-730, Dec. 1969) indicate that the steady state stress range in a low-cycle fatigue test depends on the strain rate (65 ksi for $\dot{\varepsilon} = 4 \cdot 10^{-5}$ 1/sec and 81 ksi for $\dot{\varepsilon} = 4 \cdot 10^{-4}$ 1/sec, Fig. 2 of the above reference). This result shows that for Type 304 SS rate dependent effects are important and of engineering significance. Conventional plasticity theories cannot account for this.

The above example shows that uniaxial data can be of great value and help. For a complete theory multiaxial experiments under non-proportional loading are indispensable.