NON-LINEAR CREEP ANALYSIS AT ELEVATED TEMPERATURE

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ABSTRACT

A non-linear creep analysis method is developed for solving two-dimensional structures (plane stress, plane strain and axisymmetric) subjected to time-dependent mechanical as well as thermal loadings. The numerical solution is obtained within the framework of the finite element method following an incremental procedure. In this analysis the creep behavior of metals at high temperatures is assumed to follow a mechanical equation of state in which the creep rate is assumed to be a non-linear function of stress, creep strain, temperature and time.

The creep strains are assumed to remain small throughout the deformation history and thereby no plastic strains are introduced. Two engineering theories of creep, namely the time-hardening and the strain-hardening rules, which utilize creep data from constant-stress and constant-temperature tests are considered. A special feature of the method is the treatment of temperature effects on the basis that the time-temperature shift hypothesis is valid for metals subjected to non-uniform transient temperature fields. This permits the analysis of reactor components under cyclic temperatures and hence provides the means for evaluating the effects of creep on the fatigue life of such structures.

A computer program which is capable of handling large-size problems was developed. Moreover, the material properties incorporated in the computer program can be easily updated when such information becomes available. For demonstration, numerical examples are presented to show the creep effects due to arbitrary changes in stress and temperature histories.

1. INTRODUCTION

In the design of reactor system components at elevated temperatures, creep, stress relaxation and plastic deformations are often a major concern. If a designer is to evaluate his structural design realistically, he must rely on a time-dependent analysis, which properly accounts for the structure's behavior under expected service conditions. It is for this reason that design-by-analysis approaches are being adopted for reactor structures subjected to time varying mechanical and thermal loadings.

Creep of metals at elevated temperature depends nonlinearly on the stress state, the temperature and the deformation history. Several mathematical models based on the concept that the material obeys a mechanical equation of state were proposed to predict the fundamental creep behavior of metals under transient loading conditions [1, 2]. For stress analysis purposes, however, further simplications were introduced which lead to the engineering creep theories known as the time-hardening, the strain-hardening and the life fraction methods [3]. The temperature dependence of creep was then incorporated by utilizing the time-temperature shift factor following Dorn [4].

A considerable amount of work in the creep analysis of structures has been done mostly in aircraft component and turbine engine design. Among many important contributions, Mendelson, Hirschberg and Manson [5] presented a general procedure for the incremental analysis of creep problems, which may be considered as the ground-work for treating the nonlinear time-dependent analysis. Detailed numerical solutions of creep problems with the aid of the finite element method have also been explored by many authors and applications in this area include shell structures [6-10], plane and axisymmetric solids [11-14]. In this paper the finite element method is also utilized and a general numerical formulation is presented for two-dimensional problems. Specific emphasis of the present work is placed upon the treatment of nonisothermal time-dependent loadings as well as on large design problems of practical significance.

2. CONSTITUTIVE EQUATIONS

In the following development the constitutive equations are fomulated for homogeneous isotropic material under conditions of transient stresses and temperatures. In addition, the deformations are assumed to remain small and no plastic strains are considered.

Anticipating that an incremental solution approach will be adopted, we will use the notations $\Delta\sigma_{ij}$ and $\Delta\varepsilon_{ij}$ for the incremental stress and incremental strain tensors respectively. For materials undergoing creep the strain increment within a small time interval may be expressed as the sum of the elastic, creep and thermal strains,

$$\Delta\varepsilon_{ij} = \Delta\varepsilon^{e}_{ij} + \Delta\varepsilon^{c}_{ij} + \alpha\Delta T\,\delta_{ij} \tag{1}$$

where $\Delta\varepsilon^{e}_{ij}$ is the elastic component; $\Delta\varepsilon^{c}_{ij}$, the creep strain; $\alpha\Delta T$, is the thermal strain; and δ_{ij} is the Kronecker delta.

The incremental stresses are related to the elastic strains increment by

$$\Delta\sigma_{ij} = \frac{E\nu}{(1+\nu)(1-2\nu)}\,\delta_{ij}\,\Delta\varepsilon^{e}_{kk} + \frac{E}{1+\nu}\,\Delta\varepsilon^{e}_{ij} \tag{2}$$

where E is the Young's modulus which can be a function of temperature and ν is the Poisson's ratio. Furthermore, we assume that the creep strains are incompressible, i.e.,

$$\Delta\varepsilon^{c}_{kk} = 0 \tag{3}$$

Substituting eq. (1) and eq. (3) into eq. (2), we find

$$\Delta\sigma_{ij} = \frac{E}{(1-\nu)(1-2\nu)}\,\delta_{ij}\,\Delta\varepsilon_{kk} + \frac{E}{1+\nu}\,\Delta\varepsilon_{ij} - \frac{E}{1+\nu}\,\Delta\varepsilon^{c}_{ij} + \frac{E\alpha\Delta T}{1-2\nu}\,\delta_{ij} \tag{4}$$

In the above formulation, it is easy to see that the creep strains appear as effective thermal strains.

The question remains then how to compute the creep strains defined in eq. (4). To this end the Prandtl-Reuss equations, similar to plasticity, are assumed to apply during the creep i.e.,

$$\Delta\varepsilon^{c}_{ij} = \frac{3}{2\sigma_{e}}\,S_{ij}\,\Delta\varepsilon^{c}_{e} \tag{5}$$

In eq. (5), the deviatoric stresses S_{ij} are related to the stresses σ_{ij} by

$$S_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk} \tag{6}$$

The effective stress σ_e is defined by

$$\sigma_e = \sqrt{\frac{3}{2}} (S_{ij}S_{ij})^{1/2} \tag{7}$$

and the effective creep strain is given by

$$\Delta\varepsilon_e^c = \frac{1}{\sqrt{3}} (\Delta e_{ij}^c \ \Delta e_{ij}^e)^{1/2} \tag{8}$$

where Δe_{ij}^c is the deviatoric strain component in creep. As a result of the incompressibility condition, we have

$$\Delta e_{ij}^c = \Delta\varepsilon_{ij}^c \tag{9}$$

In the unaxial state of stress σ and strain ε, the incremental effective stress and creep strain reduce to

$$\begin{aligned} \Delta\sigma_e &= \Delta\sigma \\ \Delta\varepsilon_e^c &= \Delta\varepsilon^c \end{aligned} \tag{10}$$

In view of this relationship, the creep strain $\Delta\varepsilon_e^c$ defined in eq. (5) can be characterized by uniaxial creep experiments.

As was mentioned previously, the creep of metals at elevated temperature under time-dependent loadings, depends not only on the magnitude of stress and temperature, but also on their histories. Up to the present, there is as yet no unique way of representing the creep response in a mathematical form. One approach which has been adopted by many investigators is the concept of the mechanical equation of state [1, 2]. In this approach, it is assumed that the creep rate at any given time t depends on the current stress, temperature and a certain state variables, q_i. Hence the creep rate can be expressed as

$$\dot{\varepsilon}^c = f(\sigma, T, q_1, q_2, \ldots, q_n) \tag{11}$$

and

$$\dot{q}_i = g_i(q_j), \ i = 1, 2, \ldots, k \tag{12}$$

where $(\dot{})$ represents the time derivative of a function. In a small, but finite sense, the creep strain increment is computed from

$$\Delta\varepsilon^c = \dot{\varepsilon}^c \ \Delta t \tag{13}$$

and Δt is the time increment.

The state variables in eq. (11) represent intrinsic coordinates each of which constitutes a measure of a physical phenomenon or a structural change that may take place in a material element during the creep process. In view of this, equation (11) can be made as general and as precise as desired provided the appropriate experiments are conducted. For our present purposes, however, we will consider two existing engineering creep theories which utilize the creep data from constant stress and constant temperature tests, namely, the time-hardening rule and strain-hardening rule, respectively.

2.1 TIME-HARDENING RULE

Suppose a specimen is stretched under constant temperature by a time-dependent stress

$$\sigma = \sigma(t) \tag{14}$$

The time-hardening rule assumes that the creep rate at any given time t is only a function of the current stress and time, regardless of prior stress or strain histories. Because of this, we may define the state variables in eq. (11) as follows

$$q_1 = t$$
$$q_i = 0, \; i = 2, 3, \ldots, n \tag{15}$$

and thus obtain

$$\dot{\varepsilon}^c = f(\sigma, T, t) \tag{16}$$

For example, the creep data under constant stress and temperature may be described by

$$\varepsilon^c = A(T)\, B(\sigma)\, t^n \tag{17}$$

In accordance with the time-hardening rule the creep rate is obtained simply by differentiating eq. (17),

$$\dot{\varepsilon}^c = n\, A(T)\, B\,(\sigma)\, t^{n-1} \tag{18}$$

The total creep strain at time t is then computed from

$$\varepsilon^c = \int_0^t \dot{\varepsilon}^c \, dt \tag{19}$$

2.2 STRAIN-HARDENING RULE

This rule assumes that in going from one stress level to the next, the creep rate depends on the existing accumulated creep strain in the material. Hence by taking n = 1 in eq. (11), the creep strain represents a state variable for the measure of creep rate, i.e.

$$\dot{\varepsilon}^c = f\,(\sigma, T, \varepsilon^c) \tag{20}$$

Let us again consider the creep expression given by eq. (17). Eliminating the time t between (17) and (18), the creep rate is then found to be

$$\dot{\varepsilon}^c = n\,[A(T)\, B(\sigma)]^{1/n} \cdot (\varepsilon^c)^{(n-1)/n} \tag{21}$$

where ε^c is the total creep strain defined in eq. (19).

2.3 TEMPERATURE EFFECT

In general, the creep rate increases rapidly with an increase in temperature. The basic question is in what manner should the temperature effect be incorporated in the computation of creep strains. For many metals when the experimental creep strains are plotted as a function of the logarithm of the time under test, the creep curves for the same stress and for different constant temperatures are identical excepting for parallel displacement along the time axis. In view of this property, a time-temperature parameter [4] is introduced in the expression of creep strain such that

$$\dot{\varepsilon}^c = \varepsilon^c(t, \sigma, T) \tag{22}$$
$$= g(\zeta, \sigma)$$

in which ζ is known as the temperature-compensated time, or the reduced time, and is related to the time t by

$$\zeta = \phi(T)\, t \tag{23}$$

where $\phi(T)$ is a temperature shift factor.

Furthermore, a phenomenological theory was postulated that the creep rate of metals is controlled by some process involving thermal activation. In this connection the temperature shift factor is related to the activation energy of the material in the following manner:

$$\phi(T) = e^{-\frac{\Delta H}{RT}} \tag{24}$$

where

ΔH = activation energy for creep

R = gas constant

T = absolute temperature

The activation energy defined in eq. (24) is a material parameter which could be determined from the creep tests conducted under the same stress at different temperatures.

The reduced time defined in eq. (23) is valid only for constant (or steady-state) temperature. If the temperature varies as a continuous function of time, an additional assumption must be made, namely, the reduced time depends only on the history of the temperature, not on its rate. Then the calculation of ζ may be generalized as follows:

$$\zeta = \int_0^t \phi(T)\ dt \tag{25}$$

3. NUMERICAL SOLUTION

The finite element derivations for initial strain problems is well known and need not be given here. Recasting the incremental stress-strain relations in matrix form yields,

$$\{\Delta\sigma\} = [H] \cdot \{\Delta\varepsilon\} - [H_1]\ \{\Delta\varepsilon^c\} - [H_2]\ \{\Delta\varepsilon^T\} \tag{26}$$

The matrices H, H_1 and H_2, defined in the appendix, are then used in the usual finite element procedure to obtain the element equilibrium condition within any time interval $[t, t+\Delta t]$, as follows

$$[K]\ \{\Delta v\} = \{\Delta F\} \tag{27}$$

where K is the element stiffness matrix which, if the Young's modulus is constant for all time increments, is identical to the elastic stiffness matrix; Δv is the incremental nodal displacement vector and ΔF represents the incremental nodal force vector, which consists of three parts:

$$\{\Delta F\} = \{\Delta F_1\} + \{\Delta F_2\} + \{\Delta F_3\} \tag{28}$$

In the above, ΔF_1 represents the externally applied force, ΔF_2 is the creep contribution and ΔF_3 is the thermal force vector.

The governing equations of the entire structure, obtained by the superposition of equations (27), are then solved for the nodal points displacement increment. The incremental stresses can then be computed from (26).

By the use of the finite element method and an incremental solution approach, a non-linear creep problem is then reduced to several equivalent elastic problems. In the above numerical formulations, the creep strains appear as effective body forces which depend on the current state of stress and strain. In each time step the creep strains are computed from the information at the beginning of the time interval. As a result of such an approximation, questions may arise regarding solution convergence and stability. Experience has shown that the incremental solution always converges provided that the creep strain increment is only a small fraction of the effective elastic strain, i.e.

$$\Delta\varepsilon_e^c = \kappa \frac{\sigma_e}{E} \qquad (29)$$

where κ is a constant and usually taken to be

$$0 < \kappa \leq 0.10 \qquad (30)$$

For example, if we consider the creep expression given in eq. (17), and then the time increment Δt may be found from eqs. (21) and (29)

$$\Delta t = \frac{\kappa\sigma}{nA(T)B(\sigma)\cdot t_o^{n-1}} \qquad (31)$$

where t_o has different meanings depending on which hardening rule is used. For time-hardening rule, t_o is defined as the current time. Whereas, for strain-hardening rule t_o is defined by

$$t_o = \left[\frac{\varepsilon^c}{A(T)B(\sigma)}\right]^{1/n} \qquad (32)$$

4. EXAMPLES

Although the finite element algarithm has been developed for general three dimensional state of stress and deformation, a computer program was written merely for solving two-dimensional problems, i.e., problems with plane stress, plane strain or axisyymetric deformations. The program was developed for the UNIVAC 1108 computer and it can analyze problems with the maximum size of 1800 elements and 1200 nodes and with a half-bandwidth of 80. To illustrate the method we have considered various numerical examples as follows:

4.1 UNIFORM STRETCH OF A THIN PLATE

At first we consider a plane stress model for which a plate is stretched by a uniformly applied stress as shown in Fig. 1. The plate is made of aluminum 1100 for which

Young's modulus $E = 9.01 \times 10^6$ psi

Poisson's ratio $\nu = 0.3$

Creep Strain $\varepsilon^c = .65 \times 10^{-4}(e^{.7x10^{-3}} -1)\ t^{0.5}$ (33)

at 220°C

For this problem, we consider three loading cases under constant temperature,

Case a creep under constant stress, $\sigma = 3700$ psi

Case b creep under constant stress rate, $R = 5000$ psi/hr

Case c stress relaxation under constant strain $\varepsilon_o = 0.555 \times 10^{-3}$

In case a, since the applied stress is constant, the creep curve is recovered and the results are plotted in Fig. 2. In case b, the analytical expression of the strain is found by direct integration,

$$\varepsilon(t) = \int_o^t \left[\frac{R}{E} + 0.325 \times 10^{-4}(e^{0.7x10^{-3}R\tau}-1)^{-0.5}\right] d\tau \qquad (34)$$

Because the stress field is a function of time, the strain obtained from the computer program cannot be exact.

Two different constant time increments were used to test the convergence of the numerical scheme; namely $\Delta t_1 = 0.1$ and $\Delta t_2 = 0.05$ hr. It is seen from Fig. 3 that the finer time increment gave much more satisfactory results in reference to the analytical solution.

For case c both the strain-hardening rule and the time-hardening rule were applied to find the relaxation curves. The results are compared with the analytical solutions which were computed from the uniaxial stress-strain law. As shown in Fig. 4, the difference between the analytical solution and the finite element solution cannot be distinguished within plotting errors. It is also interesting to note from the plots that after one hour of deformation the stress relaxed more than 60% of its initial value. As we may expect, the time-hardening rule predicts higher stress relaxation than the strain-hardening rule.

4.2 A THICK WALL CYLINDER

A thick-walled cylinder subjected to internal pressure is analyzed under the state of axisymmetric deformation. The material properties are assumed to be

$$
\begin{aligned}
E &= 20 \times 10^{6} \text{ psi} \\
\nu &= 0.45 \\
\varepsilon^{c} &= 6.4 \times 10^{-18} \sigma^{4.4} t
\end{aligned}
\tag{35}
$$

The inner radius of the cylinder is a = 0.16" and the outer radius is b = 0.25". The radial and hoop stress distributions are plotted in Figs. 5 and 6 respectively for the elastic and steady-state solutions. The steady-state solution of the problem was reached after 1.2 hr. of creep deformation. It is seen that as a result of the creep the stress distributions have changed quite significantly. In Fig. 7 the effective stresses for the inner and outer radii are plotted as a function of time. It is interesting to note that a pronounced stress relaxation is exhibited in the cylinder even though the applied loading is of the creep type.

This problem has also been analyzed by Greenbaum [13] with the assumption of a plane strain model. The results from both analyses agreed quite well as it should be.

4.3 A CANTILEVER BEAM

A cantilever beam subjected to an end load is analyzed with the assumption of plane stress state. The finite element model consists of 528 triangular elements and 299 nodes. An end load was applied in a manner that it is evenly distributed throughout the depth of the beam. The beam is made of Incloy 800 and its creep property at 1200°F is given by

$$
\varepsilon^{c} = 1.35 \cdot \left(\frac{\sigma}{38500}\right)^{0.052} t^{0.85} \tag{36}
$$

The time-hardening rule is utilized to calculate the cumulative creep strain. The maximum normal stress history near the fixed end and the deflection history at the point of load application are presented in Figs. 8 and 9, respectively. The results are compared with the ones obtained from the elementary beam theory. It is clear to see that the correlation between the stresses is quite good. However, a 20% discrepancy in displacement at 100 hr. is indicated between the finite element solution and the elementary beam theory. Possible numerical error was eliminated by taking sufficiently small time steps such that there was no change in solutions by further reducing the time steps. It was felt that the discrepancy could be due to the following reasons: (i) the use of constant strain elements, (ii) the difference between the deep beam and elementary beam theories, (iii) the manner in which the load is applied at the free end.

4.4 CREEP OF A SHEAR LAG

A shear lag structure subjected to two-step loadings is analyzed by the finite element method and the results are compared with the experimental data. The structure is made of aluminum 1100 and the structural configuration is described in Fig. 10. Since the structural tests were conducted at temperature 206°C, the creep data for the corresponding temperature are given by eq. (33). This problem has been analyzed in reference [12].

Initially the structure was deformed by a constant load of 1600 lb. At the end of one hour the load was increased to 2020 lb and kept constant thereafter. Because of symmetry only one quarter of the structure was analyzed. The finit element model was constructed in Fig. 11 and the analysis has been carried out by the time-hardening rule. From the analysis the peak stress was found to be at the center of the plate due to thickness effect and direct loading transfer. As a result of creep, stress re-distribution has also occurred in the plate and the peak stress relaxed as a function of time. For comparison, the strain histories at two different locations A and B as shown in Fig. 10 are plotted in Figs. 12 and 13, respectively. The solid lines represent the finite element solution whereas the circles represent the experimental data. One may see that the finite element solution agrees quite well with the experimental result up to 1 hr of creep and then discrepancy is indicated when the load is increased from 1600 lb. to 2020 lb. It is noted however that the corresponding stress state at A is 6800 psi which has already exceeded the yield limit of the aluminum, approximately at 5000 psi. Therefore, we conclude that the discrepancy is primarily contributed by the presence of plastic strain and this analysis does not consider such effect. In order to achieve a better correlation we must perform an elasto-plastic and creep analysis.

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APPENDIX STRESS-STRAIN RELATIONS

The incremental stress-strain relations have been given in eq. (26) in matrix notation,

$$\{\Delta\sigma\} = [H]\{\Delta\varepsilon\} - [H_1]\{\Delta\varepsilon^c\} - [H_2]\{\Delta\varepsilon^T\} \quad (26)$$

1. Plane Stress

The material matrix H is given by

$$[H] = \frac{E}{1-\nu^2} \cdot \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (A-1)$$

and

$$H_1 = H \quad (A-2)$$

whereas,

$$[H_2] = \frac{E}{1-\nu} [I] \quad (A-3)$$

where [I] is the identity matrix

The creep strain increments are computed from

$$\{\Delta\varepsilon^c\} = \frac{\Delta\varepsilon_e^c}{2\sigma_e} \cdot \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} \quad (A-4)$$

where the effective creep strain is equivalent to the uniaxial creep which was defined in eq. (13). The effective stress is defined by

$$\sigma_e = (\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy} + 3\sigma_{xy}^2)^{1/2} \quad (A-5)$$

2. Plane Strain

For the case of plane strain, we simply replace the definitions of Young's modulus, Poisson's ratio and thermal expension coefficient in the above by

$$\bar{E} = \frac{E}{1-\nu^2}$$

$$\bar{\nu} = \frac{\nu}{1-\nu} \quad (A-6)$$

$$\bar{\alpha} = (1+\nu)\,\alpha$$

The effective stress is redefined by

$$\sigma_e = [\sigma_{xx}^2 + \sigma_{yy}^2 - (1+2\nu-2\nu^2)\,\sigma_{xx}\sigma_{yy} + 3\sigma_{xy}^2]^{1/2} \quad (A-7)$$

3. AXISYMMETRIC DEFORMATION

The material matrices H, H_1 and H_2 are given by

$$[H] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \quad \text{(A-8)}$$

and

$$[H_1] = \frac{E}{1+\nu} \quad [I] \quad \text{(A-9)}$$

$$[H_2] = \frac{E}{1-2\nu} \quad [I] \quad \text{(A-10)}$$

The creep strain increments are computed from

$$\{\Delta\varepsilon^c\} = \frac{\Delta\varepsilon_e^c}{2\sigma_e} \cdot \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix} \begin{Bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \sigma_{rz} \end{Bmatrix} \quad \text{(A-11)}$$

where the effective creep has the same meaning as before. However, the effective stress is redefined by

$$\sigma_e = \frac{1}{\sqrt{2}} [(\sigma_{rr} - \sigma_{\theta\theta})^2 + (\sigma_{\theta\theta} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{rr})^2 + 6\sigma_{rz}^2]^{1/2} \quad \text{(A-12)}$$

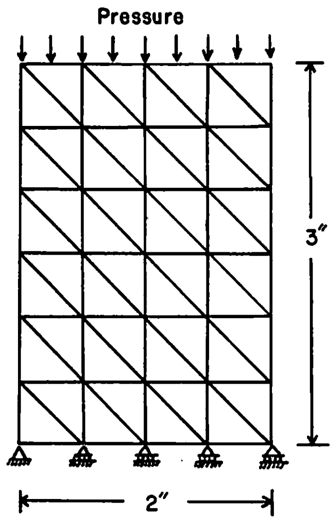

Fig. 1 Uniform Stretch of A thin Plate.

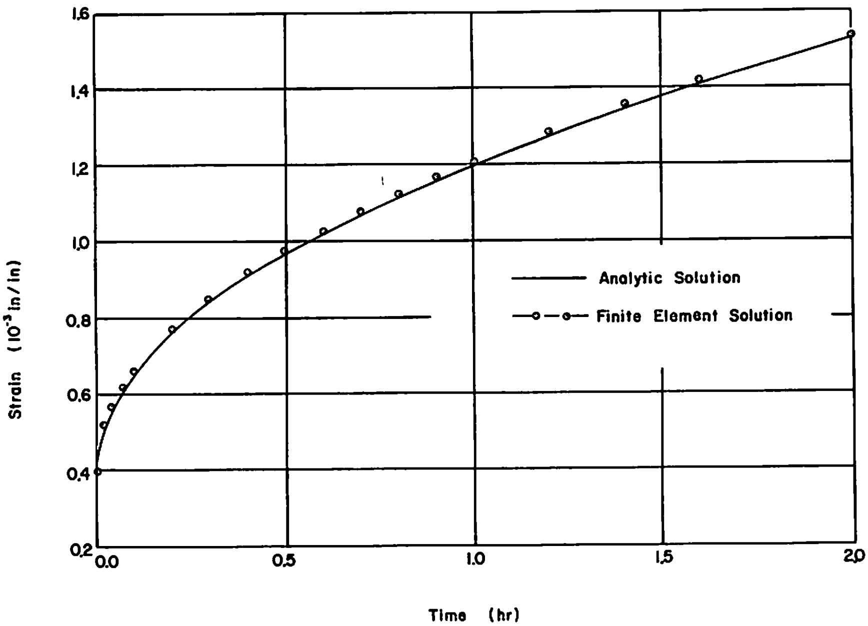

Fig. 2 Creep of an Aluminum - 1100 plate under a stress of 3700 psi and a temperature of 220° C.

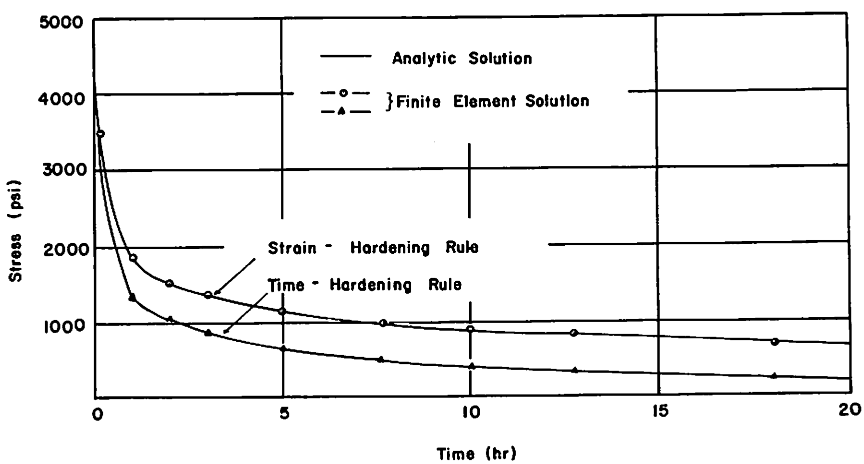

Fig. 3 Creep strain under constant stress rate, R=5000 psi/hr.

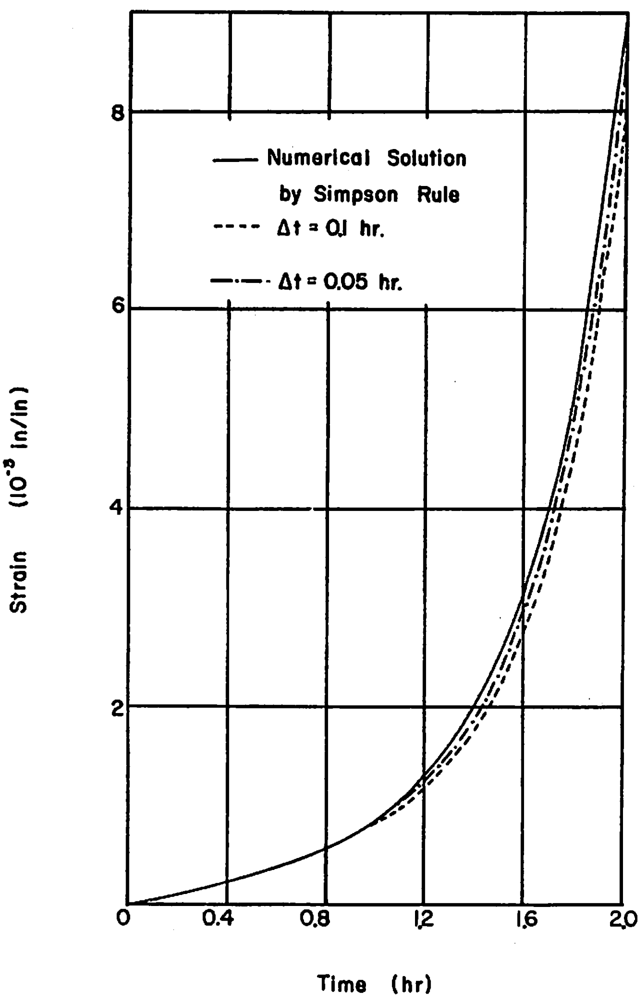

Fig. 4 Relaxation Curves.

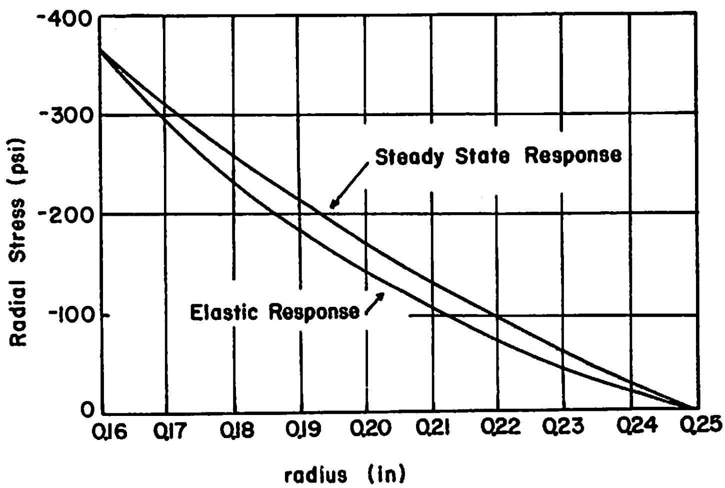

Fig. 5 Radial stress distribution.

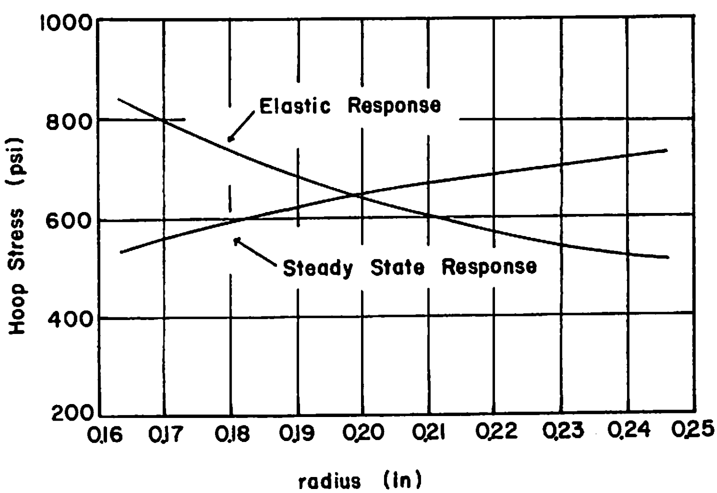

Fig. 6 Hoop stress distribution.

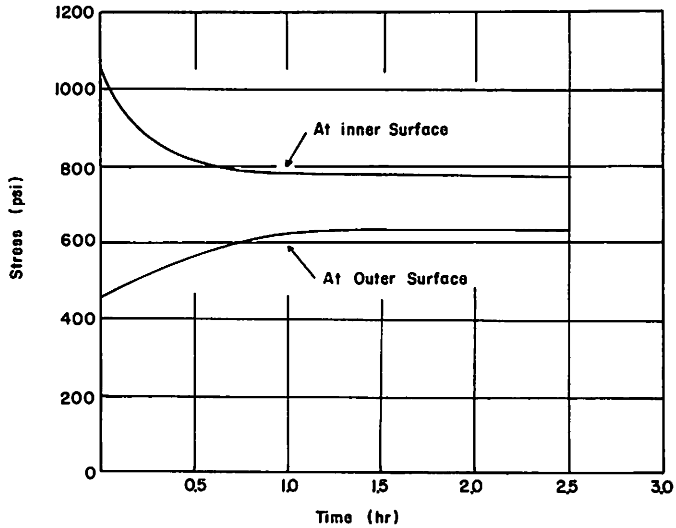

Fig. 7 Effective stress history.

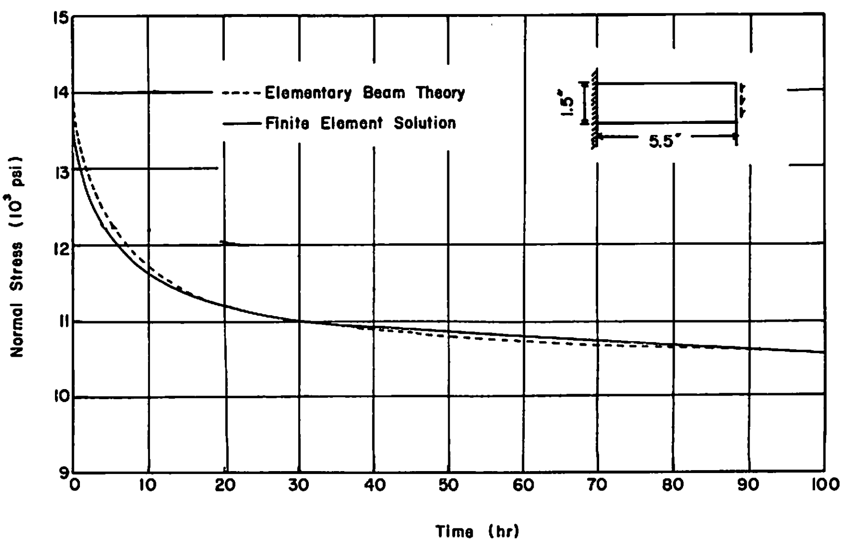

Fig. 8 Maximum normal stress history near the built-in end of the beam.

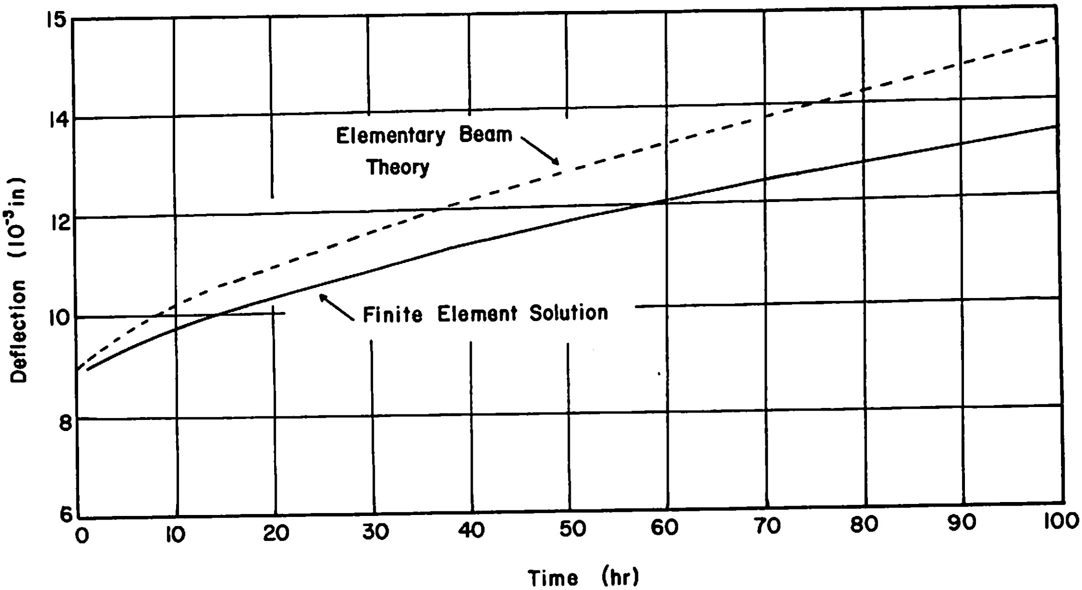

Fig. 9 Deflection history at the free end of the beam.

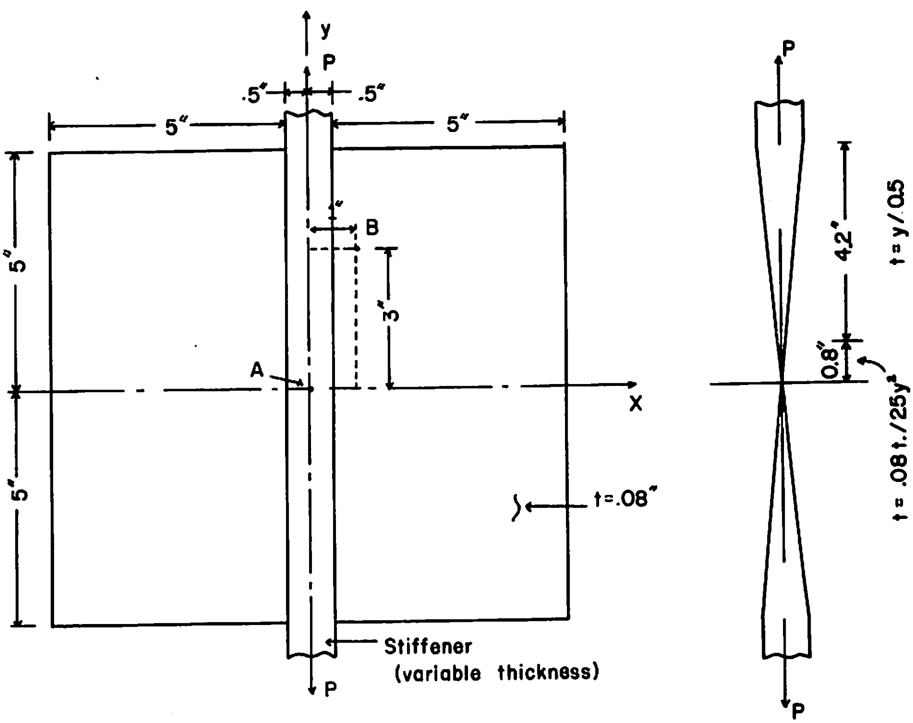

Fig. 10 A shear lag specimen.

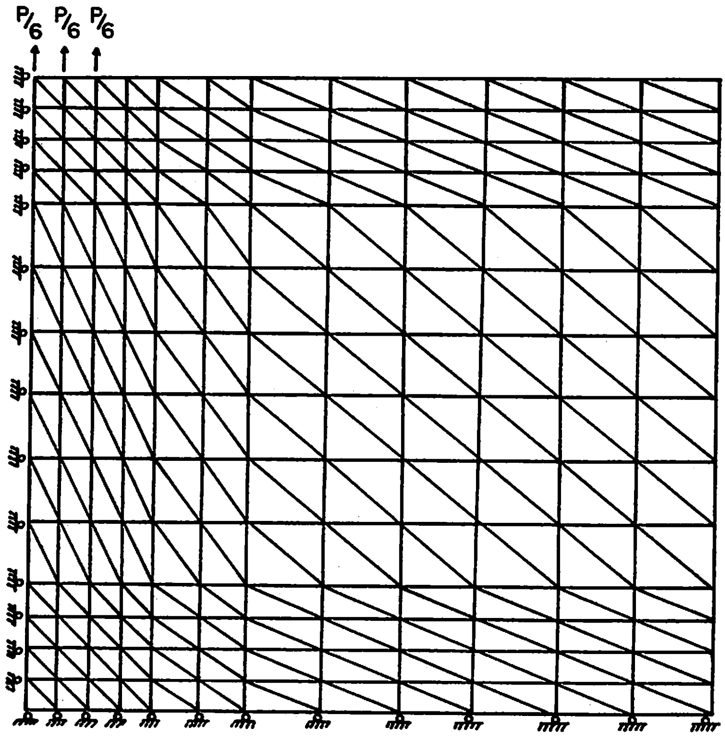

Fig. 11 Finite element model for the shear lag.

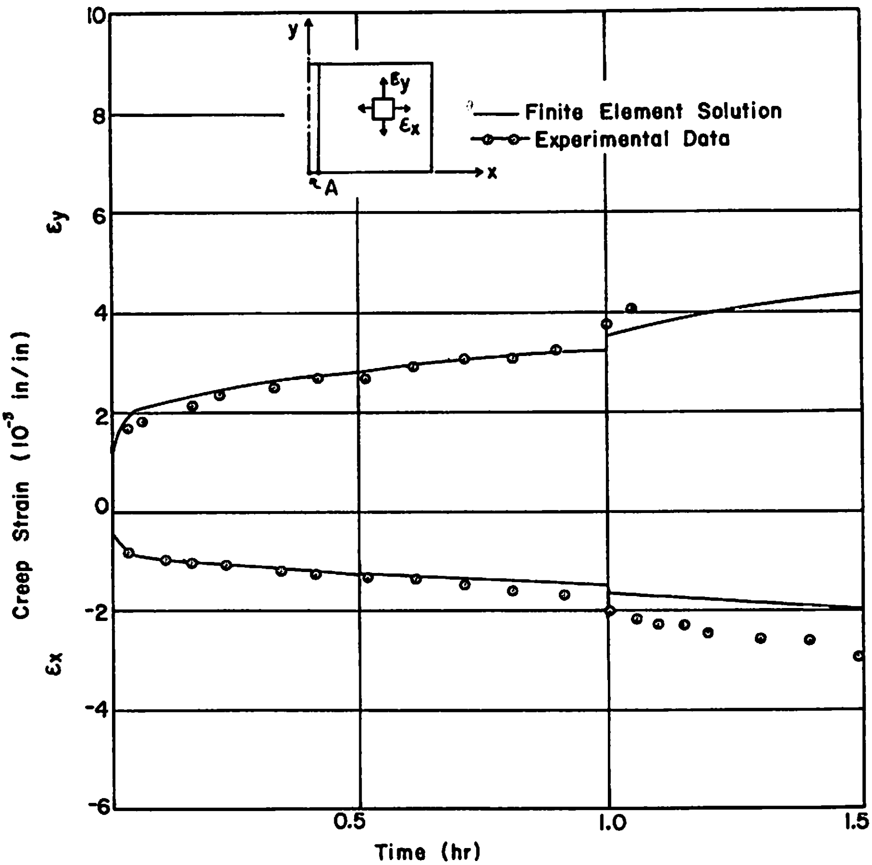

Fig. 12 Strain history at location A (center of the plate).

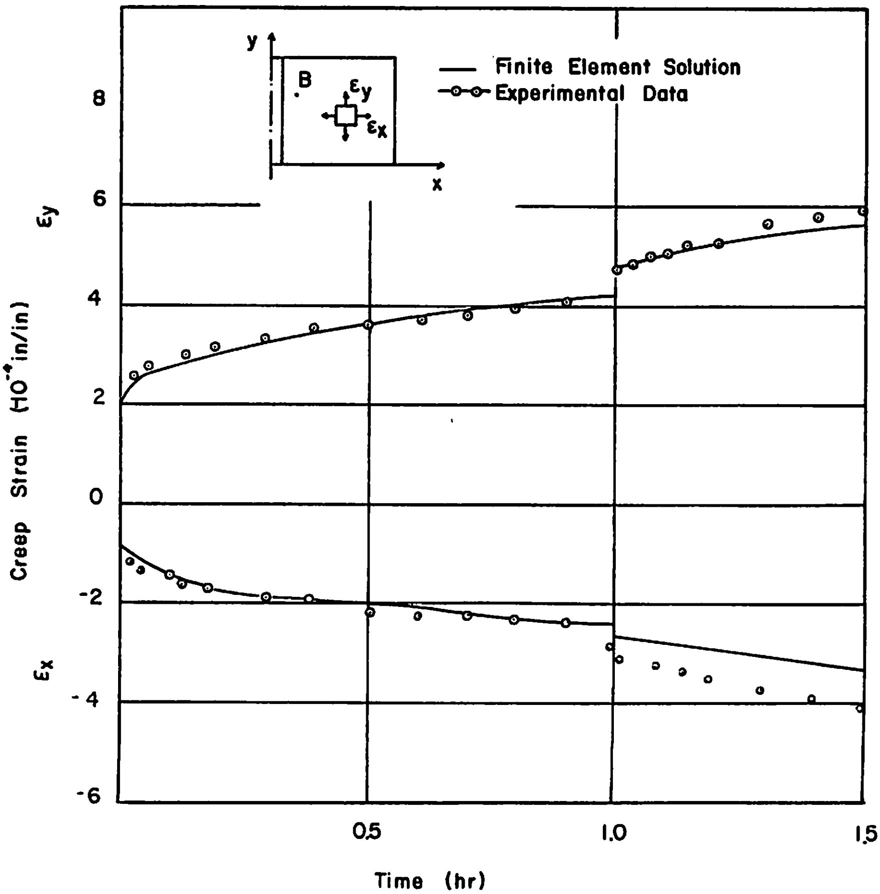

Fig. 13 Strain history at location B (x=1", y=3").

DISCUSSION

Q

T. MALMBERG, Germany

You mentioned that the heavy steel section is penetrated by a series of holes and that you adjusted the elastic and the creep material parameters in such a way that the axisymmetric finite element calculation accounts for this fact. Could you comment on this ?

A

T. Y. CHANG, U. S. A.

In our creep analysis the material properties were adjusted to account for the penetration effect in a similar manner as it was customarily done for an elastic analysis. This adjustment may cause an unrealistic stress state around the holes. However, since our primary interest is to find the stress history near the plate-cylinder junction where is considered to be the critical area and also is some distance away from the penetration, we feel that the approach we have used would provide useful results for design purposes.