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DESIGN OF MILD STEEL STRUCTURES UNDER LOW-ENDURANCE FATIGUE CONDITIONS

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ABSTRACT

The response of determinate, mild steel beams is observed under conditions of fully reversed moment where strain and load are controlled respectively. Moment-curvature models are developed for the two cases under consideration paying particular attention to creep effects which arise in the load control situations. These models are used to check the response of cantilever beams to tip loads.

INTRODUCTION

Many investigations [1-3] have been carried out to examine the behaviour of metals under low endurance fatigue conditions; these investigations have established the existence of a unique coupling between the value of cyclic life, N_f, and true plastic strain-range through an equation of the type,

$$\Delta e_p \gamma N_f = C \tag{1}$$

The investigation of material behaviour under conditions of low endurance fatigue has been more extensive than the study of this type of behaviour on structural components. A survey of these studies reveal that very little attempt has been made to correlate the material behaviour with the structural performance. The work after Royles [4] throws some light on the response of simple and continuous beams under a cyclic constant deflection range, formulating empirical moment-curvature relationships for mild steel beams subjected to alternating deflections. This work has been applied to predicting the behaviour of structural components by using values of moment-amplitude and strain-range (used to construct the moment-curvature relationship) pertaining to the half life state of a single specimen under pure reversed bending. This is justified by the argument that the specimen will be in a state unaffected either by initial transient phenomena (residual stresses) or the presence of cracks. The authors [5] in their earlier work have developed an analytical technique to predict the performance of structural components under reversed bending at various states of its cyclic life; the method was based upon developing cyclic moment-curvature relationships which are discrete functions of instantaneous cyclic history. However, these investigations deal only with the prediction of load-deformation behaviour of structural components under fully reversed deflections. The work will be complete and more practical if it is also extended to embrace the behaviour of structural components under a constant load range. As most of the studies reported on this subject are inconclusive, the present investigation is aimed at studying the failure of mild steel beams under low-endurance fatigue conditions as influenced by the following variables:

- (a) type of loading,
  - i) pure bending, and ii) moment gradient.
- (b) type of control,
  - i) load, and ii) deformation.

The loading on the beam is so chosen to produce failure at about 10^5 cycles. Cyclic moment-curvature and failure criteria are established from cyclic tests on beams under pure bending. These relationships are then employed in predicting the load-deformation behaviour and failure of cantilever beams under tip loads. A digital computer programme is developed which, given the distribution of load, geometry of the components and type of control, will predict the load-behaviour characteristic of structural components.

MATERIAL PROPERTIES AND TEST SPECIMENS

Fully killed 1020 steel was used in making the pure bending and cantilever beam specimens. The chemical properties are tabulated below.

S_i	S	P	Mn	C	Cn	Fe
0.037	0.025	0.004	0.47	0.143	0.01	Remainder %

The pure bending and cantilever specimens used in the cyclic tests are sketched in Fig. 1. The pure bending test piece had a uniform section ¼" square and approximately 4" long about the middle section; the two ends were larger in cross-section. The transition from the middle section to the end was uniform and smooth to avoid stress concentrations. The cantilever beam specimens were approximately 12" long and ½" and ¼" in cross-section. The built-in end of the beam was 2½" long and ½" x 1" in cross-section. The smooth transition between these two portions reduces stress concentration. The span of the cantilever beam is 6½".

The specimens were cut roughly to the required size. They were then soaked at 1600°F for about 30 minutes and allowed to cool in air. Great care was taken in machining to reduce the contact stresses. A finish of four micro-inches was attained in order to achieve a fairly correct estimate of life.

MECHANICAL PROPERTIES

Axial tension tests were carried out using an Instron Universal Testing Machine on two pieces selected at random from the specimens in order to evaluate Young's Modulus, upper and lower yield stress, yield strain and range of ductility. A representative curve is given in Fig. 2(a) and typical values are indicated.

Two beams, 9" long by ¼" square, were subjected to four point loading to determine the lower yield stress in bending. The specimens were ground and subjected to the same heat treatment as the pure bending and cantilever specimens. A special bending rig was used to adapt the Instron machine for this type of test. The test was halted when the load deflection curve in Fig. 2(b) became flat. The average lower yield stress in bending was found to be 33.3 ksi.

TEST EQUIPMENT AND PROCEDURE

The experiments were carried out in a servo-controlled testing machine. The system was augmented by special bending fixtures and transducers suitable to control either load or deformation. The testing machine and the various fixtures are fully explained elsewhere [6].

For the pure bending tests described here the fixtures shown in Figs. 3(a) and 3(b) were used. The rig in Fig. 3(b) is a modified version of the fixture in Fig. 3(a). Curvature was measured by a transducer shown in Fig. 4(a).

The fixture in Fig. 4(b) was used to test the cantilever specimens. The load on the specimen was measured by the load cell and the measurement of the deflection was achieved by an LVDT (Linear Voltage Displacement Transducer), which is an integral part of the testing system. A strip-chart recorder and X-Y plotter were used to record the load and deformation, respectively.

EXPERIMENTS

With the experimental set-up described previously, it is possible to undertake an examination of the behaviour of mild steel beams under fully reversed bending in the inelastic range. Fig. 5 represents the type of loading and control imposed on the test specimens.

Pure Bending:

It has been established by Topper [7] that the stress-strain characteristic of mild steel under cyclic axial load fall into three categories: i) purely elastic, ii) in the range between the yield point and ± 2.5 percent strain, and iii) above ± 2.5 percent strain. From reversed bending tests, Royles [4] has concluded that the influence of axial extension is predominant when the strain exceeds ± 2.5 percent and that the moment amplitude values have to be adjusted accordingly. After considering the results obtained by these investigators, the study in this paper was limited to strains varying from about ± 0.1 percent to ± 2.5 percent.

Eight strain-controlled and seven moment-controlled tests were carried out on mild steel specimens under reversed pure bending; the controlled strain varied from ± 0.1 percent to ± 2.2 percent while the applied bending moment varied from ± 90.0 inch lbs. to ± 160.00 inch lbs. A typical output loop is given in Fig. 5, for each case.

The cyclic variation of the peak values of the moment amplitudes and strain amplitudes were noted for strain controlled and moment controlled tests, respectively. The failure of the specimens was also observed.

Varying Bending (Cantilever Beam):

These tests were carried out on cantilever beams with rectangular cross-section; both deflection and load controlled tests were performed. The controlled deflection varied from ± 0.1 inch to ± 0.7 inch and the controlled load on the beams ranged from ± 72 lbs. to ± 140 lbs.

The variation of the load and deformation was observed for deformation and load controlled tests, respectively. Failure of the beams was also observed.

The test speed varied from 3 to 30 cpm depending upon the amount of load or deformation imposed on the beams.

ANALYSIS OF TEST RESULTS

Pure Bending (Strain Control): The peak values of the moment amplitudes from the tests are plotted against the number of cycles as indicated in Fig. 6(a). It can be seen from the plot that after an initial period of either softening or hardening, the beams eventually settle down to a "stable" condition until cracks develop to such an extent that the load carrying capacity of the specimen is reduced. The "stable" state usually extends from 25 to 75 percent of the life to failure (N_f). Hence, a moment-curvature curve, Fig. 6(b), can be constructed from Fig. 6(a), by plotting the moment and the corresponding strain amplitudes pertaining to the half-life state for several different beams. This curve is based on the assumption that the cross-section of the beam remains constant until significant cracking develops to reduce substantially the sectional area. The experimental points can be represented by a mathematical function of the form,

$$k = a_1(m)^{b_1} \tag{2}$$

where $k = k/ky$; $m = \frac{M}{My}$ and a_1 and b_1 are geometrical constants which can be computed from Fig. 6(b).

Pure Bending (Moment Control): The cyclic variation of strain range in the beams under fully reversed moment is plotted in Fig. 7(a). When the moment range is held constant initially the material either softens or hardens, as seen in the case of strain cycling, and then settles down to a "stable" condition. However, unlike the case of strain control tests, the material "creeps" under constant moment range cycling as shown in Fig. 5(b). This, hereafter, will be referred to as "cyclic creep". A curve coupling moment amplitude and the corresponding maximum strain amplitude, Fig. 7(b), pertaining to the half life stage can be constructed to represent the material behaviour under constant moment cycling. It can be represented mathematically by a power law,

$$k_{max} = a_2(m)^{b_2} \tag{3}$$

where $k_{max} = \frac{k_{max}}{ky}$, $m = \frac{M}{My}$ and a_2, b_2 are geometrical constants which can be computed from Fig. 7(b).

It is important to note the difference between Eqs. (2) and (3). Since no "cyclic creep" occurs in the case of strain cycling, maximum and minimum moment amplitudes are equal in magnitude. Thus, a curve (Eq. 2) coupling moment amplitude ($\frac{\Delta m}{2}$) and strain (curvature) amplitude will be sufficient to represent the material behaviour. But, due to the presence of "cyclic creep" in the case of moment cycling, maximum and minimum strain amplitudes are not equal in magnitude. Hence maximum strain amplitude is coupled with moment amplitude as in Eq. (3).

Varying Bending (Cantilever Beam):

The moment curvature relationships [Eqs. (2) and (3)] are now employed in predicting the load-deformation behaviour of cantilever beams; both constant deflection and constant load cycling problems are investigated. The analytical method to derive the load-deformation behaviour is described elsewhere [6] in detail and will not be included here.

It can be seen that cantilever beams also shake down to a "stable" state after an initial period of softening or hardening (Figs. 8 and 9). Hence, it is possible to construct a "stable" load-deformation curve from values pertaining to the half life states for different beams.

The moment-curvature relationship of Eq. (2) is employed in predicting the load-deformation curve for cantilever beams under constant strain cycling and compared with experiments [Fig. 10(a)]. The load-deformation characteristic is also derived for cantilever beams under moment control by applying Eq. (3) and compared with experiments as in Fig. 10(b). The theory is seen to correlate reasonably well with experiments.

DISCUSSION AND CONCLUSIONS

The load versus deformation response of mild steel beams under low cycle fatigue loading is studied. Two types of control are imposed on the beams: i) load control, and ii) deformation control.

It has been observed that beams under cyclic load settle down to a "stable" state, which usually extends from 25 percent to 75 percent of the life of failure. When beams are subjected to load cycling, "cyclic creep" was observed as indicated in Figs. 7(a) and 9(a). Moment-curvature relationships pertaining to the half life stage are derived from experiments on beams under pure bending. These relationships are applied to predicting the "stable" load-deformation behaviour of cantilever beams under load and deformation control. Use of moment-curvature relationships pertaining to the half life stage of various beams are justified by the argument that the specimen will be unaffected either by initial transient phenomena (residual stresses) or the presence of cracks.

A digital computer programme is written to facilitate the computations. The experiments correlate well with theoretical predictions.

ACKNOWLEDGEMENT

This investigation was carried out in the Department of Civil Engineering, University of Waterloo, under National Research Council of Canada Grant No. A-1582.

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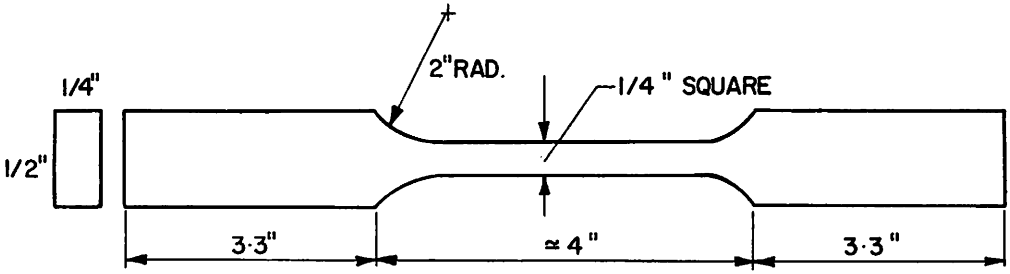
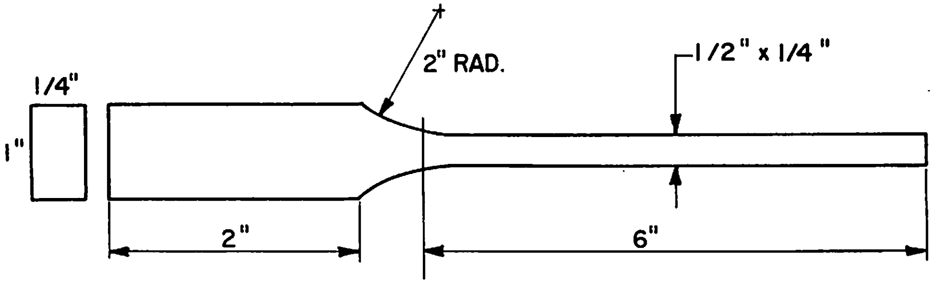


(a) PURE BENDING SPECIMEN



(b) CANTILEVER SPECIMEN

FIG. I Specimens

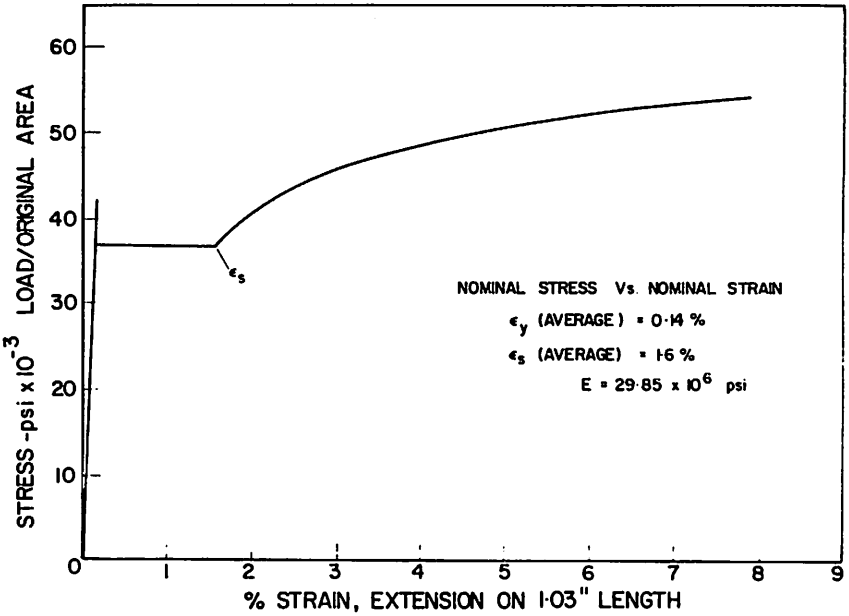


FIG. 2 (a) Stress-Strain Relationship

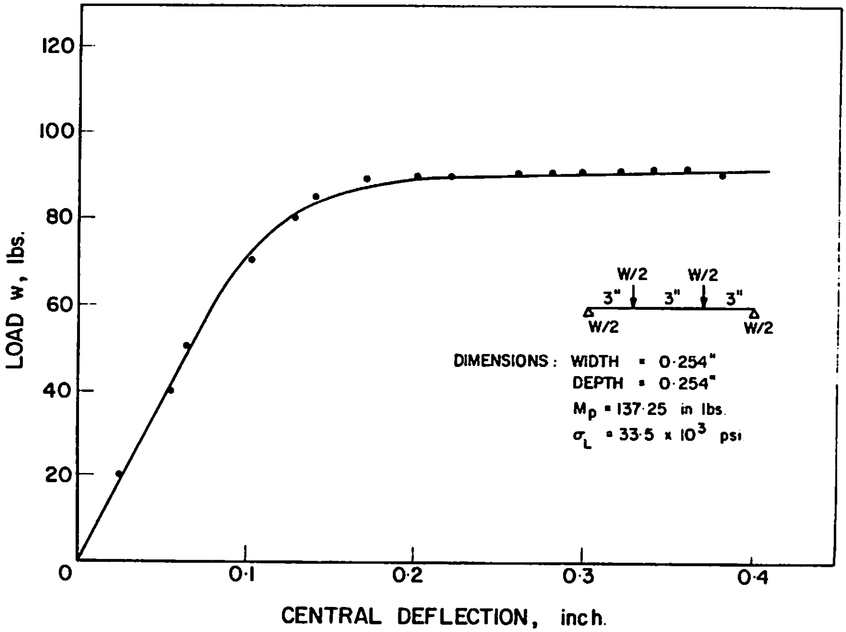


FIG. 2 (b) Load-Deflection Relationship

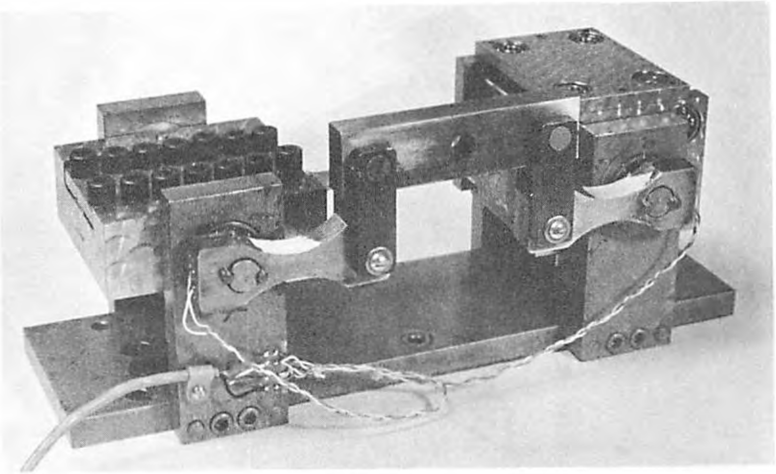


FIG. 3 (a) Pure Bending Fixture - Original

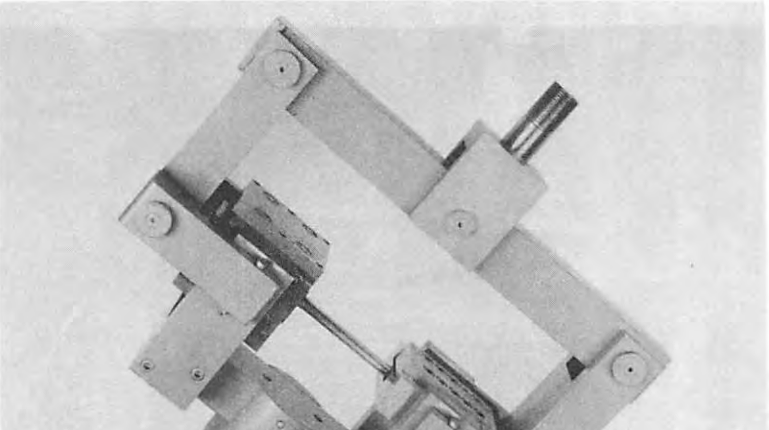


FIG. 3 (b) Pure Bending Fixture - Modified

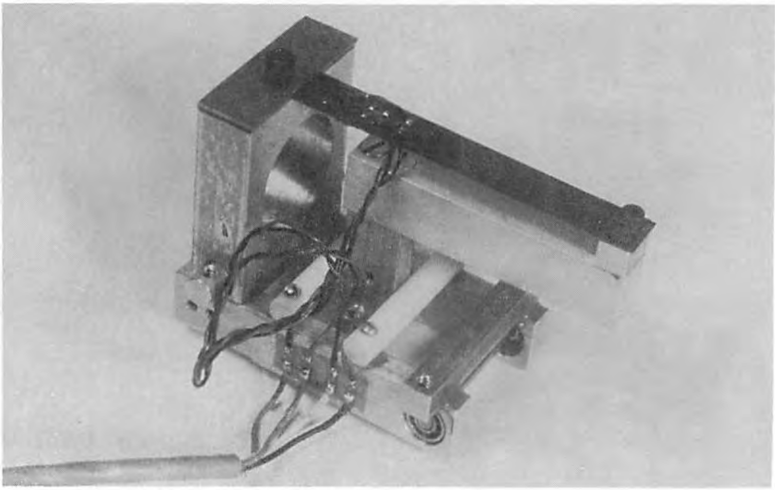


FIG. 4 (a) Curvature Meter

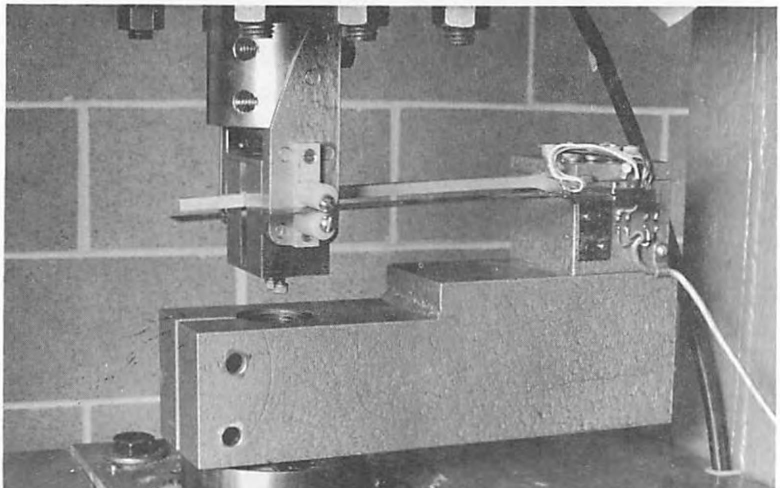
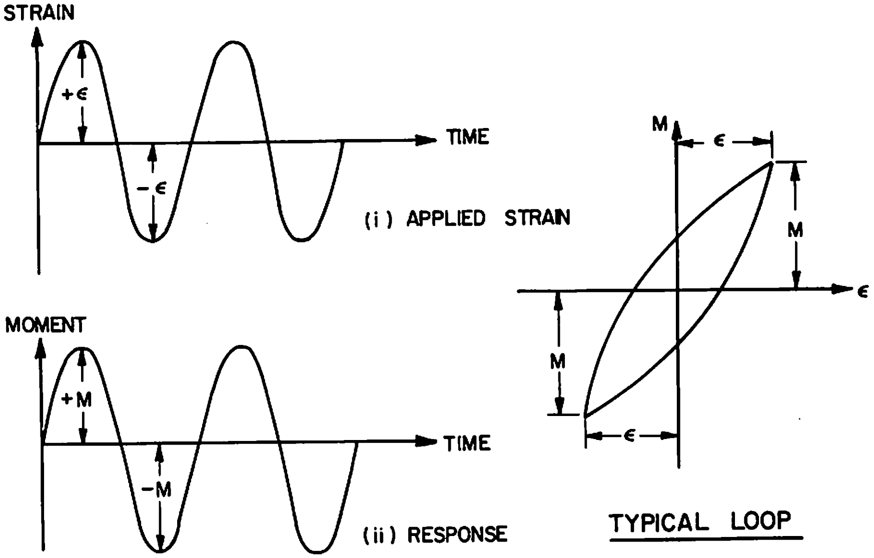
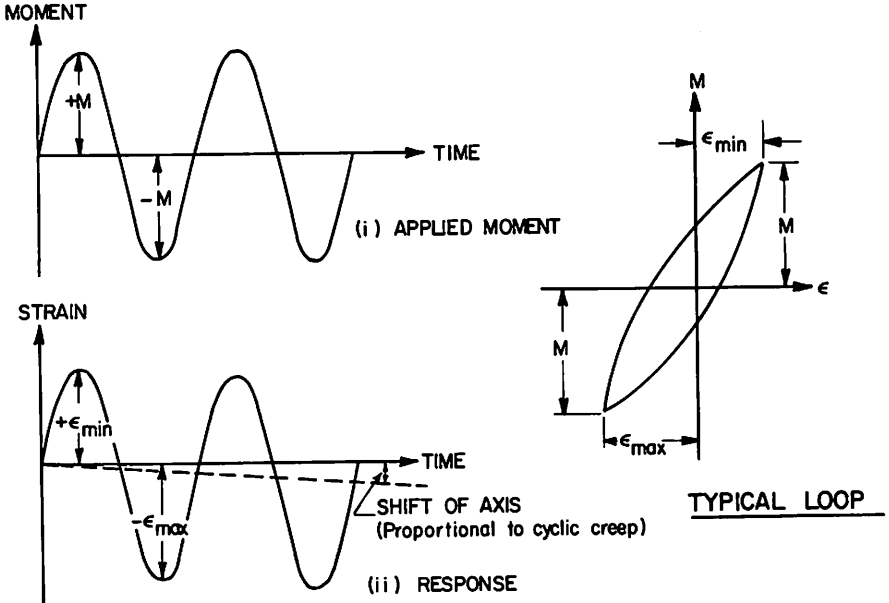


FIG. 4 (b) Cantilever Beam Fixture



(a) FULLY REVERSED STRAIN



(b) FULLY REVERSED MOMENT

FIG. 5 Controlled Strain and Moment Inputs

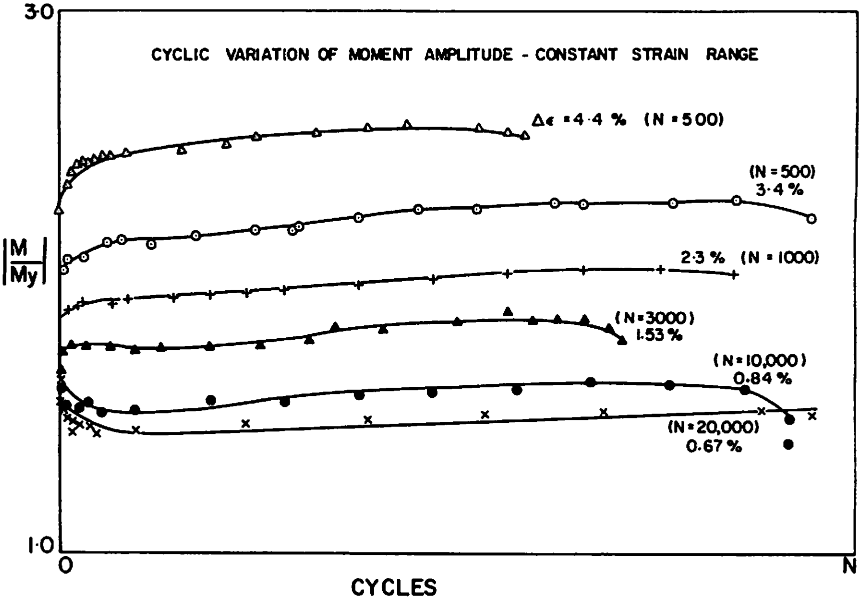


FIG. 6 (a) Cyclic Variation of Moment Amplitude - Constant Strain Range

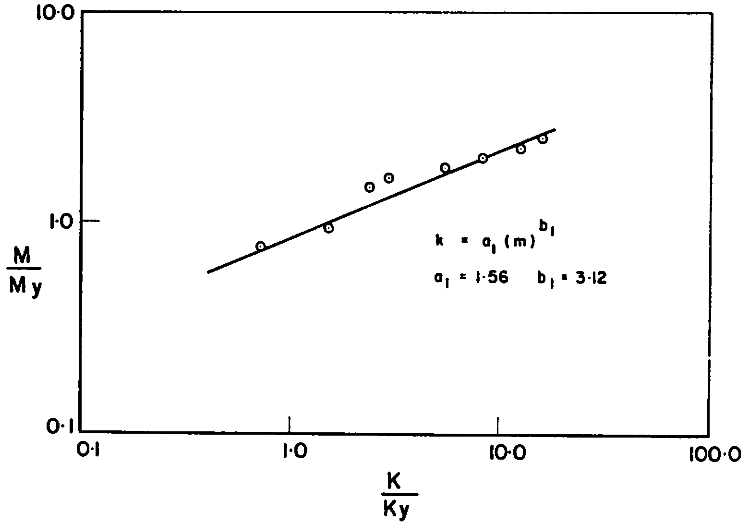


FIG. 6 (b) Stable Moment-Curvature Relationship - Half Life Values from Constant Strain Range Tests

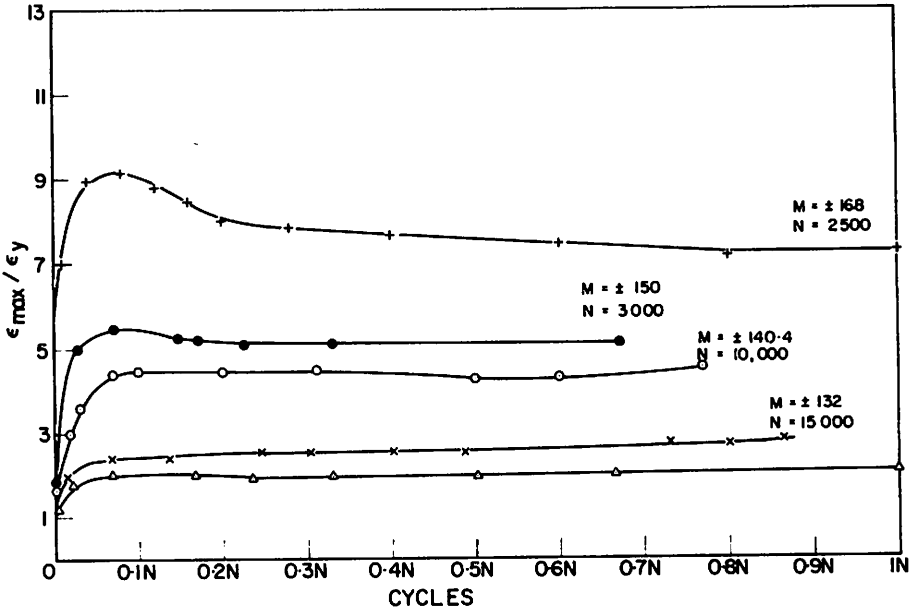


FIG. 7(a) Cyclic Variation of Strain Amplitude - Constant Moment Range

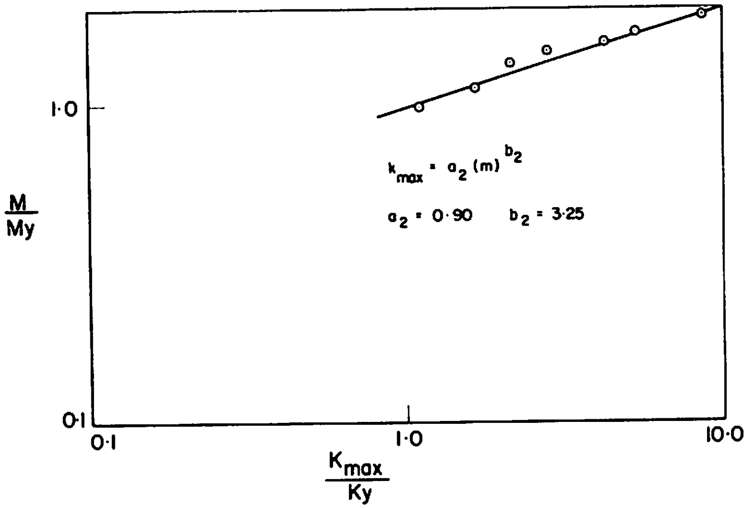


FIG. 7 (b) Stable Moment-Curvature Relationship - Half Life Values from Constant Moment Range Tests

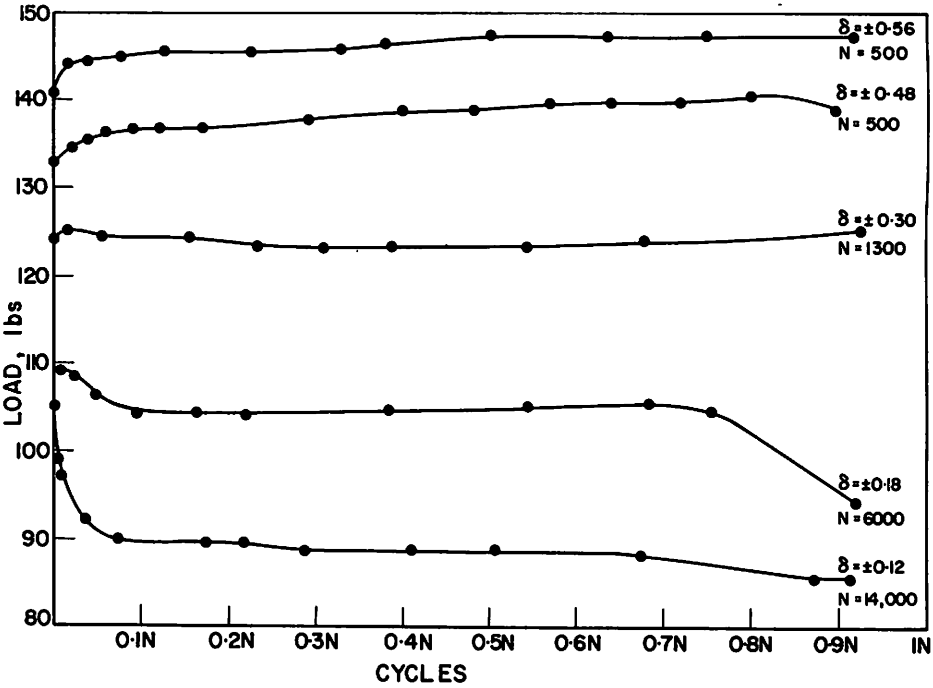


Fig. 8 Cyclic Variation of Load under Constant Deflection Range

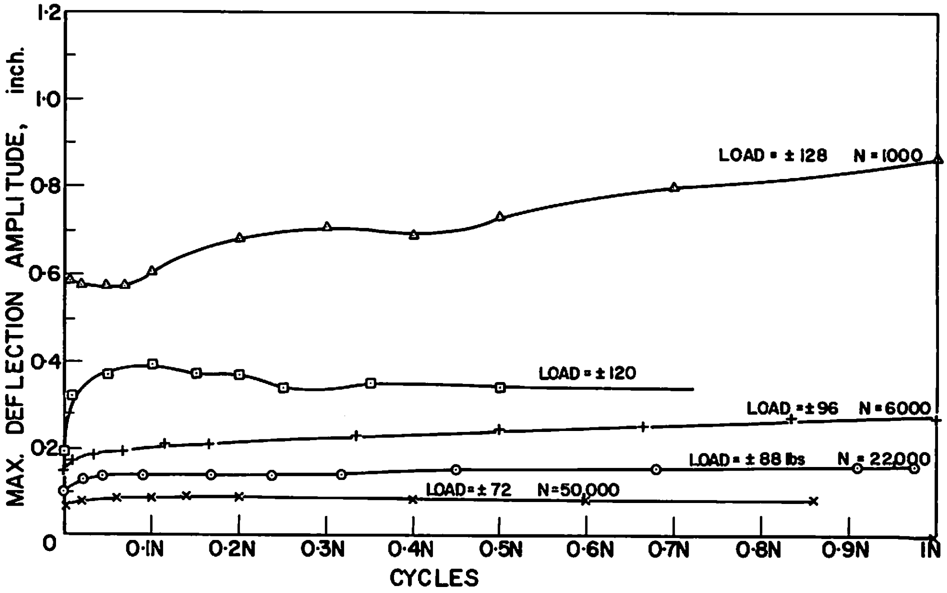


Fig. 9 Cyclic Variation of Deflection under Constant Load Range

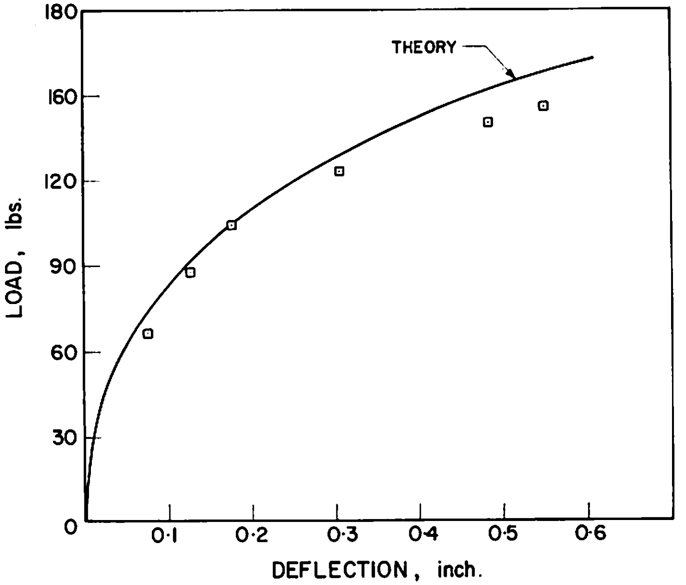


FIG. IO (a) Load-Deflection Plot for Cantilever Beam under Constant Deflection Range

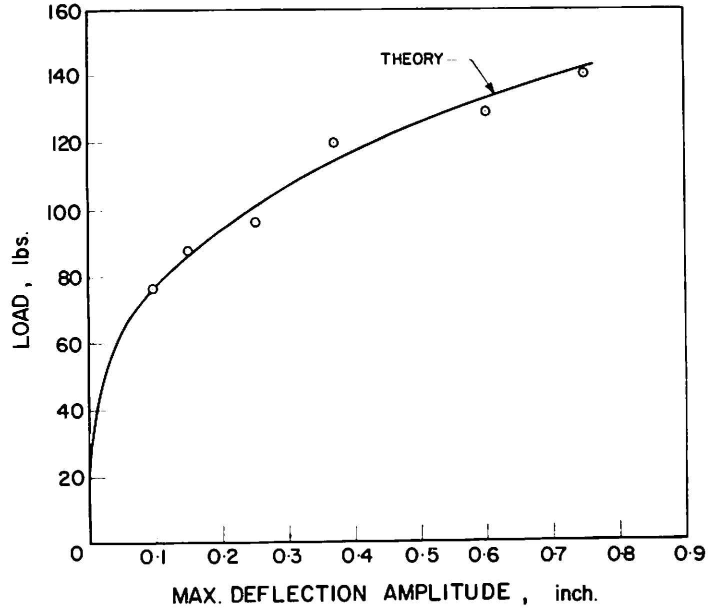


FIG. IO (b) Load-Deflection Curve for Cantilever Beam under on Constant Load Range