A FRACTURE MECHANICS APPROACH TO THE PROPAGATION OF FATIGUE CRACKS IN REACTOR COMPONENTS BY ACOUSTICALLY INDUCED STRESSES

K. JERRAM,

Central Electricity Generating Board,
Berkeley Nuclear Laboratories, Berkeley, Gloucestershire, United Kingdom

ABSTRACT

Fatigue crack propagation caused by high frequency cyclic stresses whose peak values conform to a Rayleigh distribution has been theoretically investigated. It is shown that the two crucial crack propagation parameters for predicting crack propagation under such conditions are the minimum propagation rate and the criterion for non-propagation. Simple formulae are derived for the permitted r.m.s. stress and the permitted life for a component containing a crack. The simple formulae are shown to give values in good agreement with those calculated using a crack propagation summation analysis for an edge cracked beam subjected to more than 5×10^{10} random amplitude stress cycles.

NOMENCLATURE

a	crack length
a_1	initial value of crack length
a_2	final value of crack length
C	material constant (see equations 1 and 4)
$f(a)$	function of crack length relating stress intensity factor to stress
K	stress intensity factor
ΔK	stress intensity factor range
ΔK_o	critical stress intensity factor range below which no crack propagation occurs
$K_{\sigma 1}$	r.m.s. stress intensity factor at crack length a_1
$K_{\sigma 2}$	r.m.s. stress intensity factor at crack length a_2
n	exponent (see equation 4)
N	number of cycles
N_{12}	number of cycles required for crack to propagate from length a_1 to a_2
$p(x)$	probability of a stress peak lying between x and $x + \delta x$
r_o	minimum possible propagation rate ($= C\Delta K_o^n$)
x	magnitude of positive stress peak
σ	r.m.s. stress

1. INTRODUCTION

The effect of the presence of a crack-like defect in a structure tends to be assessed by whether it may cause failure by fast or brittle fracture, either on the initial loading of the structure or after the defect has been propagated by fatigue loading. Fatigue crack propagation has, therefore, received a considerable amount of attention in recent years.

The fracture mechanics approach of Paris and Erdogan [1] seems to be the most suitable method for assessing fatigue crack propagation rates. They showed that, for repeated tension loading and quite a wide range of stress intensity factors, the crack propagation rate was governed by the equation:

$$\frac{da}{dN} = C\,\Delta K^4 \tag{1}$$

Equation (1) implies that fatigue crack propagation will occur for all cyclic loadings, no matter how small the value of ΔK. However, non-propagating fatigue cracks occur in practice. Forst [2], [3], [4] and Frost and Greenan [5], [6] have extensively investigated criteria for fatigue crack propagation. From experimental work, Frost concluded that fatigue cracks propagated only if a critical value of the parameter $\sigma^3 a$ was exceeded, the actual value being dependent upon the material. Recently Frost, Pook and Denton [7] have analysed published data using fracture mechanics concepts, and shown that no fatigue crack propagation occurs if the range of stress intensity factor is less than a critical value ΔK_o.

It may, alternatively, be postulated that the equation governing fatigue crack propagation is of the form:

$$\frac{da}{dN} = C\,(\Delta K - \Delta K_o)^n \tag{2}$$

Taking n as 4, ΔK_o as 3 ksi $\sqrt{in}$, and C as 10^{-11} klbf^{-4} in^7 per cycle, equations (1) and (2) are compared in Figure 1 with experimental scatter bands of fatigue crack propagation of a large number of steels (Crooker and Lange [8]). In the region for which the bulk of the experimental data was collected there is little difference between the two equations, both adequately representing the observed behaviour. However at low values of ΔK, the propagation rates predicted by the two equations differ significantly. For instance, at $\Delta K = 4$ ksi $\sqrt{in}$, equation (1) predicts a propagation rate 300 times greater than that derived from equation (2).

At present there are insufficient experimental data on fatigue crack propagation to determine whether equation (1) or equation (2) best represents fatigue crack propagation behaviour. For design purposes, it is therefore prudent to use equation (1) (with a cut-off at ΔK_o), as it predicts higher propagation rates than equation (2).

In the gas circuits of gas cooled nuclear reactors it has been demonstrated (Rizk and Seymour [9], Armstrong, Lawson, Townley and White [10]) that components vibrate at frequencies in the order of 1 kHz with the magnitudes of individual stress peaks following a Rayleigh distribution. It is possible to estimate fatigue crack propagation caused by random amplitude stresses by carrying out crack propagation summation calculations, assuming that each stress peak causes the same amount of propagation as it would under constant amplitude cycling. The calculations involve dividing the stress spectrum into a series of small increments, evaluating the amount of propagation caused in each and summing all such propagation increments to obtain the total fatigue crack propagation. In this paper easily usable approximate equations are derived for estimating fatigue crack propagation caused by very large numbers (10^{11} - 10^{12}) of Rayleigh distributed stress peaks. The equations are suitable for initial assessment of the acceptability or otherwise of cracks or crack like defects. In a borderline case, the full crack propagation analysis might, however, still be required.

2. ANALYSIS

Consider a crack in a body subjected to random stresses of Rayleigh distribution of peaks. At a point on the body where the r.m.s. stress is σ the probability of a positive stress peak lying between x and x + δx is

$$p(x) = \frac{x}{\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right) \delta x \tag{3}$$

Fatigue crack propagation is assumed to be governed by equation 1 for values of ΔK exceeding the critical values of ΔK. Hence

$$\begin{aligned} \frac{da}{dN} &= C\,(\Delta K)^n \qquad \text{for } \Delta K > \Delta K_o \\ \frac{da}{dN} &= 0 \qquad\qquad\quad \Delta K < \Delta K_o \end{aligned} \tag{4}$$

(N.B. the exponent 4 of equation (1) has been replaced by n).

It is assumed that every positive peak precedes a negative peak of equal magnitude and that under variable amplitude loading, a single peak will cause the same propagation as it would in constant amplitude loading. In addition the stress intensity factor for the crack is taken to be related to the stress peak x by the equation:

$$K = x\, f(a)$$

Hence

$$\Delta K = 2x\, f(a) \tag{5}$$

Crack propagation will be zero for $x < \dfrac{\Delta K_o}{2f(a)}$

By combining equations (3), (4) and (5) the propagation rate at a crack length a is given by

$$\frac{da}{dN} = \int_{x=\frac{\Delta K_o}{2f(a)}}^{\infty} C\left[\frac{x}{\sigma^2}\right]\left[2x\, f(a)\right]^n \exp\left(\frac{-x^2}{2\sigma^2}\right) dx \tag{6}$$

Hence

$$\frac{da}{dN} = -C\,\{2\sigma\, f(a)\}^n e^{\frac{-x^2}{2\sigma^2}} \Bigg\{ \left(\frac{x}{\sigma}\right)^n + n\left(\frac{x}{\sigma}\right)^{n-2} + n(n-2)\left(\frac{x}{\sigma}\right)^{n-4} + \dots$$

$$+ n(n-2)(n-4)\dots(n+4-2r)\left(\frac{x}{\sigma}\right)^{n+2-2r} + \dots \Bigg\}_{x=\frac{\Delta K_o}{2f(a)}}^{\infty}$$

$$= C\, e^{-\frac{1}{2}\left[\frac{\Delta K_o}{2\sigma f(a)}\right]^2} \Bigg\{ (\Delta K_o)^n + n\left[2\sigma f(a)\right]^2 (\Delta K_o)^{n-2} +$$

$$n(n-2)\left[2\sigma f(a)\right]^4 (\Delta K_o)^{n-4} + \dots \Bigg\}$$

$$\frac{da}{dN} = C(\Delta K_o)^n e^{-\frac{1}{2}\left[\frac{\Delta K_o}{2\sigma f(a)}\right]^2} \left\{1 + n\left[\frac{2f(a)}{\Delta K_o}\right]^2 + n(n-2)\left[\frac{2\sigma f(a)}{\Delta K_o}\right]^4 + \dots\right\} \tag{7}$$

Now $\frac{\Delta K_o}{2f(a)}$ = critical peak/r.m.s. ratio above which crack propagation will occur.

The limited data available for steels indicate that the crack propagation rate at stresses just above the non-propagating level is of the order of 4×10^{-8} cm/cycle (10^{-9} in/cycle). In most applications the amount of crack propagation that can be tolerated is likely to be less than 25 mm (1 in.). Where very large numbers of stress cycles are considered ($> 10^{11}$), it is thus apparent that fewer than 1% of the stress peaks can be above the critical level and actually cause propagation. For a Rayleigh distribution about 1% of peaks exceed a peak/r.m.s. ratio of 3, and about 0.1% of peaks exceed a peak/r.m.s. ratio of 4. The value of the exponent n in the crack propagation equation is usually about 4, although it can vary between about 2 and 6. Now if n is an even integer, the integral of equation (6) becomes exact, and only the first $(\frac{n}{2} + 1)$ terms of equation(7)are required. Thus in the bracket $\{1 + n\,[\frac{2f(a)}{\Delta K_o}]^2 + n(n-2)\,[\frac{2\sigma f(a)}{\Delta K_o}]^4 + \dots\}$, taking the value of $[\frac{\Delta K_o}{2\sigma f(a)}]$ as 3, the unity term is the largest for all even integer values of n up to 6. If $[\frac{\Delta K_o}{2\sigma f(a)}]$ is taken as 4 (corresponding to 10^{12} cycles), the unity term is dominant for $n = 2$ and $n = 4$, and double the sum of the remaining terms for $n = 6$.

Hence to a first approximation

$$\frac{da}{dN} \simeq C(\Delta K_o)^n e^{-\frac{1}{2}\left[\frac{K_o}{2\sigma f(a)}\right]^2} \tag{7a}$$

The number of cycles, N_{12}, required to propagate a crack from a length a_1 to a length a_2 may be determined by integrating equation(7):

$$N_{12} = \int_{a_1}^{a_2} \frac{e^{-\frac{1}{2}\left[\frac{\Delta K_o}{2\sigma f(a)}\right]^2} da}{C(\Delta K_o)^n\left\{1 + n\left[\frac{2\sigma f(a)}{\Delta K_o}\right]^2 + n(n-2)\left[\frac{2\sigma f(a)}{\Delta K_o}\right]^4 + \dots\right\}} \tag{8}$$

Providing that the stress intensity factor within the range from a_1 to a_2 is never greater than the value at a_2, a lower bound to N is given by:

$$N_{12} = \frac{(a_2 - a_1)\, e^{\frac{1}{2}\left[\frac{\Delta K_o}{2\, f(a_2)}\right]^2}}{C(\Delta K_o)^n\left\{1 + n\left[\frac{2\sigma f(a)_2}{\Delta K_o}\right]^2 + n(n-2)\left[\frac{2\sigma f(a_2)}{\Delta K_o}\right]^4 + \dots\right\}} \tag{9}$$

Making the last-mentioned assumption, equation (7a) may also be integrated:

$$N_{12} \simeq \frac{(a_2 - a_1)}{C(\Delta K_o)^n} e^{\frac{1}{2}\left[\frac{\Delta K_o}{2\sigma f(a_2)}\right]^2} \quad (9a)$$

Noting from equation (4) that $C(\Delta K_o)^n = r_o$ and from equation(5) that $\sigma f(a_2) = K_{\sigma 2}$ = stress intensity factor at a_2 crack length, equation (9) becomes

$$N_{12} \simeq \left(\frac{a_2 - a_1}{r_o}\right) e^{\left[\frac{\Delta K_o^2}{8K_{\sigma 2}^2}\right]} \quad (10)$$

For design purposes, the number of cycles is usually known, and it is required to calculate the permissible r.m.s. stress. Hence, rewriting equation (10):

$$K_{\sigma 2} = \frac{\Delta K_o}{2\left\{2 \log_e \left[\frac{r_o N_{12}}{a_2 - a_1}\right]\right\}^{\frac{1}{2}}} \quad (11)$$

and

$$\sigma = \frac{\Delta K_o}{2f(a_2)\left\{2 \log_e \left[\frac{r_o N_{12}}{a_2 - a_1}\right]\right\}^{\frac{1}{2}}} \quad (12)$$

Equations (10), (11) and (12) may thus be used for estimating fatigue crack propagation caused by very large numbers ($> 10^{11}$) of Rayleigh distributed stress peaks, (such as occur in nuclear reactor gas circuits), assuming that no crack propagation occurs below a critical stress intensity factor range, ΔK_o, and that above ΔK_o, the equation $\frac{da}{dN} = C(\Delta K)^n$ governs crack propagation.

It is interesting to note that the exponent n in the crack propagation equation does not appear in the simple equations derived, and that the minimum propagation rate, r_o, and the critical stress intensity factor range for propagation, ΔK_o are the important crack propagation parameters.

3. COMPARISON OF DERIVED FORMULAE WITH CRACK PROPAGATION SUMMATION CALCULATIONS

To assess the accuracy of the equations derived in the preceding section, comparison has been made with values calculated by crack propagation summation for a particular crack propagation law and geometry. Crack propagation has been assumed to obey the equations:

$$\frac{da}{dN} = 10^{-11} (\Delta K)^4 \qquad \text{for } \Delta K > 3 \quad (13)$$

and

$$\frac{da}{dN} = 0 \qquad \text{for } \Delta K < 3 \quad (14)$$

(These equations are typical of those for steel under a tensile mean stress, expressed in kilopound force and inch units.)

The geometry considered is a beam of unit depth and width, containing an edge crack initially of length 0.45 in., and subjected to a random amplitude bending stress of magnitude σ_{rms}.

The number of cycles required to propagate the crack from its initial length of 0.45 in. by 0.1 in. to a final length of 0.5 in. has been calculated using equations (13) and (14) in conjunction with the stress intensity factor at the mean crack length. (A stress intensity factor calibration for an edge cracked beam in bending has been presented by Brown and Srawley [11]). The random amplitude stress spectrum was divided into stress intervals corresponding to peak/r.m.s. ratio intervals of 0.1. For a given value of the r.m.s. stress the probability of a single stress peak occurring within a given peak/r.m.s. interval was determined from the Rayleigh distribution equation. The amount of crack propagation that would be caused by a single peak at the mean value of the interval considered was then determined from equations (13) and (14). The probable crack propagation per cycle was obtained by summing for all the peak/r.m.s. ratio intervals the product of the propagation per cycle and the probability of a single peak lying within the interval. The number of cycles required to propagate a crack by a given amount was the total propagation divided by the probable propagation per cycle.

The r.m.s. stress versus number of cycles relationship predicted by the crack propagation summation analysis described above for the particular case considered is shown by the full line of Figure 2. The relationship predicted by equations (10) and (12) is shown by the dashed line. For endurances greater than about 7×10^{10} cycles, equations (10) and (12) are seen to predict lower values of stress than the values given by the full crack propagation summation, the maximum error being about 12%. However the values of endurance and stress predicted by equations (10) and (12) respectively are conservative for endurances greater than about 7×10^{10} cycles, but become increasingly unconservative for lower endurances and higher values of stress. However in nuclear reactor gas circuits, the zero crossing frequency is normally of the order of 1 kHz, corresponding to some 10^{12} cycles in a life of about thirty years. The simple formulae derived are therefore suitable for assessing the acceptability of crack-like defects in reactor gas circuits.

The simple formulae derived lead to conservative values at the longer lives because the crack propagation is estimated from the stress intensity factor when the crack has propagated by the amount considered, not the value at the mean crack length. If equations (10) and (12) are evaluated on the basis of the average crack length, substituting $\frac{1}{2}(K_{\sigma 1} + K_{\sigma 2})$ for $K_{\sigma 2}$ and $f(\frac{a_1 + a_2}{2})$ for $f(a_2)$, the predicted behaviour is shown by the chain-dashed line of Figure 2. While the predicted values become more accurate for endurances greater than about 3×10^{11} cycles, they are at all times unconservative. The formulae as derived, based on the final crack length, are therefore to be preferred for design and defect assessment purposes.

4. CONCLUSIONS

1. Simple formulae have been derived for use in considering fatigue crack propagation caused by high frequency cyclic stresses whose peak values follow the Rayleigh distribution:

The number of cycles required to propagate a crack from a length a_1 to a length a_2 is:

$$N_{12} = \left\{\frac{a_2 - a_1}{r_o}\right\} e^{\left[\frac{\Delta K_o^2}{8K_{\sigma 2}^2}\right]}$$

The r.m.s. stress required to propagate a crack from length a_1 to length a_2 in N_{12} cycles is:

$$\sigma = \frac{\Delta K_o}{2f(a_2)\left\{2\log_e\left[\frac{r_o N_{12}}{a_1 - a_2}\right]\right\}^{\frac{1}{2}}}$$

2. Values deduced from the formula for a mild steel bar in bending have been shown to be in satisfactory agreement with values obtained from crack propagation summation calculations for endurances greater than 5×10^{10} cycles.

3. The formulae are suitable for initial assessment of the acceptability or otherwise of cracks and crack-like defects subjected to long exposure ($\sim 10^{11}$ cycles) to Rayleigh distributed stress peaks.

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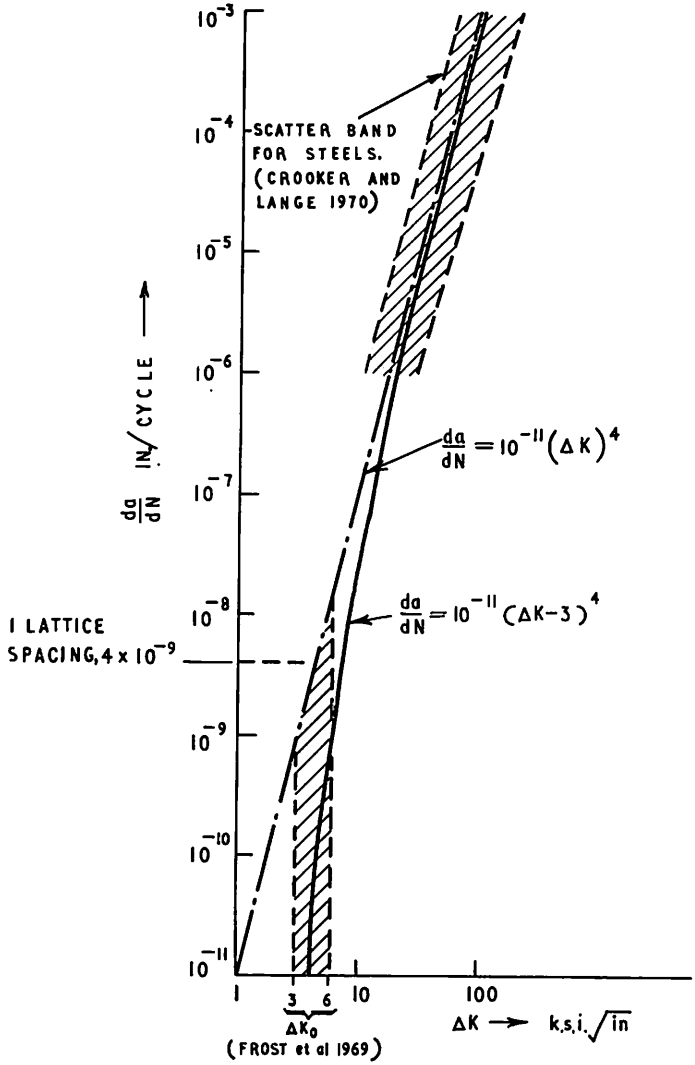

Figure 1. Crack propagation relationships

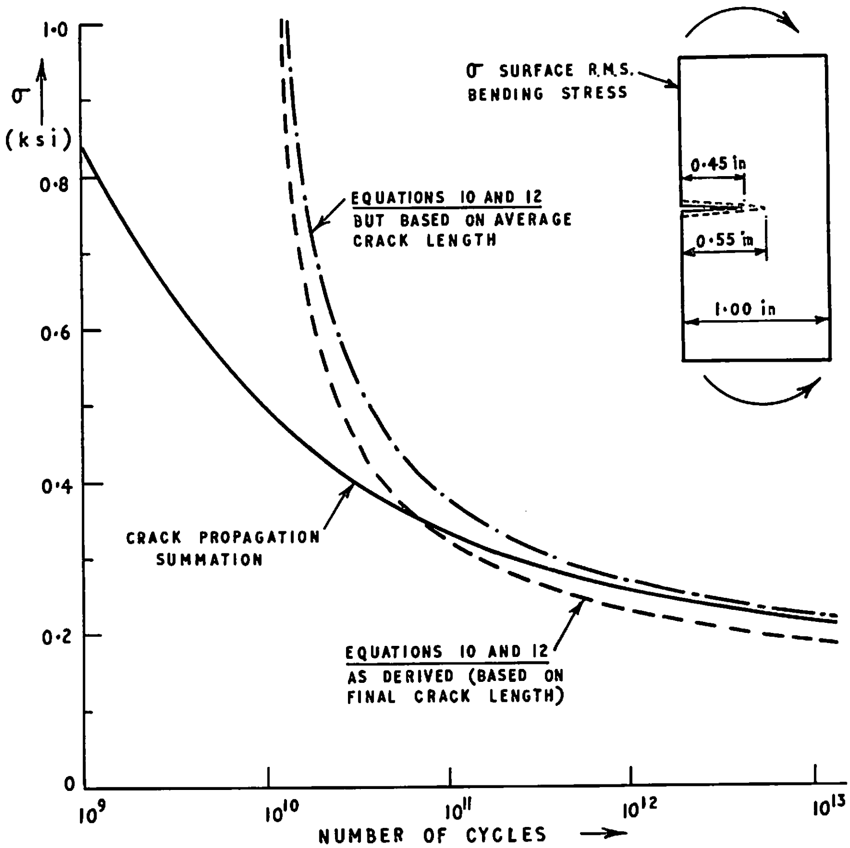

Figure 2. Comparisons of derived equations with crack propagation summation calculations for edge cracked beam subjected to random amplitude bending.

DISCUSSION

Q F. ERDOGAN, U. S. A.

1. What is the definition of "cycle" used in your work ?
2. What kind of cumulative damage theory was used in analysing the fatigue results under random loads ?

Q A. COWAN, U. K.

The method appears to offer a very useful analysis for this type of cyclic stressing. In gas cooled reactors failures by fatigue are most likely to occur at fillet welds; are analyses for stress intensity available for these geometries ?

A K. JERRAM, U. K.

Before answering the two specific points raised by Prof. Erdogan and Dr. Cowan, I would like to describe how the equations derived in my paper may be further simplified. In the analysis of Section 2, it is assumed that the crack always propagates at its rate when it reaches its final crack length a_2. Thus if the value of a_2 is assumed to be large, unduly conservative values of endurance or permitted stress will be obtained. For an infinite plate the maximum permitted stress, σ , is obtained when

$$\frac{a_2}{a_1} \quad 1.10.$$

If this value is assumed to be valid for other geometries, equations (10) and (12) become:

$$N_{12} \simeq \left(\frac{a_1}{10r_o}\right) e^{\left[\frac{\Delta K_o^{\,2}}{8K_{\sigma 2}^{\,2}}\right]}$$

$$\sigma \simeq \frac{\Delta K_o}{2f(1.1a_1)\left\{2 \log_e \left[\frac{10r_o N_{12}}{a_1}\right]\right\}}$$

In answer to Dr. Cowan's query, the analysis in my paper is intended for application to problems involving a specific crack or crack like defect. One of the problems can be determining the stress intensity factor. In some cases the problem under consideration can be represented by a geometry for which a stress intensity calibration is available in the literature, e. g. (14). Alternatively finite element stress analysis methods can be used to determine the stress intensity factor (15, 23). The method described in the paper should be applicable to fillet welds with lack of penetration, present either by design or poor manufacture. It is probable that finite element techniques would be required to determine the stress intensity factor, although it could be estimated from published solutions.

In reply to Prof. Erdogan's first query, the analysis in my paper is for the case of narrow band ramdom loading, i. e. the probability of an individual stress peak having a given peak/ r. m. s. ratio is governed by the Rayleigh distribution relationship, equation (3) of the paper. This type of waveform is obtained by passing white noise through a small bandwidth filter. Thus the stressing considered in the paper occurs at a constant frequency, say f_o, so that individual stress peaks on the same side of the zero mean level will occur at intervals of $1/f_o$. A graphical illustration of the waveform is given by Lewszuk and White (12). With regard to Prof. Erdogan's second point on cumulative damage theory for random loading, the assumption made was the obvious one, namely that a single cycle caused the same amount of propagation when preceded and followed by random amplitude cycles as it would have done if preceded and followed by cycles of the same amplitude. Recent work (13) at Berkeley Nuclear Laboratories has confirmed the validity of the assumption.

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