ON THE FINITE ELEMENT MATRIX DISPLACEMENT METHODS FOR TIME-DEPENDENT STRESS ANALYSIS OF REACTOR MATERIALS IN TWO AND THREE DIMENSIONS

A. JEZERNIK,
Central Electricity Generating Board, London,

A. ALUJEVIC, J.L. HEAD,
Imperial College of Science and Technology, University of London, London, United Kingdom

Abstract

The aim of this paper is to present a time-dependent stress analysis of some reactor materials by comparative matrix displacement methods.

A viscoelastic step-by-step stress model has been used and is derived for two and three dimensional analyses. The solution is advanced in small quasi-steady time intervals, however the reactor parameters and material data vary in time and space. The validity of this approach when extended for use in the finite element method has been confirmed by comparison with earlier analytical one-dimensional numerical integration codes and experimental evidence.

For time-dependent stress analysis of reactor components, the accuracy of the method beside the required computer memory and computing time becomes an important consideration since the repeated use of the finite element procedure and the computer round-off errors at each time step may influence the stability of results. The matrix displacement methods currently used to solve different structural problems in nuclear reactor engineering use various solution techniques as Gauss-Seidel iteration, direct triple-band and frontal approaches. By solving several complex two-dimensional stress problems the relative suitability of different methods is analysed, results compared and some conclusions are drawn.

Results are presented for generalised plane strain and axisymmetrical 2-D cases for components made of graphite and UO₂. The stress/time history in multichannel graphite blocks under temperature and damage fast neutron flux tilt with 19 holes in pentagonal and hexagonal shapes, is presented and the overall distribution and buildup of stresses in the critical regions and in ligaments studied using coarse and fine meshes. Next, the stress against time in the graphite sleeves and coated particle fuel cartridges of a tubular fuel pin (hollow rod) under temperature and flux tilt are given. Stresses/time in graphite blocks and in the tubular fuel pin are calculated for the reactor at power (operating stresses) and reactor shut down (residual stresses). Further in an axisymmetrical case the radial and longitudinal displacements of a UO₂ pellet and S/S canning are examined in order to determine the axial dishing.

A qualitative analysis of the above 2-D results is performed and some conclusions regarding the design of a nuclear reactor core are drawn.

The outline of the 3-D time-dependent code is given and two typical examples for its application, the teledial fuel element and caps and a reactor concrete pressure vessel are discussed.

Finally the possibility of extension of step-by-step model to calculate stresses in some other reactor materials for which different flow rules as time hardening and strain hardening apply and again in parallel the suitability of use of various solution techniques are discussed.
1. Introduction

The time-dependent analysis of stresses in reactor components by matrix displacement methods, taking account of creep, is in general considerably more complex than elastic analysis. The characteristics and input requirements of the various matrix displacement methods must be considered, creep data for the material must be obtained experimentally and the computation may have to be repeated at a number of time intervals, allowing for changes of external loading, temperature distribution and material properties, which may be time dependent. Two-dimensional creep calculations require a large amount of computer time and storage. In three dimensions, creep calculations appear to be extremely costly and limited more severely than two-dimensional calculations by the present size of commercial computers. For these reasons it is the aim of this paper to present time-dependent stress analyses of some reactor components undergoing creep and to examine and discuss simultaneously the matrix displacement methods using different solution techniques to solve the system of equations for the whole structure for nodal displacements. Results are presented of two-dimensional plane strain analyses of a hollow-rod fuel pin with a cross-pin temperature gradient, an axisymmetric analysis of the end region of a tubular fuel pin, plane strain analyses of pentagonal and hexagonal multi-channel graphite moderator blocks and axisymmetric analysis of a UO₂ fuel pellet. The paper also includes an outline description of a three-dimensional time-dependent programme. A form of the initial strain approach is used in the analyses. The analyses are mainly of graphite core components of High Temperature Reactors (HTR), since the programmes were developed originally for this purpose. However, the programmes are in general suitable to solve creep problems of other materials, but different creep laws appropriate to the material behaviour have to be used.

2. The Basic Theory and Computer Programs

2.1 The Theory

In an HTR core, the graphite components will be subject to temperature gradients and to spatial variations of neutron energy spectrum. There will therefore be spatial variations of the irradiation induced dimensional changes (Wigner strains) and of the thermal strains. Stresses will develop, analogous to thermal stresses in a body which is not at a uniform temperature. The stresses in the graphite components will be modified by an irradiation-induced creep of the graphite.

The basic theory is derived in three dimensions assuming a transversely isotropic material. From these equations, two-dimensional cases can be deduced.

The basic equations describing stress and deformation of a solid are:

(a) Equilibrium \[ \text{div} \left[ \sigma \right] + \mathbf{F} = 0 \text{ with } \sigma_{ij} = \sigma_{ji} \]  \hspace{1cm} (1)

(b) Constitutive law \[ \sigma = [\mathbf{D}] \left( \varepsilon - (\varepsilon^n) \right) \]  \hspace{1cm} (2)

(c) Strain-deformation \[ \varepsilon = \frac{1}{2} \left( \mathbf{\nabla} \mathbf{u} + \mathbf{\nabla} \mathbf{u}^T \right), \varepsilon_{ij} = \varepsilon_{ji} \]  \hspace{1cm} (3)

(d) Compatibility \[ \text{curl} \left( \text{curl} \left[ \varepsilon \right] \right)^T = 0 \]  \hspace{1cm} (4)

Here

\[ \left[ \sigma \right] = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} \quad \left[ \varepsilon \right] = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{bmatrix} \]  \hspace{1cm} (5)

and

\[ \left( \sigma \right) = \left( \sigma_{11} \sigma_{22} \sigma_{33} \sigma_{12} \sigma_{13} \sigma_{23} \right) \text{ and } \left( \varepsilon \right) = \left( \varepsilon_{11} \varepsilon_{22} \varepsilon_{33} \varepsilon_{12} \varepsilon_{13} \varepsilon_{23} \right) \]  \hspace{1cm} (6)
The nonelastic strain components can be expressed as follows:

\[
\{\varepsilon^n\} = \{(\varepsilon + \varepsilon^w)\} \delta_{ij} + \{\varepsilon^c\}
\]  

(7)

where \(\varepsilon\) and \(\varepsilon^w\) are experimentally determined as functions of temperature and neutron dose (time elapsed).

Also

\[
\{\varepsilon^c\} = \frac{\gamma}{2} \begin{bmatrix} C & 0 \end{bmatrix} \Delta Y
\]

(8)

is to be calculated incrementally with time.

For transversely isotropic materials, \(\varepsilon^r = \varepsilon^\phi \neq \varepsilon^z\), \(\varepsilon^w = \varepsilon^w \neq \varepsilon^w\), and the above matrices \([D]\) and \([C]\) each include 5 independent constants, for example the elements of the \([D]\) matrix may be expressed in terms of \(E_r\), \(E_z\), \(v_{r\phi}\), \(v_{rz}\), \(G_{rz}\) and the elements of the \([C]\) matrix in terms of \(C_r\), \(C_z\), \(C_{r\phi}\), \(C_{rz}\), \(C_{r\phi}^z\).

The elasticity matrix \([D]\) is then as follows:

\[
[D] = \begin{bmatrix} \frac{1}{E_r} - \frac{v_{r\phi}^2}{E_r} - \frac{v_{rz}}{E_z} \\ \vdots \\ \frac{1}{E_r} - \frac{v_{rz}}{E_z} \\ \text{symm.} \end{bmatrix}^{-1} \begin{bmatrix} 0_3 \end{bmatrix} = \begin{bmatrix} [A] [0_3] \end{bmatrix} [0_3] [r] \]

(9a)

where

\[
[A] = \frac{E_r}{(1+v_{r\phi})(1-v_{r\phi}^2)2v_{rz}E_z} \]

(9b)

\[
[r] = \begin{bmatrix} G_{rz} & G_{rz} & \frac{E_r}{2(1+v_{r\phi})} \end{bmatrix}
\]

(9c)

The matrix of creep constants \([C]\) is

\[
[C] = \begin{bmatrix} [M] & [0_3] \\ [0_3] & [L] \end{bmatrix}
\]

(10a)

where

\[
[M] = \begin{bmatrix} \begin{bmatrix} C_r - C_r\mu_{r\phi} - C_{r\phi}C_z \end{bmatrix} & \vdots \\ \text{symm.} \end{bmatrix} \quad \text{and} \quad [L] = \begin{bmatrix} C_{rz} & C_r^z(1+\mu_{r\phi}) \end{bmatrix}
\]

(10b)
If we assume that creep occurs at constant volume 

\[ (C_r C_{ij}^c = 0) \]

\[ C_r - C_r u_{r\phi} - C_z u_{rz} = 0 \text{ and } C_z - 2 C_z u_{rz} = 0 \] (11)

and \( u_{rz} = 0.5 \) and \( u_{r\phi} = 1 - \frac{C_z}{C_r} u_{rz} \) (12)

therefore

\[
\begin{bmatrix}
C_r - C_r + C_z/2 - C_z/2 \\
\vdots \quad C_r - C_z/2 \\
\text{asyn.} \quad C_z
\end{bmatrix}
\]

(13)

and

\[
\begin{bmatrix}
\int C_r & C_z & C_r - C_z
\end{bmatrix}
\]

(14)

In the case of completely isotropic materials, the elasticity matrix \([\mathcal{E}]\) reduces to

\[
\begin{bmatrix}
\lambda & -\frac{\lambda}{\nu} & 0 \\
-\frac{\lambda}{\nu} & \frac{\nu}{\nu} & 0 \\
0 & 0 & \frac{\nu}{\nu}
\end{bmatrix}
\]

where \( \lambda \) is the 1st Lamé constant \((\lambda = E\nu/(1+\nu)(1-2\nu))\) and \( G \) (2nd Lamé constant) is shear modulus \((G = E/[2(1+\nu)])\) (15)

The creep matrix \( C \) in this case is as follows:-

\[
\begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix}
\]

(16)

The computer programs have been developed for anisotropic, transversely isotropic and fully isotropic materials, but since for the examples presented in this paper only two elastic constants \((E \text{ and } \nu)\), and one creep constant were available, the equations (11) and (16) have been used. However the coefficient of thermal expansion and the irradiation induced dimensional changes are available for both transverse and longitudinal directions and have therefore been used in the analyses. The above differential equations can be solved by the finite element method using the well known calculus of variations. The displacement field in each individual element is expressed in terms of nodal values

\[
u_j = \sum_{i=1}^{N_1} N_i u_{ij} \quad (j=1,2,3)
\]

(17)

so that the "stiffness" equation for the whole structure becomes

\[
\{K\} \{\nu\} = \{R\}
\]

where

\[
\{R\} = \sum_{k=1}^{n} \left( \{F\}_{k}^{e} + \{F_{n}\}_{k}^{e} \right)
\]

(19)

and

\[
\begin{bmatrix}
\mathcal{E} \\
\mathcal{E}^T
\end{bmatrix} = \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T
\]

(20)

Here

\[
\begin{bmatrix}
\mathcal{E} \\
\mathcal{E}^T
\end{bmatrix} = \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T
\]

(21)

and

\[
\{F_{n}\}_{k}^{e} = \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T \quad \int \mathcal{E} \mathcal{E}^T
\]

(22)
can be determined by numerical procedures. The matrix \([B]\) has to be expressed in terms of derivatives of known shape functions \(N_i\), i.e. \(\partial N_i / \partial \eta_j\) \((j=1,2,3, i=1, \ldots, n)\). The force calculation is performed by assuming \((e^N) = \sum N_i (e^N)_i\). When stiffness equations system is solved by the computer for nodal displacements:

\[
(u) = [K]^{-1} (R)
\]  

(23)

the total strains and related stresses can be obtained from

\[
(e) = [B] \varepsilon (u) \quad \text{and} \quad (\sigma) = [D] [B] \varepsilon (u) - \\
[D] (e^N)
\]

(24)

2.2 The Computer Programmes

The matrix displacement methods currently used to solve different structural problems in nuclear reactor engineering use mainly two approaches to solve the system of equations (18). One, referred to as the direct method, uses Gaussian elimination. The other, called the iterative method, uses Gauss-Seidel iteration. Currently the most commonly used programmes based on these techniques are direct-band, iterative and frontal solution programmes (see for example Zienkiewicz [1], Argyris [2], Irons [3]).

In general when calculating the time-dependent stresses in reactor components, the material properties, the external loads and geometrical dimensions all vary in space and time. In this general case the stiffness matrix \([K]\) has to be recalculated and its inverse found at each time step. If the loads only change with time and the material properties and geometry can be assumed to be constant, only the right hand side of the equation (18) has to be recalculated at each time interval and the stiffness matrix \([K]\) and its inverse have to be calculated only once.

Two versions of a two-dimensional time-dependent stress analysis code STAG, one using the iterative the other the direct-band technique have been developed (Jezernik [4]) for calculating time (or neutron dose) dependent stresses in graphite components of an HTR. In the most general case, when the stiffness matrix \([K]\) has to be recalculated, the iterative version was found to be faster by a factor about 2.5 for the same level of accuracy. For this reason the iterative version was used for the analyses described in the paper.

A three dimensional time-dependent code TRISTAN, (Fig. 12) is being currently developed (Alujevic [5]). A resolution (changing the loads only), taking account of thermal, shrinkage and creep strains, is considered to be an economical proposition. Otherwise the three dimensional code will require computing time of well over 1 hour in terms of a CDC6600 installation. For example the elastic analysis of the end of a teledial fuel pin (fig. 11) requires approximately 1,000 secs. using 30 H32 elements (approximately 500 nodal points), and 330 secs. using 30 H20 elements (approximately 300 nodes). By comparison, a time-dependent calculation in two dimensions (plane strain) using the iterative programme, with 852 constant stress triangular elements and 639 nodes (Fig. 7) and material properties changing in time and space requires approx. 200 secs. of CDC6600 time for 10 time intervals.

3. Basic Data

It is not possible to present a detailed account of all data used in the analyses, but references are given to the sources of the data. All mesh data used in the analyses have been calculated using automatic mesh generation subroutines or programmes.
The temperature distributions for the hollow-rod fuel pin and the UO₂ fuel pellet (Figs. 3,10) have been chosen arbitrarily. The temperature distributions for the axisymmetric calculation of the fuel pin end (Fig. 6) and for the pentagonal graphite block (Fig. 1) were provided by Kinkead [9]. These temperature distributions have been calculated using finite element codes and correspond to the start of life for the fuel pin end and to a neutron dose 10 x 10²⁰ n/cm² Ni-Dido (6–8 months in the reactor) for the pentagonal block. The temperature distribution used in the analysis of the hexagonal block (Fig. 8), having a control rod inserted in the middle, is similar to a temperature distribution provided by Allen [7], which was used in an HTR design study. For all components except the hollow-rod fuel pin it is assumed that the temperature distribution remains unchanged throughout the life in the reactor, although the program permits the element temperature to be re-read as frequently as required.

For graphite components the neutron dose distribution has to be calculated or read in at each time (dose) interval. It has been assumed that the neutron dose received by the graphite has a constant value throughout all graphite components analysed. In some cases this is an unrealistic assumption but at the time of making the analyses, the spatial distributions of neutron dose were not available. Only a part of the life of the graphite components in the reactor has been covered. Calculations for the tubular fuel pin end and the hollow rod extend to a dose 17 x 10²⁰ n/cm² Ni-Dido, corresponding to approximately 1 year's life in the reactor. The calculations of the graphite blocks have been extended to a dose 9 x 10²⁰ and 12 x 10²⁰ n/cm² Ni-Dido respectively. A dose (time) step of 1 x 10²⁰ n/cm² Ni-Dido was used in all calculations.

The graphite data used in the analyses (thermal expansion coefficient, irradiation induced dimensional changes, elastic constants and creep data), which relate to Giscocarbon graphites, were provided by Everett et al. [3,9]. UO₂ data were taken from a paper by Belle [10].

Results

Previous analyses of stresses in graphite components have been reviewed by Head and Kinkead [11]. The results presented in this paper refer to some more complex reactor and loading conditions covering especially regions of flux and temperature perturbations, for example, core/reflector boundary regions of an HTR.

4.1 Fuel Pins

The authors previous work includes analyses of the fuel tubes of a hollow-rod fuel pin under axi-symmetric loading (Jezernik and Head [12]), a teledial fuel pin and a graphite fuel tube under temperature tilt (Jezernik [4], Head and Jezernik [13]) and a tubular interacting fuel pin (Alujevic and Head [14]). The results presented in this paper represent an extension of this work.

The stress distributions in the hollow-rod fuel pin (Fig. 2,3) with a cross-pin temperature gradient and assuming that the pin cannot bow, are similar in pattern to the stress distributions with axi-symmetrical temperature distributions except for the axial stresses. The axial stresses reach high values, being compressive at the hotter side and tensile at the cooler side of the pin and thus generating a bending moment (Fig. 4). It is apparent that the graphite tube and fuel compact will be only partially restricted from bowing and therefore the axial stresses will have lower values than those calculated. The amount of bowing of the pin depends on the clearance between the channel and the fuel pin and on the extent of bowing of the graphite block.
The effect of differential irradiation shrinkage, the shrinkage rate being greatest in the hotter regions of the fuel pin, is to generate a reversed stress pattern, the tensile stresses developing on the hotter side of the fuel pin and compressive stresses on the cooler side. Shutting down the reactor, and allowing the fuel pin to reach a uniform temperature, leads to an increase of stress because the thermal expansion effect, which opposes the effect of dimensional changes, vanishes (Fig. 4). The most dangerous conditions appear on shut down of the reactor after prolonged irradiation causing the stresses to rise to very high values. In the case of the tubular fuel pin, the area of interest is the end cap, since this part of the pin is exposed to severe temperature gradients and to edge loadings, due to different expansions and shrinkages of the inner and outer graphite tubes. The highest stresses develop at the inner edges of the top cap (Table I, Fig. 7) being initially tensile at the right hand inner corner and compressive at the left hand corner.

The stresses in other areas of the fuel pin are in general lower. As in the case of the hollow rod fuel pin, the effect of differential irradiation shrinkage is to cause a reversal of the initial (thermal) stress distribution. Shut down of the reactor causes the stresses to rise.

4.2 Graphite Blocks

The stresses in the multichannel graphite blocks are in most cases lower than those developing in the fuel pins since they are subjected to lower temperatures. However, at core reflector boundaries and in cases where a control rod is placed in the middle of the block, substantial temperature and neutron dose gradients may develop resulting in high stresses.

In the analysis of the pentagonal block, a coarse mesh of 455 elements and 322 nodes has been used (Fig. 1). The temperature distribution used in the analysis is not shown in detail, but there is a temperature difference across the block, from about 870°C at the right hand side to about 770°C at the left hand side. The highest stresses are axial stresses being initially compressive at the hotter right hand side and tensile at the cooler left hand side. Lower stresses develop in x-y plane (Table II).

In the hexagonal block, a control rod inserted in the middle will cause substantial temperature gradients, with highest temperatures at the outer boundary of the block and lowest temperatures in the centre (Fig. 8). The highest stresses develop in the axial direction. High stresses develop also in the ligaments, which are initially mainly under tension (Table III, Fig. 9).

In both cases, in the pentagonal and the hexagonal blocks, the operating stresses are rapidly relaxed due to differential shrinkage and creep and reversed stress distributions are established. Stresses again rise if the reactor is shut down. It is apparent that the stresses are higher in the hexagonal block due to temperature gradients caused by the presence of the control rod. It can be concluded that a radial variation of neutron dose across the structure of a similar form as temperature tilt will amplify the calculated stress pattern.

4.3 UO₂ Pellet

UO₂ pellet end swelling has been calculated for zero time only using the body of revolution option of STAG (Jezernik and Alujevic [15]). Dose-dependent properties can be incorporated giving the variation of the swelling (bamboo) effects with time. The calculated axial displacements are used to calculate the minimum dishing required, taking into account the longitudinal displacements of the fuel can (see also Guyette [14]). The real dishing is
usually larger than the calculated dishing. The displacements of a non-dished pellet of
1.0 cm diameter x 1.0 cm long are presented in Fig. 10.

5. Conclusions

A finite element stress analysis model for the solution of creep problems, based on the
initial strain approach has been demonstrated by two-dimensional analyses of some nuclear
reactor components. A three-dimensional version of the programme has been outlined. Two-
dimensional versions of the programme have been written, based on direct (Gaussian elimina-
tion) and iterative (Gauss-Seidel) solution techniques. In the most general cases, where the
temperature distribution, material properties and external loads all vary in space and time,
and the stiffness matrix [K] for the structure has to be calculated and inverted at each time
step, the iterative solution technique of solving for nodal displacements has been found to
be faster and more suitable than the direct-band method for the two-dimensional problems
presented in the paper. It is thought that if only loads have to be modified at each time
step, the computer time required by the iterative and direct-band programmes may well be
comparable for two-dimensional problems. As far as accuracy and stability of results are
concerned, one programme does not show definite advantages over the other. However the
limited capacity of the iterative method for the solution of large scale engineering problems,
by the computer storage requirements of the programme. It has not been possible yet to compare
the programmes with the frontal solution programmes, but it seems that a further comparison with
frontal solution techniques, and using in addition higher order elements, is desirable
because time dependent calculations require a considerable amount of computer storage and
time.

The core and computer time requirements are further increased in three-dimensional
calculations where a front solution method and higher order elements appear to be the best
preposition. It is possible that some conclusions from comparative analyses of matrix
displacement methods could contribute to the development of the time-dependent three-
dimensional work.

Finally some conclusions can be drawn regarding the relative suitability of the reactor
components which have been analysed.

A substantial temperature tilt across a hollow rod fuel pin causes high axial stresses
if the pin is restricted from bowing. It is probable that the axial stresses will be reduced
by bowing of fuel pins, the extent of the bowing being determined by the initial clearances
between the fuel pins and the graphite moderator block and by the bowing of the block itself.
The stresses in the fuel pin end caps are largely influenced by the magnitude of any tempera-
ture difference between the inner and outer graphite tubes. The stresses in x-y plane of the
hollow rod are only slightly influenced by the temperature (or neutron dose) tilt and their
values and pattern are similar to those for symmetric loading.

For a graphite block under temperature tilt the axial stresses are highest. Over the
dose (time) range presented, the highest are the initial (thermal) stresses, but it can be
assumed that the residual axial stresses are most severe after a prolonged irradiation in the
reactor. As in the case of the hollow-rod fuel pin, it is the amount of bowing which will
determine the magnitude of the maximum stress levels. The graphite block with a central
control rod and having therefore severe radial temperature and flux gradients, represents by
far the most severe stress situation since the axial stresses are not relieved by deformation.
In all the HTR graphite components analysed the highest stresses are the stresses occurring when the reactor is shut down after a prolonged period of operation. Neutron dose tilts in addition to temperature tilts will amplify the existing stress pattern.

The stress levels in the fuel pins and graphite blocks can be reduced by reducing respectively the thickness and diameter of the fuel pins or reducing the size of the blocks, but this is clearly possible only to a limited extent and may be in contradiction with other design requirements.

The programmes described have been mainly demonstrated by analysing the graphite components of high temperature reactors, but they can be used also for other materials assuming different creep laws, as for example, time hardening or strain hardening.
6. Notation

A ... auxiliary matrix
B ... shape matrix
C ... creep matrix, creep constant
D ... elasticity matrix
d, d' ... operators
E ... Young's modulus
F ... force
G ... shear modulus
I ... identity matrix
J ... Jacobian matrix
K ... stiffness matrix
L ... auxiliary matrix
M ... auxiliary matrix
N ... shape functions
O ... zero matrix
r ... radius, positional vector
R ... resultant
t ... time
u ... displacement
V ... volume
Z ... auxiliary matrix
\alpha ... coefficient of thermal expansion
\gamma ... equivalent neutron dose
\Gamma ... auxiliary matrix
\Delta ... difference
\delta ... Kronecker symbol
e ... strain
\eta ... coordinate
\theta ... temperature
\lambda ... 1st Lame constant
\mu ... creep ratio
\nu ... Poisson's ratio
\sigma ... stress
\Sigma ... transformation matrix

Subscripts and superscripts

c ... creep
e ... element
i, j ... 1, 2, 3
k ... count
n ... nonelastic, number of nodes per element
m ... number of elements
r ... radial, perpendicular, transverse
z ... axial, parallel, longitudinal
Subscripts and superscripts (cont'd.)

# ... circumferential, perpendicular, tangential
T ... transposed
v ... Wigner's irradiation damage

Symbols

[] ... square matrix
[ ] ... trace matrix
{ } ... column matrix
| | ... determinant
→ ... vector
- ... average, mean
References

5. Alujevic, A., Thesis to be published, University of London.

Acknowledgements

Thanks are due to Dragon Project, Winfrith, England and Central Electricity Generating Board for providing some of the data used in the analysis and assistance in presenting this paper.
## TABLE I

### STRESSES IN THE FUEL PIN END

<table>
<thead>
<tr>
<th>Element</th>
<th>Operating Stresses (MN/m²)</th>
<th>Residual Stresses (MN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma_{\text{max}} )</td>
<td>( \sigma_{\text{min}} )</td>
</tr>
<tr>
<td>ELEM. 236</td>
<td>7.13</td>
<td>1.86</td>
</tr>
<tr>
<td>298</td>
<td>5.39</td>
<td>0.25</td>
</tr>
<tr>
<td>143</td>
<td>1.22</td>
<td>-2.37</td>
</tr>
<tr>
<td>161</td>
<td>1.19</td>
<td>-4.61</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>0.54</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>0.46</td>
</tr>
<tr>
<td>141</td>
<td>-1.13</td>
<td>-3.82</td>
</tr>
<tr>
<td>142</td>
<td>-0.62</td>
<td>-3.85</td>
</tr>
</tbody>
</table>

Dose 4x10²⁰ n/cm² Ni-Dido (Int. 5)

<table>
<thead>
<tr>
<th>Element</th>
<th>Operating Stresses (MN/m²)</th>
<th>Residual Stresses (MN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma_{\text{max}} )</td>
<td>( \sigma_{\text{min}} )</td>
</tr>
<tr>
<td>ELEM. 236</td>
<td>-0.11</td>
<td>-1.98</td>
</tr>
<tr>
<td>298</td>
<td>0.02</td>
<td>-1.60</td>
</tr>
<tr>
<td>143</td>
<td>1.96</td>
<td>-0.55</td>
</tr>
<tr>
<td>161</td>
<td>1.45</td>
<td>-0.08</td>
</tr>
<tr>
<td>2</td>
<td>-0.1</td>
<td>-0.49</td>
</tr>
<tr>
<td>4</td>
<td>-0.09</td>
<td>-0.44</td>
</tr>
<tr>
<td>141</td>
<td>1.52</td>
<td>0.61</td>
</tr>
<tr>
<td>142</td>
<td>1.49</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Dose 9x10²⁰ n/cm² Ni-Dido (Int. 10)

<table>
<thead>
<tr>
<th>Element</th>
<th>Operating Stresses (MN/m²)</th>
<th>Residual Stresses (MN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma_{\text{max}} )</td>
<td>( \sigma_{\text{min}} )</td>
</tr>
<tr>
<td>ELEM. 236</td>
<td>-2.04</td>
<td>-3.76</td>
</tr>
<tr>
<td>298</td>
<td>-0.12</td>
<td>-2.80</td>
</tr>
<tr>
<td>143</td>
<td>2.32</td>
<td>-0.37</td>
</tr>
<tr>
<td>161</td>
<td>1.96</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>-0.52</td>
<td>-0.78</td>
</tr>
<tr>
<td>4</td>
<td>-0.50</td>
<td>-0.75</td>
</tr>
<tr>
<td>141</td>
<td>1.94</td>
<td>1.33</td>
</tr>
<tr>
<td>142</td>
<td>1.77</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Dose 1x10²¹ n/cm² Ni-Dido (Int. 15)

<table>
<thead>
<tr>
<th>Element</th>
<th>Operating Stresses (MN/m²)</th>
<th>Residual Stresses (MN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma_{\text{max}} )</td>
<td>( \sigma_{\text{min}} )</td>
</tr>
<tr>
<td>ELEM. 236</td>
<td>-3.00</td>
<td>-4.24</td>
</tr>
<tr>
<td>298</td>
<td>-0.62</td>
<td>-2.86</td>
</tr>
<tr>
<td>143</td>
<td>1.82</td>
<td>-0.12</td>
</tr>
<tr>
<td>161</td>
<td>1.78</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>-0.71</td>
<td>-0.77</td>
</tr>
<tr>
<td>4</td>
<td>-0.71</td>
<td>-0.78</td>
</tr>
<tr>
<td>141</td>
<td>1.76</td>
<td>1.33</td>
</tr>
<tr>
<td>142</td>
<td>1.27</td>
<td>-0.14</td>
</tr>
</tbody>
</table>

The highest stresses develop initially in el’s 236, 298, 143, 161, 2, 4, 141, 142 and from each element the underlined value represents the highest numerical value of stress.
<table>
<thead>
<tr>
<th>Dose 0 (Int. 1)</th>
<th>( \sigma_\text{max} )</th>
<th>( \sigma_\text{min} )</th>
<th>( \sigma_z )</th>
<th>( \sigma_\text{max} )</th>
<th>( \sigma_\text{min} )</th>
<th>( \sigma_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELEM. 84</td>
<td>1.71</td>
<td>-0.11</td>
<td>3.79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>293</td>
<td>1.52</td>
<td>-0.03</td>
<td>-0.16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>109</td>
<td>-0.06</td>
<td>-1.73</td>
<td>-0.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>104</td>
<td>-0.07</td>
<td>-1.67</td>
<td>-0.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>117</td>
<td>0.86</td>
<td>-0.01</td>
<td>4.77</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>443</td>
<td>0.05</td>
<td>-0.01</td>
<td>-5.97</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dose 4x10^{20} n/cm² Ni-Dido (Int. 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELEM. 84</td>
</tr>
<tr>
<td>293</td>
</tr>
<tr>
<td>109</td>
</tr>
<tr>
<td>104</td>
</tr>
<tr>
<td>117</td>
</tr>
<tr>
<td>443</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dose 9x10^{20} n/cm² Ni-Dido (Int. 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELEM. 84</td>
</tr>
<tr>
<td>293</td>
</tr>
<tr>
<td>109</td>
</tr>
<tr>
<td>104</td>
</tr>
<tr>
<td>117</td>
</tr>
<tr>
<td>443</td>
</tr>
</tbody>
</table>

The highest stresses develop initially in element 84, 293, 109, 104, 117, and 443 and for each element the underlined value represents the highest numerical value of stress.
### TABLE III

**STRESSES IN THE HEXAGONAL GRAPHITE BLOCK (FIG. 8)**

<table>
<thead>
<tr>
<th>OPERATING STRESSES</th>
<th>RESIDUAL STRESSES</th>
</tr>
</thead>
<tbody>
<tr>
<td>(MN/m²)</td>
<td>(MN/m²)</td>
</tr>
<tr>
<td>(a_{\text{max}})</td>
<td>(a_{\text{min}})</td>
</tr>
<tr>
<td>---------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td><strong>Dose 0 (Int. 1)</strong></td>
<td></td>
</tr>
<tr>
<td>ELEM. 249</td>
<td>7.67</td>
</tr>
<tr>
<td>242</td>
<td>7.42</td>
</tr>
<tr>
<td>45</td>
<td>-0.64</td>
</tr>
<tr>
<td>781</td>
<td>-0.34</td>
</tr>
<tr>
<td>60</td>
<td>5.31</td>
</tr>
<tr>
<td>233</td>
<td>6.02</td>
</tr>
<tr>
<td>167</td>
<td>5.23</td>
</tr>
<tr>
<td>238</td>
<td>4.58</td>
</tr>
<tr>
<td>358</td>
<td>3.78</td>
</tr>
<tr>
<td>359</td>
<td>3.20</td>
</tr>
<tr>
<td><strong>Dose 4x10^{20} n/cm² Ni-Dido (Int. 5)</strong></td>
<td></td>
</tr>
<tr>
<td>ELEM. 249</td>
<td>5.00</td>
</tr>
<tr>
<td>242</td>
<td>4.94</td>
</tr>
<tr>
<td>45</td>
<td>-2.31</td>
</tr>
<tr>
<td>781</td>
<td>-2.28</td>
</tr>
<tr>
<td>60</td>
<td>2.30</td>
</tr>
<tr>
<td>233</td>
<td>3.67</td>
</tr>
<tr>
<td>167</td>
<td>3.12</td>
</tr>
<tr>
<td>238</td>
<td>2.97</td>
</tr>
<tr>
<td>358</td>
<td>1.57</td>
</tr>
<tr>
<td>359</td>
<td>0.54</td>
</tr>
<tr>
<td><strong>Dose 8x10^{20} n/cm² Ni-Dido (Int. 10)</strong></td>
<td></td>
</tr>
<tr>
<td>ELEM. 249</td>
<td>2.96</td>
</tr>
<tr>
<td>242</td>
<td>3.01</td>
</tr>
<tr>
<td>45</td>
<td>-2.29</td>
</tr>
<tr>
<td>781</td>
<td>-1.63</td>
</tr>
<tr>
<td>60</td>
<td>1.04</td>
</tr>
<tr>
<td>233</td>
<td>1.67</td>
</tr>
<tr>
<td>167</td>
<td>2.27</td>
</tr>
<tr>
<td>238</td>
<td>1.91</td>
</tr>
<tr>
<td>358</td>
<td>0.82</td>
</tr>
<tr>
<td>359</td>
<td>-0.21</td>
</tr>
</tbody>
</table>

The highest stresses develop initially in el's 249, 242, 45, 781, 60 and 233 and for each of these elements the underlined value represents the highest numerical value of stress. Stresses which develop in el's 167, 238, 358 and 359 represent typical values of stresses in x-y plane in ligament regions (see Fig. 8).
Figure 1  Some reactor components.
Figure 2  Hollow rod fuel pin-mesh.
Figure 3  Hollow rod - temperature distribution (°C).

Figure 4  Hollow rod-radial distributions of principal stresses at time intervals 1 and 10. Reactor at power.
Figure 5  Tubular fuel pin end – mesh.
Figure 6 Tubular fuel pin end - temperature distribution (°C).

Figure 7 Tubular fuel pin end - variation of principal stresses on elements 236 and 141. Reactor at power and shut-down.
Figure 8 Hexagonal graphite block - mesh, temperature distribution and highly stresses regions.
Figure 9 Hexagonal graphite block - variation of principal stresses on elements 60 and 249. Reactor at power and shut-down.

Figure 10 Radial and axial displacements of UO₂ undished pellet.
Figure 11  Three-dimensional mesh and two-dimensional basic plane mesh.
Figure 12  Flow diagram of three dimensional code TRISTAN.
Q

J. M. DONEA, JRC Ispra, Italy

In a solution for stresses in a graphite structure, is it necessary to recalculate the stiffness matrix at each time step, i.e., thus Young's modulus varies much or can it be assumed that $E(D) = 2E_0$?

I should take immediately the saturated value.

A

A. JEZERNIK, U. K.

The assumption that saturated value of Young's modulus (i.e. irradiated value) is approximately: $E(D) = 2E_0$ is correct for some graphites at a certain temperature. However the changes of Young's modulus with irradiation often assume a more complex pattern. For example for a particular graphite under consideration for high temperature reactors - pressed Gilso-carbon graphite - the changes of Young's modulus under irradiation are as shown below.

Therefore it is necessary to recalculate the stiffness matrix at each time step and the assumption that $E(D) = 2E_0$ may be satisfactory only for some specific examples.

Q

Z. ZUDANS, U. S. A.

Would iterative technique be of advantage if materials requiring incremental constitutive relations were required?
A. JEZERNIK, U. K.

The incremental constitutive relations are used in the analysis presented in the paper M 5/8. The iterative technique appears to be of advantage for time-dependent calculations where elastic constants and/or structure dimensions are changing in time and space and stiffness matrix for the structure must be recalculated at each time step. If the elastic constants and structure dimensions do not vary with time, only a resolution of the system of equations is required at each time step but not the recalculation of the stiffness matrix for the structure. (The stiffness matrix need to be calculated only once for the whole computation). For this particular case the direct solution techniques may well use a comparable amount of computing time as iterative procedure.