INTRODUCTION

The shakedown design method is used in the UK for guarding fast reactor components against progressive deformation or ratchetting and for assessment of fatigue and creep damage (Rose, 1986). The rules are applicable to level A type loadings and because shakedown methods assess steady cyclic state behaviour can take account of creep as well as plasticity.

This paper summarises the results of test cases based on three examples of structures subjected to repeated cyclic loading containing different levels of primary and secondary stresses. The aim was to check by comparison with inelastic solutions whether shakedown assessments were sufficiently conservative while also being more economical than complete inelastic analysis.

SHAKEDOWN METHOD

Shakedown is defined as a state where after a few cycles of loading plasticity ceases and no further change in the dimensions of the structure takes place. The stress–strain response is wholly elastic for strict shakedown although some small regions of plasticity are admissible in overall shakedown where the material undergoes cyclic plasticity but is constrained by the surrounding material. A shakedown state is therefore reached when a residual stress field develops such that the superposition of elastically calculated stresses causes at most localised yielding.

The method uses the elastic analysis of a load cycle and requires the estimation of a constant residual stress field which is used to obtain a reference stress for creep damage estimates and local estimates of fatigue damage. Successful shakedown analyses have been performed using a manual trial and error basis on simple cases to identify a residual stress field. This approach is being superseded by computer methods and a residual stress post processor ADAPT has been developed (Rose, White, 1988) to interface with the ABAQUS finite element program as an aid in deciding whether complete or overall elastic shakedown occurs. The results described here have involved the application of the post processor ADAPT which is being successively updated and improved as the range of test examples widens. Three different versions have been used to date each one giving improvement of convergence characteristics.

The following sections contain results for three different problems. A brief description of the test cases and results are given and where applicable comparison is made with inelastic solutions and damage assessments are performed.
Description of the Problem

The structure is a thin tube of length \( L = 300 \, \text{mm} \), thickness \( h = 1.4 \, \text{mm} \) and mean radius \( R = 76 \, \text{mm} \) (Fig. 1(a)). The geometry and loading was based on a UK ratchetting experiment. The tube is subjected to a combination of mechanical and thermal loading. The mechanical loading consists of an axial tensile load which is held constant throughout the entire loading history. The thermal loading comprises a downshock followed by a slow heating up period. The tube is originally at a uniform temperature of 632.5°C and subjected to the downshock over a narrow band \( 2a = 10 \, \text{mm} \). This is achieved in the experiment by means of a narrow spray of cold water. The minimum quench down temperature of the tube was 380°C (Fig. 1(b)).

The loading on the tube was simulated by matching an analytical temperature solution to measured values from the experiment. The heating up period back to 632.5°C was approximated by assuming temperature varies linearly with time. A number of example cases were studied in which the primary stress was varied between 55 and 70 MPa in 5 MPa increments.

The material properties assumed were for type 316 stainless steel. The expansion coefficient was scaled down by a factor of 2.1 from values for the steel. This was in order to reduce the severity of the real problem in the experiment and to allow shakedown with design values of yield stress. The same values of mechanical properties were used for a shakedown assessment using ADAPT and for cyclic inelastic analysis using ABAQUS based on an elastic-perfectly plastic material model.

Eight time points from the thermal downshock calculations were used for the stress calculations (chosen to ensure that stresses in between could be approximated by linear interpolation). The reheat phase was treated as a proportional change from full shock to starting conditions.

Details of the Shakedown Analysis

At 55 MPa end load shakedown was found within 3 iterations using ADAPT compared with 5 complete cycles of inelastic analysis using ABAQUS. At 60 MPa end load global shakedown was found within 4 iterations and again after 5 cycles of inelastic analysis with ABAQUS. Figs. 1(c) and 1(d) shows results for the 60 MPa end load at two positions. At 65 MPa ADAPT was unable to find shakedown within 12 iterations. The iterated residual stress distribution had converged to a solution which had failed local shakedown at several points in the model and was beyond or close to the limit through certain sections of the tube. This was interpreted as beyond shakedown in any acceptable sense. The cyclic elastic-plastic solution was pursued to six cycles and at quite a number of points the stress state was still changing however it was not clear whether the ultimate state would be of shakedown or slight ratchetting.

In view of this result one further cyclic analysis was performed using 70 MPa end load and this solution showed a steady ratchetting state after 5 loading cycles. The form of the ratchet involved inward radial movement.

The effort required to perform the cyclic inelastic calculations proved so great that it became impractical to pursue them all to their ultimate conclusion. Nevertheless the results indicate that ABAQUS and ADAPT agree about strict shakedown of the problem up to 60 MPa end load with ADAPT perhaps slightly more conservative at 65 MPa. Significant advantages were also shown in terms of reduced numerical effort.
Description of the Problem

The structure consists of two plates welded together in a T-type joint. The weld and parent materials were assumed to have the same properties and only the concentration effect due to the geometry of the butt weld was examined. Finite element calculations using ABAQUS were carried out on a 2D generalised plane strain idealisation with only half of the cross-section of the two plates modelled because of symmetry conditions (Fig. 2(a)). In the region adjacent to the upper fillet, a maximum stress concentration factor of 2.1 was obtained. The accuracy of the finite element discretisation was verified by comparing elastic results with different degrees of refinement. The material properties assumed were for type 316 stainless steel. For inelastic calculations plasticity parameters of a form appropriate for ORNL constitutive laws were used.

The loading cycle consists of a variable bending moment applied at different temperatures. The moment loading M is applied at the vertical and horizontal legs by a combination of concentrated forces and kinematic constraints to simulate pure bending. It can be defined by the nominal bending stress \( \sigma_{\text{nom}} \) on the vertical leg. The temperatures were assumed to change proportionately with mechanical load from 540°C to 370°C and sufficiently slowly that no thermal stresses were generated. Three test cases were examined, the details of which are given in Table 1. Problems 1 and 2 were cases where the nominal stress range was 86% of the strict shakedown limit (evaluated by the elastic-perfectly plastic analysis). Problem 3 had a nominal stress range well above this strict shakedown limit. This case was also examined by inelastic analysis first with no creep effects and then with a 1000 hr hold period included at the hot condition, using plasticity properties including hardening.

<table>
<thead>
<tr>
<th>Problem No.</th>
<th>( \sigma_{\text{H, nom}} ) (MPa)</th>
<th>( \text{TH} ) (°C)</th>
<th>( \sigma_{\text{C, nom}} ) (MPa)</th>
<th>( \text{TC} ) (°C)</th>
<th>ADAPT No. of Iterations</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>540</td>
<td>6</td>
<td>370</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>130</td>
<td>540</td>
<td>36</td>
<td>370</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>131</td>
<td>540</td>
<td>-40</td>
<td>370</td>
<td>7 to 11</td>
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Table 1 T-Butt Weld Joint, Details of Test Problems

Details of Shakedown Analysis

The numbers of iterations needed to achieve shakedown are given in Table 1. For cases 1 and 2 comparisons were made with finite element solutions based on elastic/perfectly plastic behaviour. The result for the steady cyclic stress path at the peak condition in case 2 is shown in Fig. 2(b) and required seven iterations (equivalent to elastic solutions). This compares favourably with the inelastic analysis which required three times as many solutions. For case 3 shakedown was achieved in an overall sense and convergence was found for practical purposes after about 7 iterations although there were minute changes in the estimated non-shakedown area up to 10 or 11 iterations, this area being conservatively represented with respect to all the inelastic analyses (Fig. 2(c)).

Fatigue damage estimates were made for case 3 using the strain enhancement
FIGURE 1  TUBE WITH DOWNSHOCK AND END LOAD

Fig. 1(a)  
Temperatures During Downshock on Inner Surface

Fig. 1(b)  
Temperatures During Downshock on Inner Surface

Fig. 1(c)  
Elastic and Shakedown Stress Path Z = 6.8 mm on Inner Surface

Fig. 1(d)  
Elastic and Shakedown Stress Path at Centre (z = 0) on Inner Surface

FIGURE 2  TEE–BUTT WELD PROBLEM

Fig. 2(a)  Finite Element Model

Fig. 2(b)  Stress Paths for Case No. 2

Fig. 2(c)  
Non–Shakedown for Case 3
procedures and compared with estimates from elasto-plastic-creep analyses including hardening. Comparisons were also made with results from elastic assessment routes in design codes ASME N-47 and RCC-MR. The various estimates of strain range are summarised as follows.

<table>
<thead>
<tr>
<th></th>
<th>Cycle with no dwells</th>
<th>Cycle with dwells</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE inelastic analysis</td>
<td>0.167</td>
<td>0.169 (6th cycle)</td>
</tr>
<tr>
<td>Shakedown method</td>
<td>0.219</td>
<td>0.279</td>
</tr>
<tr>
<td>RCC-MR</td>
<td>0.233</td>
<td>0.362</td>
</tr>
<tr>
<td>N-47</td>
<td>0.186</td>
<td>0.378</td>
</tr>
</tbody>
</table>

**Comparison of Strain Ranges (%)**

For the cycle with no dwells, the N-47 estimate is closest to the value obtained in the finite element analysis. This occurs due to the different specification of cyclic stress-strain curves. When creep is significant both N-47 and RCC-MR are more conservative.

**THERMAL STRESS PROBLEM WITH MOVING SODIUM LEVEL**

**Description of the Problem**

The structure is a thin cylinder of thickness 20 mm and inside radius of 1067 mm. On the outside of the cylinder is hot sodium at a uniform temperature of 547°C. On the inside of the cylinder is cold sodium with a stratified temperature layer rising from 395°C to 480°C with argon gas above (Fig. 3(a)). The test problem considers the case where the cold sodium rises rapidly by 1000 mm in a time of 20 seconds. The rate was assumed logarithmic and faster in the earlier stages. The stratified temperature profile remains unchanged relative to the pool surface over the 20 s period. The sodium either side of the cylinder then cools down to 395°C over a long period of time. The argon gas is assumed to be a perfect insulator for the purposes of the calculation. The initial temperature distribution is therefore one of through thickness and axial gradients below the cold pool level and constant temperature of 547°C above.

As the cold sodium rises the outer surface is thermally shocked experiencing temperature reductions up to 60°C in the first 5 seconds while the temperature on the hot side changed little. Elastic stress results were obtained at a small number of time points to represent the stress history with linear interpolation of stress assumed between each. The material properties assumed for the calculations were for type 316 stainless steel.

**Details of Shakedown Analysis**

So far only preliminary shakedown calculations have been performed. Cases with three and six time points representing the elastic history have been studied. The result after four iterations was nearly the same in both cases with only marginal differences at mid-thickness. The result for the case with three time points is given in Figs. 3(b) and 3(c) in terms of shakedown ratio (the maximum equivalent value of elastic plus residual stress divided by yield stress). This ratio changed little after the first estimate of equilibrated residual stress.
The requirements for strict shakedown (i.e. complete elastic behaviour) was not satisfied however the regions of non-shakedown were less than 20% of any section hence the requirement for overall shakedown was satisfied. It is envisaged that a slightly more accurate prediction would be obtained with an increased number of time points defining the stress history.

CONCLUSIONS

Shakedown assessments can be made for a wide variety of loadings at less computational effort and complexity than complete inelastic analysis.

Fatigue estimates are reasonably conservative when compared with estimates from inelastic analysis.

Acknowledgement

The authors wish to thank colleagues who have contributed to the work and their companies NNC Ltd and GEC-ER C for permission to publish the paper.

REFERENCES


Fig. 3(c) Through Thickness Variation at A-B

Fig. 3(b) Shakedown Ratio

Fig. 3(a)

FIGURE 3 CYLINDER WITH MOVING TEMPERATURE FRONT