Boundary Constraints in Ribbed Shells Analysis

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INTRODUCTION

This paper presents the general theory of shells containing internal ribs in the scope of the theory of boundary constraints. There has been used the Hu-Washizu functional completed with constraints put on the rib lines of components of displacements field between external shell surfaces. We have obtained fundamental equations for shells along with compatibility conditions on the rib lines.

THEORY

The formulating of the general theory of body with the internal constraints we obtain by means of (Kutyłowski, 1986) , where there were shown equations for the shells. We have got all these equations by means of the first form of Hu-Washizu functional, which was given in the following form : / shear deformations has been taken into consideration /

$$\Pi\left(\sigma^{ij}, e_{ij}, v_i\right) = \frac{1}{2}\int_V C^{ijkl} e_{ij} e_{kl}\, dV + \int_V \sigma^{ij}\left[\frac{1}{2}\left(v_{i,j} + v_{j,i}\right) - e_{ij}\right] dV - \int_V v_i X^i\, dV - \int_{\Omega_\sigma} v_i q_o^i\, dS - \tag{1}$$

$$- \int_{\Omega_u}\left(v_i - v_{oi}\right)\sigma^{ij}\nu_j^*\, dS - \int_{S+} v_i q_+^i\, dS_+ - \int_{S-} v_i q_-^i\, dS_- ,$$

where stress, displacement and deformation fields were described in three - dimentional base of initial configuration / fig. 1 /. Surfaces S+ and S- are external surfaces, which are charged by q_+^i , q_-^i . Ω_σ and Ω_u are boundary surfaces which are charged by boundary forces q_o^i or boundary displacement v_{oi} . Vector ν_j^* is a unit normal vector to Ω and C^{ijkl} is an elasticity tensor.

We define the internal forces as follows (Naghdi , 1963):

$$N^{\alpha\gamma} = \int_{h_\xi} \mu \sigma^{\alpha\beta} \mu_\beta^\gamma d\xi , \qquad N^{\alpha 3} = \int_{h_\xi} \sigma^{\alpha 3} \mu \, d\xi ,$$

$$M^{\alpha\gamma} = \int_{h_\xi} \mu \sigma^{\alpha\beta} \mu_\beta^\gamma \xi d\xi , \tag{2}$$

$$M^{\alpha 3} = \int_{h_\xi} \sigma^{\alpha 3} \mu \xi d\xi , \qquad N^{33} = \int_{h_\xi} \sigma^{33} \mu \, d\xi ,$$

where μ_β^γ is a translator, and $h_\xi = \langle -H, -h \rangle \cap \langle h, H \rangle$ is shown in Fig. 2. We determine Hu-Washizu functional by two dimentional fields defined on the fundamental surface:

$$e_{\alpha\beta} = \frac{1}{2} (\mu_\alpha^\gamma \varepsilon_{\gamma\beta} + \xi \mu_\alpha^\gamma \varkappa_{\gamma\beta} + \mu_\beta^\gamma \varepsilon_{\gamma\alpha} + \xi \mu_\beta^\gamma \varkappa_{\gamma\alpha}) ,$$

$$e_{\alpha 3} = \frac{1}{2} (\xi \ \varepsilon_{\alpha 3} + \gamma_\alpha) , \tag{3}$$

$$e_{33} = \gamma_3 ,$$

where $\varepsilon_{\gamma\beta}$, $\varepsilon_{\alpha 3}$, $\varkappa_{\gamma\beta}$, γ_α , γ_3 are deformations on the fundamental surface described by no more than the first derivatives of displacement fields.

In order to take into account the influence of the ribs we put on fundamental surface along the line L / fig. 3 / the following constraints:

$$\gamma_{(\tau)} = \gamma_\alpha \tau^\alpha = (\delta_\alpha - \beta_\alpha) \tau^\alpha = 0, \tag{4}$$

$$\gamma_3 = \delta_3 = 0,$$

(τ) no summing in brackets.

The determined functional for S' area treats forces on the line L from S'' area as the external forces for S' area.

They are

$$N^{''\alpha\beta}, \quad M^{''\alpha\beta}, \quad N^{''\alpha 3}, \quad M^{''\alpha 3} .$$

If we complete functional Π, by functional Π', which we define:

$$\Pi' = \int_L \Big[\lambda_3 \, \delta_3 + \lambda(\tau) \, \tau^\alpha (\delta_\alpha + u_{3,\alpha} + B_\alpha^\beta \, u_\beta) -$$

$$- N^{''\alpha\beta} \nu_\alpha \, u_\beta - M^{''\alpha\beta} \nu_\alpha \, \delta_\beta - \tag{5}$$

$$- N^{''\alpha 3} \nu_\alpha \, u_3 - M^{''\alpha 3} \nu_\alpha \, \delta_3 \Big] dL ,$$

we shall obtain $\Pi^* = \Pi + \Pi'$, the functional which describes our problem. $\lambda(\tau)$, λ_3 are Lagrange multipliers, and are reaction forces connected with constraints. Using the du Bois-Reymond lemma we obtain all the fundamental equations for shells together with compatibility equations which we describe on the lines of the ribs.

Equations of equilibrium:

$$\begin{aligned} & N^{\alpha\beta}|_\alpha + F^\beta - N^{\alpha 3} B_\alpha^\beta = 0 , \\ & N^{\alpha\beta} B_{\beta\alpha} + F^3 + N^{\alpha 3}|_\alpha = 0, \\ & M^{\alpha\beta}|_\alpha + L^\beta - N^{\beta 3} = 0 , \\ & M^{\alpha\beta} B_{\beta\alpha} + L^3 + M^{\alpha 3}|_\alpha - N^{33} = 0 . \end{aligned} \tag{6}$$

Geometric equations:

$$\begin{aligned} & \varepsilon_{\beta\alpha} = u_{\beta|\alpha} - B_{\beta\alpha} \, u_3 , \quad \varkappa_{\beta\alpha} = \delta_{\beta|\alpha} - B_{\beta\alpha} \, \delta_3 , \\ & \varepsilon_{\alpha 3} = \delta_{3|\alpha} , \quad \gamma_\alpha = \delta_\alpha - \beta_\alpha , \quad \gamma_3 = \delta_3 . \end{aligned} \tag{7}$$

Constitutive equations:

$$N^{\alpha\beta} = E_6^{\alpha\beta\tau\varepsilon}\,\varepsilon_{\tau\varepsilon} + E_7^{\alpha\beta\tau\varepsilon}\,\varkappa_{\tau\varepsilon} + E_1^{\beta\alpha 33}\,\gamma_3 ,$$

$$M^{\alpha\beta} = E_7^{\alpha\beta\tau\varepsilon}\,\varepsilon_{\tau\varepsilon} + E_8^{\alpha\beta\tau\varepsilon}\,\varkappa_{\tau\varepsilon} + E_2^{\beta\alpha 33}\,\gamma_3 ,$$

$$M^{\alpha 3} = E_3^{\alpha 3\gamma 3}\,\varepsilon_{\gamma 3} + E_4^{\alpha 3\gamma 3}\,\gamma_\gamma , \tag{8}$$

$$N^{\alpha 3} = E_4^{\alpha 3\gamma 3}\,\varepsilon_{\gamma 3} + E_5^{\alpha 3\gamma 3}\,\gamma_\gamma ,$$

$$N^{33} = E^{3333}\,\gamma_3 + E_1^{\alpha\beta 33}\,\varepsilon_{\alpha\beta} + E_2^{\alpha\beta 33}\,\varkappa_{\alpha\beta} .$$

Kinetic and static boundary conditions:

$$\left.\begin{array}{llll} u_\beta = u_{0\beta} , & \delta_\beta = \delta_{0\beta} , & & \\ & & & \text{on } C_u \\ u_3 = u_{03} , & \delta_3 = \delta_{03} , & & \\ & & & \\ N^{\alpha\beta}\nu_\alpha = T_1^\beta , & M^{\alpha\beta}\nu_\alpha = T_2^\beta , & & \\ & & & \text{on } C_\sigma \\ N^{\alpha 3}\nu_\alpha = T_3 , & M^{\alpha 3}\nu_\alpha = T_4 . & & \end{array}\right. \tag{9}$$

N^{ij} , $M^{\alpha i}$, T_i^α are respectively generalized internal forces, or boundary loading. $E_1^{\alpha\beta 33}$ and others are elasticity tensors. Compatibility equations we describe as follows:
Kinetic conditions:

$$\begin{aligned} \delta_3 &= 0 , \\ (\delta_\alpha - \beta_\alpha)\,\tau^\alpha &= 0 . \end{aligned} \tag{10}$$

Static conditions:

$$
\begin{aligned}
&(N^{\alpha\beta} - N''^{\alpha\beta})\,\nu_\alpha + B^\beta_\alpha\,\lambda_{(\tau)}\,\tau^\alpha = 0\,, \\
&(M^{\alpha\beta} - M''^{\alpha\beta})\,\nu_\alpha + \lambda_{(\tau)}\,\tau^\beta = 0\,, \\
&(M^{\alpha 3} - M''^{\alpha 3})\,\nu_\alpha + \lambda_3 = 0\,, \\
&(N^{\alpha 3} - N''^{\alpha 3})\,\nu_\alpha + \frac{\partial}{\partial L}\left(\lambda_{(\tau)}\,\tau^\alpha\,\overline{\varepsilon}_{\alpha\gamma}\,\nu^\gamma\right) = 0\,.
\end{aligned}
\qquad (11)
$$

Based on this no published yet work there were prepared the following papers (Kutyłowski , 1988 , Myślecki , 1988). In this work there were shown equations for the shells with constraints put on the lines of ribs in normal direction. In paper (Myślecki , 1988) there were shown equations for constraints put on the lines of ribs in tangent direction.

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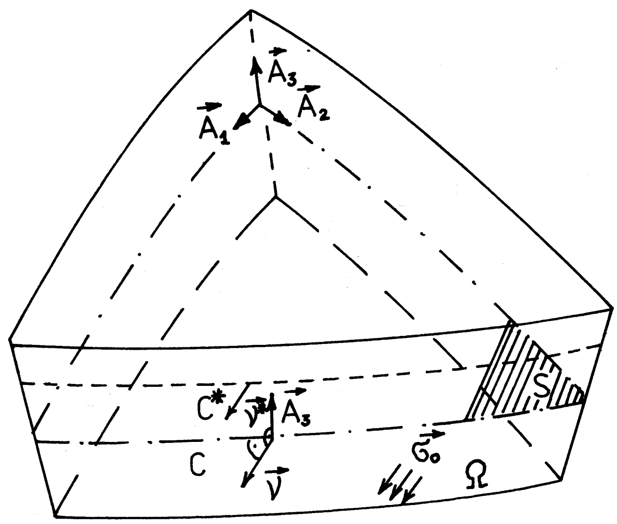

Fig. 1.

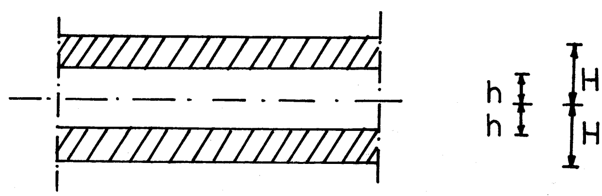

Fig. 2.

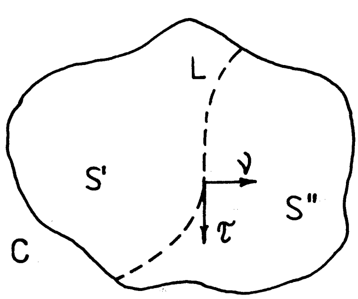

Fig. 3.