Structural Damage Analysis Using Sensitivity Method

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ABSTRACT

This paper deals with several methods wich, knowing experimental dynamic data of the actual structure and the computed dynamic characteristics of the theoritical initial structure, estimate the modification of physical parameters as cross-section of beam, thickness of shell, ...

This process consists in three parts. The first one is a correlation analysis between numerical results and experimental data. The next one is the location of errors in the finite element model. For this purpose, many different techniques are used so as to give a greater degree of confidence in the error localisation. The last one is to tune physical parameters to update numerical modelisation of the structure. For this tuning, an iterative process updating structural parameters coupled with a sensitivity method is performed. These techniques are based on the combined use of natural frequency sensitivity, orthogonality conditions of the eigenmodes and the minimization of a functional built by a least square method.

The capabilities shall be implemented in a next release of SYSTUS general purpose computer code developed by FRAMASOFT.

I - EXTENSION OF EXPERIMENTAL MODES AND CORRELATION ANALYSIS

We assume that the experimental eigenmodes and eigenvalues are real, and the measured components of eigenmodes are located at some nodes of finite element mesh used for the numerical analysis.

The extension of measured eigenmodes can be obtained by solving a linear system.

$$(K - \omega_e^2 M) \begin{bmatrix} F_{ex} \\ F_{exm} \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix} \qquad (1)$$

Where K is the numerical stiffness matrix

M is the numerical mass matrix

F_{exm} is the measured components of the experimental eigenmodes

F_{ex} is the extended components of the experimental eigenmodes

ω_e is the measured eigenvalue

R is the modal reaction

ω_e must not be an eigenvalue of the structure with fixed degrees of freedom where the eigenmodes are measured. In practical applications it is not the case.

An other algorithm to obtain an extension of experimental eigenmodes is to project experimental modes on the set (or a subset) of numerical eigenmodes and to use a least square method as it is described below.

$$\left.\begin{aligned} F_{exm} &= V \ \alpha \\ \alpha &= (V^T V)^{-1} \ V^T \ F_{exm} \end{aligned}\right\} \Longrightarrow \ F = \alpha V \qquad (2)$$

F : measured modal vector
V : numerical modal vector

A measured and analytical modal vector describing the same mode shape are perfectly proportional to each other. Based upon this consideration one can define a modal scale factor and a correlation coefficient, called modal assurance criterion

$$\text{MSF} \ (V, \ F) = \frac{V^T \ F}{F^T \ F} \qquad \text{MAC} \ (V, \ F) = \frac{\left| V^T \ F \right|^2}{(V^T V) \ (F^T F)}$$

The MSF parameter is an average proportionality factor between two vectors. The MAC parameter can take any value between 0 and 1. A value close to 1 indicates a good consistancy. These parameters can help to identify which experimental mode is correlated to an given numerical mode.

II - ERROR LOCALISATION ANALYSIS

To localise the modifications made to the theoritical model we can use several analyses we describe here. In an industrial application it is advisable to perform several different analysis so as to have a good estimation of error localisation.

2.1. Modal residue

Using equation (1) the modal residues R allow to calculate an error on the measured degrees of freedom for each mode.

$$g_i = \sup_k \left| \ r_{ik} \right| / \ n$$

with k : number of the eigen mode
i : number of measured degree of freedom
n : a normalisation factor given by :

$$n = \sup_i \ (\sup_k \left| r_{ik} \right| \ )$$

with these values we can obtain a first approximation of the area where the error is dominant.

2.2. Modal difference vector

The modal difference vector is given by :

$$\text{MDF} = F - \text{MSF} * V$$

The absolute values of the difference vectors for each mode may be summed to obtain an approximation of the error area.

Before applying this formula we must sure that there is a good consistency between the measured mode and the numerical mode. For this purpose, we use MAC parameter.

2.3. Flexibility error matrice

The flexibility error matrice is given by :

$$FEM = \sum_{i=1,N} F_i \, \omega_{ei}^{-2} \, F_i^T - V_i \, \omega_{ai}^{-2} \, V_i^T$$

We assume in this formula all eigenmodes are normalized in respect to the mass matrix. The greatest extradiagonal terms of FEM give som indications of the error area.

2.4. Results of error localisation analysis

We can build the error area by examinating which nodes occur most often in the results of the different error localisation methods which have been performed.

At this step, we can have a good idea of kind of error :

systematic error
boundary conditions error
localised error

If a systematic error is analysed, a global tuning of some physical parameters must be performed firstly.

III - UPDATING PROCEDURE

As for error localisation analysis several methods can be used for modifying some parameters of the numerical model.

3.1. Method using modal residues

This method used the equation (1) and need only to solve only static problems. The purpose is to minimize the modal residues of equation (1). We can write (1) on the following form :

$$(K + \sum_j \lambda_j K_j - \omega_e^2 M) F_k = \begin{bmatrix} 0 \\ R_k \end{bmatrix} \qquad (3)$$

Where λ_j represent the error on the mechanical parameters and K_j an elementary stiffness matrix in the error area determined previously. We assume here we have modifications only on the stiffness matrix, but the same process can be employed with mass matrix modification. We obtain the values of λ_j so as to R vanishe by an iterative process.

At the iteration p, R is linearised in respect to λ_j :

$$\left. \begin{aligned} R_k^{(p+1)} &= R_k^{(p)} + \nabla R_k^{(p)} \quad \Delta\Lambda(p) \\ \Delta\Lambda_{(p)}^T &= [\Delta\lambda_1^{(p)}, \Delta\lambda_2^{(p)}, \ldots \Delta\lambda_m^{(p)}] \end{aligned} \right\} \qquad (4)$$

We minimize the functional J

$$J = \sum_{k} R_k^{T(p+1)} \; R_k^{(p+1)} \qquad (5)$$

using a Newton algorithm : $(\sum_k \nabla R_k^{T(p)} \; \nabla R_k^{(p)}) \; \Delta\Lambda_{(p)} = -\sum \nabla R_k^{T(p)} \; R_k^{(p)}$ (6)

At each iteration, the gradient $\nabla_j R_k$ is obtained by derivaiton of (3).

$$\frac{\partial R_k}{\partial \lambda_j} = K_j \, F_k$$

This process can be expensive if we use a great number of parameters.

3.2. Methods using sensitivities

The global stiffness matrix K and mass matrix M depend on some parameters p. The parameters can be physical one or proportional coefficients applied on a set of elementary stiffness or mass matrices. We wish to obtain by updating parameters of dp the relations induced by orthogonality properties of modal vector :

$$F_i^t \; K \; (ps + dp) \; F_j = \omega_{ei}^2 \quad \delta_{ij} \qquad F_i^t \; M \; (ps + dp) \; F_j = \delta_{ij}$$

Using a first order taylor expansion

$$M \; (ps + dp) = M \; (ps) + \frac{\partial M}{\partial p_k} \quad dp_k$$

$$K \; (ps + dp) = K \; (ps) = \frac{\partial K}{\partial p_k} \quad dp_k$$

We obtain :

$$\delta_{ij} = m_{ij} + \sum F_i^T \; \frac{\partial M}{\partial p_k} \; F_j \; dp_k$$

$$\omega_{ei}^2 \; \delta_{ij} = k_{ij} + \sum F_i^T \; \frac{\partial K}{\partial p_k} \; F_j \; dp_k \qquad (7)$$

A other set of equation can be constructed using eigenvalue :

$$\omega_{ei} = \omega_{ai} + \sum (\partial\omega_i / \partial_{pk}) \; dp_k \qquad (8)$$

Equations (7), (8) can be written together in the following form

$$q = qs + S \; dp$$

qs represents the state for current values of the parameters ps

q the desired state

S sensitivity matrix of the current state for parameter change dp.

Before to build sensitivity matrix, we must select significant parameters.the error localisation analysis is an efficient tool for this selection. We shall eliminate also parameters associated to a low energy in respect to the global energy of the structure.

In the system (9), some parameters are relative to the eigenvalue and other ones to eigenmodes, also we must normalize these equations and introduce adimensional values.

A first solution is to use a least square method for solving (9) by minimisation of :

$$\| qs + S \; dp - q \|_2$$

Industrial applications show that this solution is not the best one, it has a propensity to give a global solution where composants correponding to unsensible area are surestimated. We can observe also some compensation phenomena : the increase of parameters in an area involving a decrease of the same parameters in the neighbourhood.
An other solution is to select a subset of parameters p by evaluation of

$$e = \| q - qs \|_2 / \| qs \|_2$$

The evolution of e in respect to the choosen parameters give good informations on the influency of each parameter. A significant residual value of e can come from a systematic error.
The best solution is to take into account the initial values of the parameters and to minimise the following functional :

$$\omega e \; \| qs + S\, dp - q \|_2 + Wa \; \| ps + dp - p0 \|_2$$

We and Wa are diagonal matrices which express confidence in each experimental data and in the initial value (i.e. numerical value) of the parameters.

All the sensitivity methods must be included in an iterative process where the S matrix is updated at each iteration.

IV - CONCLUSIONS

There is not only one efficient method to correlate experimental data and numerical results but the set of the methods presented here is a good tool for understanding the discrepancy between the two approaches and the actual behavior of the structure.

Industrial applications of these techniques can be :

Structural damage analysis
Analysis of structure modification
Parametric optimisation

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