The Impact of Boundary Conditions on the Buckling of Cylindrical Shells

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INTRODUCTION

During the last ten year, many theoretical as well as experimental studies for designing the first large size Liquid Metal Fast Breeder Reactor (LMFBR) have been achieved. Many of the sensitive parts of this reactor are thin shells subjected to high temperatures and loads. Special care has been given to buckling because it often governs design.

The aim of this study is to observe, by means of numerous experiments and f.e.m calculations the influence of boundary conditions on the buckling of different types of cylindrical shells.

Two cases are studied : first, the elastic buckling of corrugated shells under external lateral pressure and second, the plastic buckling of axially compressed cylindrical shells. Note that corrugated shells are often proposed in the design of LMFBR's projects because they are space-saving.

During the tests, two types of boundary conditions are studied. One being a near perfect clamping, the other being a mechanical grip allowing very slight axial displacements on axisymmetrical mode n=0 but also on modes n≠0.

The aim of this paper is to see if the boundary conditions which are generally taken into account in buckling design calculations are realistic or, at least, conservative.

EXPERIMENTAL METHOD

The cylindrical shells are obtained by electrodeposition of nickel on a machined support which is then dissolved (for more details, see WAECKEL 1984).

The two types of boundary conditions mentioned earlier, are given fig. 1. The first one (fig. 1a) is a mechanical grip. In the case of lateral pressure loading it allows very slight axial displacements (a few microns), and in the case of axial compression it is not sufficient to avoid any influence of small flatness imperfections of the edges due to manufacturing process.

In the opposite, if rigid metallic rings are intergrated to both extremities of the specimens during the electrodeposition process, the clamping conditions are near perfect (i.e. CL1 conditions \( w = w_{xx} = x = xy = 0 \) or,

SS1 conditions \( w = w_{xx} = x = xy = x_0 \)).

All the tested shells are free from initial geometrical imperfections. So, only the influence of boundary conditions is studied.

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ELASTIC BUCKLING OF CORRUGATED SHELLS UNDER LATERAL PRESSURE

Initial geometry

The geometries of the different tested shells are given in the Table 1. The corrugated shells, which present coaxial cylindrical and conical parts, are supposed to give a better critical pressure than a simple straight cylindrical shell with similar geometrical characteristics (Aflak, 1987). They are also interesting because they are space-saving in design of the compact reactor block.

Results

The load-displacement curves are given fig. 2 and 3. $P_{EO}$ is the critical pressure of a straight cylindrical shell obtained with a simplified formula (Yamaki, 1984).

By comparing the fig. 2 and 3, it can be shown that the effect of boundary conditions may be much greater than the effect of geometrical imperfections, especially for double-curved shells under external pressure. The slight axial play, in mechanical grip, leads to reduce drastically the critical load values. This means that critical load can be reduced from 20% for a straight cylindrical shell to 44% and 75% for a convex and a concave shell of Euler's theoretical load! (fig. 2). In the opposite, with clamped end conditions (fig. 3), critical load values approach Euler's to within 2% for straight cylindrical shells, 5% for convex shells and 8% for concave shells.

Numerical interpretation

The buckling occurs in the elastic range. When the end conditions are well known (i.e. clamped conditions CL1, fig. 3), linear Euler buckling load computations are sufficient to approach experimental values with a very good agreement.

In the opposite, if slight axial displacements are allowed at both extremities of the shells, the end conditions can be modelized with elastic springs whose stiffnesses are analytically determined and then introduced in the computations (Debbaneh, 1988). Results are in good agreement with experiments.

From a design point of view, it is preferable to use the following method to calculate thin structures which have not well defined end conditions: the computations are performed by using the CL3 end conditions (i.e. the axial displacements are free on circumferential modes $n\neq 0$ but equal to zero on axisymmetrical mode $n=0$) with the quasi axisymmetric element "COMU" of INCA code (Combescure, 1985). The results given in fig. 4 show that such approach is conservative.

PLASTIC BUCKLING OF AXIALLY COMPRESSED CYLINDRICAL SHELLS

The tested specimens are straight cylindrical shells with the following properties: Radius $R = 75 \times 10^{-3}$ m, height $L = 150 \times 10^{-3}$ m and thickness $t = 600 \times 10^{-6}$ m.

Experimental results

In the case of plastic buckling of axially compressed cylindrical shells, a non-homogeneous stress distribution linked to imperfect shell/support contact, influences global behaviour of the shell such as slight decrease in critical load and heavy decrease in initial axial stiffness.

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The effect of boundary conditions on critical load is slight. The experimental results approach the theoretical buckling load of Gerard's formula (Gerard, 1956) to within 13% with 9% of scattering for a mechanical grip and 5% with 3% of scattering for a near perfect clamping (fig. 5).

In the opposite, if we compare the axial stiffness during the pre-buckling range for the two types of boundary conditions (fig. 6 and 7), we observe that slightly imperfect ends conditions (fig. 6) lead to under-estimate the real axial stiffness of the shell according to the Young modulus of the material.

**Numerical interpretation**

The non-linear computations with the INCA code (Combescure, 1985) give numerical values (buckling loads and initial axial stiffness) which are in good agreement with experimental results for the near-perfect clamped conditions (fig. 7).

As the shell/support contact imperfections linked to the manufacturing process, are very difficult to modelise, we do not attempt to simulate them. These contact imperfections, even if they are very slight, lead the shell to behave as independant columns. The non-axisymmetric shortenings create a non-homegeneous stress distribution in the shell increasing whith plastification. This mechanism reduces the buckling resistance of the structure and increases the pre- and post-buckling deflections.

From a design point of view, to modelise imperfect ends conditions (general cases) by clamping conditions (CL1 or SS1) may lead to under-estimate the deformations of the structure. Nevertheless, in the case of axially compressed cylinders (125 < R/t < 450), the buckling load itself is not really affected by the boundary imperfections. On the rebound, it is necessary to take into the initial geometrical imperfections.

**CONCLUSION**

Designing thin S curve shells under external pressure without carefully taking into account the real boundary conditions can be dangerous.

It is shown that little axial displacements of the edges (a few microns is enough !) lead to decrease drastically the critical pressure value. In certain cases (inward directed curves) critical load can be reduced to 75% of Euler's theoretical load computed with clamped edge ! A conservative design method consists in computing elastically the shell, assuming that the edges are allowed to displace axially on circumferential modes n≠0. This method requires a quasi-axisymmetric element if 2D calculations are performed.

In the case of plastic buckling of axially compressed cylindrical, it is shown that non homogeneous stress distribution linked to imperfect shell/support contact decreases slightly the critical load but reduces strongly the axial stiffness. Consequently, one must be conscious, that computing structures with ideal clamped boundary conditions can lead to under-estimate the deflection of the shell during the pre-buckling range. This can be incompatible with functional requirements.

**ACKNOWLEDGEMENTS**

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REFERENCES


Combescur, A. (1985). Static and dynamic buckling of large thin shells SMIRT 8


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**FIGURE 1  BOUNDARY CONDITIONS**

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**Table 1 - Geometries**

<table>
<thead>
<tr>
<th>L (m)</th>
<th>R (m)</th>
<th>(a_{ew} (m))</th>
<th>(L_{^1} (m))</th>
<th>(L_{^2} (m))</th>
<th>(L_{^3} (m))</th>
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<td>straight</td>
<td>150</td>
<td>81.5</td>
<td>0.175</td>
<td>-</td>
<td>-</td>
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<tr>
<td>convex</td>
<td>150</td>
<td>81.5</td>
<td>0.160</td>
<td>35</td>
<td>40</td>
</tr>
<tr>
<td>concave</td>
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<td>81.5</td>
<td>0.163</td>
<td>35</td>
<td>40</td>
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</table>
\[ P_{E0} = 1.3775 \frac{E}{C} R L \left( \frac{t}{R} \right)^{5/2} \]  

Figure 2 - Load-displacement curve - Mechanical grip

Figure 3 - Load-displacement curve - Clamped edges
Figure 4 - Load-displacement curves

Figure 5 - Experimental results - Axially compressed cylindrical shells.

Figure 6 - Load-displacement curve - Mechanical grip

Figure 7 - Load-displacement curve - Clamped edge