Crucial Effect of Boundary Conditions on the Buckling of Shells

Alain Combescure
CEA–CEN Saclay, Gif sur Yvette, France

INTRODUCTION

We want to show here that the boundary conditions can have a drastic effect on the prediction of buckling loads and that this influence can even be more drastic that the initial imperfection in some cases.
This is a very important feature because the design rules in case of buckling take into account the effect of initial imperfections but do not consider the effect of unknown boundary conditions.
We shall first give two examples showing this drastic effect and then developp a methodology to prevent the designer against over estimation of buckling loads due to a non conservative choice of boundary conditions.

Two examples

* Auto stiffened cylinder under constant external pressure.
The first case consists of a so called "auto-stiffened" cylinder.
The geometry is defined on figure 1.
The cylinder consists in fact of two cylindrical parts separated by two conical shells with a very low conicity.
The material is behaving elastically and has the following properties :
Young's modulus : 172 300 MPa. Poisson's ratio : 0.3.
The experimental buckling pressure is 0.021 MPa. If one computes the elastic buckling pressure with the normal boundary condition (both ends clamped) one obtains a critical pressure of 0.078 MPa. This buckling pressure is associated with the Fourier mode 7.
You can observe the buckling mode on figure 2. The calculation is performed with the INCA code of CASTEM system.
One can imagine that the difference with experimental behaviour is due to imperfections. The prediction of the behavior of the imperfect non axisymmetric structure has been performed with the INCA program using the special COMU element [2]. The same initial imperfection as the measured one has been introduced. Large displacement and elastoplasticity have been taken into account. With all these inputs the critical pressure is decreased only by 10 %. From this analysis one must admit that the difference between experimental and theoretical prediction of buckling pressure is not due to either plasticity nor initial imperfection. It is also not due to the combination of the two effects.
The stability analysis is then rerun with different boundary conditions at the extremities of the cylinders.
This time the structure is allowed to have a free axial displacement while buckling. With this hypothesis, called CL3, the critical elastic pressure drops to 0.012 MPa (Mode 5).
The buckling mode is shown on figure 3. This shows that the shell is very sensitive to the axial restraint boundary conditions. The experimental boundary condition is closer to the free end one than the clamped one.
Figure 4 shows the comparison of experimental buckling mode with the different predicted buckling mode. We see here that the free end buckling mode is much closer to experimental one.

If one adds an axial stiffness at both ends of the cylinder one finds exactly the experimental observed buckling pressure.
The figure 5 shows the final prediction taking into account initial imperfection and plasticity that coincides perfectly with experimental observation.
One sees here clearly that in the case of cylinder under external pressure the boundary condition can decrease the buckling load much more than the initial imperfection.

* Cylinder under wind load.

a) Problem

A short cylinder under wind load is then studied. The diameter if 20 m. The high 5 m and the thickness 3.4 mm.

The material has the following properties : Young's modulus 2.1 10^{11} Pa
Poisson's ratio 0.3

The cylinder is loaded by the wind which is considered as a pressure constant along the axial direction and varying along the the circonferential one.
Figure 6 gives an approximation of the wind load. This shape can be decomposed in Fourier series and a good approximation is obtained with the 5 first symmetric cosine Fourier modes. This leads to the following coefficients for the cosine functions :

Fourier mode N°	0	1	2	3	4
Coefficient	- 1.1	.56	1.8	.94	- .2

b) Stress analysis

Two stress analysis are performed. The first one is performed with a clamped basis and produces an initial stress field that we shall call σ_1.

The second one is performed with a free axial boundary condition on all non axisymmetric modes. This one produces a stress field σ_2. The COMU element of INCA program is used for the computation.

c) Stability analysis

A stability analysis is then performed with the two initial stress fields. The buckling analysis is also performed with INCA program of CASTEM system. Two buckling analyses are performed. One with campled basis, the other with free axial boundary condition on all non zero Fourier modes. We shall call CL1 the clamped boundary conditions and CL3 the free axial one.

We obtain the following results, for critical pressure and critical mode basis :

Prestress / Boundary Conditions	σ_1	σ_2
CL1	3.16 (15-29)	/
CL3	0.725 (1-15)	0.79 (2-16)

Figure 7 shows the mode associated with CL1 and σ_1.

Figure 8 shows the mode associated with CL3 and σ_2.

We see clearly from this table and from the observation of the deformed shape that the axial boundary condition have a big influence on the buckling load due to the wind load. There is a facotr of roughly 4 between the two boundary conditions. The comparison shows also clearly that this drastic drop is due to a drop stiffness and not a change in the stresses, because the critical load with CL3 and σ_1 is close to the load associated with CL3 and σ_2 (less thant 10 % difference). The clamped axial boundary conditions stops the possibility of buckling with low order Fourier's modes and stiffens a lot this short cylinder.

In ref. [3] experimental results are given to asses the design of such shell. The shell is a waste storage tank, built on a soil fondation. One has certainly a bad knowledge of the soil axial stiffness.

The shell is modelled by a cylinder clamped on a very thick plate and put in a wind tunnel. This experimental study provides without any doubt good informations on the buckling loads and buckling pressure fields. But this experiment can be very far from actual behaviour of the storage tank. In experimental set up the shell is clamped at its base. In reality one can think that the boundary condition is closer to free end than clamped one. This shell being very sensitive to axial boundary conditions, the model could very well be not representative of the true behavior.

Design Methodology

* Influence of boundary conditions on buckling.

If one wants to predicts by calculation the buckling load, one generally makes a mesh of the part of the structure which is supposed to buckle. The problem arises because one has to choose an appropriate boundary condition at the limit of the part of the structure modelised.

* Methodology

a) The rational way would be to built one super element at the end of the modelised part. This procedure is possible, but some times heavy to handle in particular in three dimensional situations. It may also be rather unrealistic because the ideal modelised boundary conditions is not very representative of the reality.

b) Simplified analysis. Two sets of boundary conditions are to be taken into account for the bifurcation analysis.

First the clamped boundary condition must be studied.

Second tangential free displacement must be evaluated. The buckling load must be compared in the two extreme cases. If there is no much difference between the two buckling loads one can use any of them to make the prediction of buckling taking into account plasticity and initial imperfections. If the structure is sensitive to the boundary condition the free end boundary condition must be taken as valid for the more detailed non linear buckling analysis.

CONCLUSION

The boundary condition has been shown to have a drastic effect on the buckling of cylinders under external pressure or on more complex structure. The decrease in buckling load can be of a factor 4 between the clamped end case and the axial free end one. The free end prediction is closer to observed experimental buckling pressure.
It is therefore proposed to introduce in the design methodology against buckling, a procedure that gives the good boundary condition to be used for realistic predictions. This method compares well with experimental results.

REFERENCES

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[3] Stability of plate and shell structures - Proceeding of International Colloquium Held 6-8 April 1987 Ghent - Belgium - pp 529-537 - Stability of Large diameter thin walled steel tanks subjected to wind loadings.

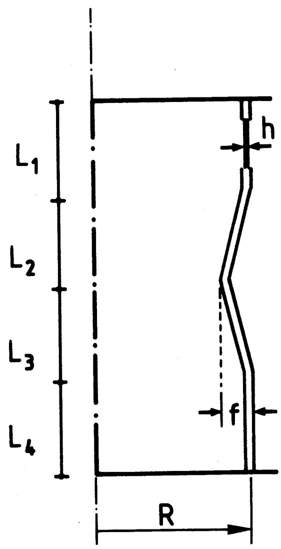

Fig. 1 : Geometry 10^{-3} m
$L_1 = L_4 = 35 \quad h = .17$
$L_2 = L_3 = 40 \quad f = 2$
$R = 75$

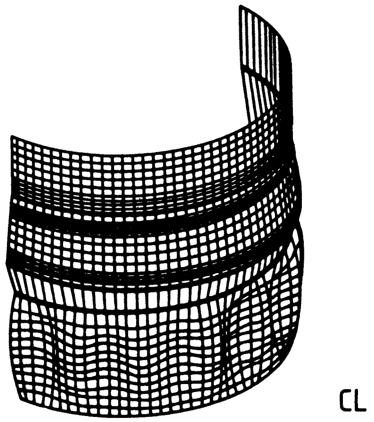

Fig. 2 : Computed clamped buckling mode

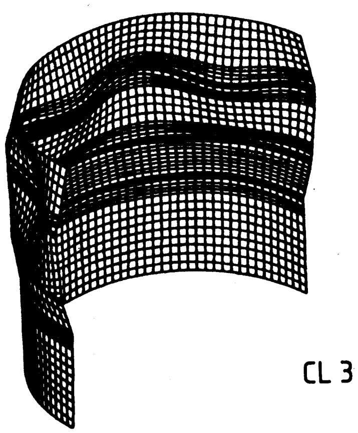

Fig. 3 : Computed axial free end buckling mode

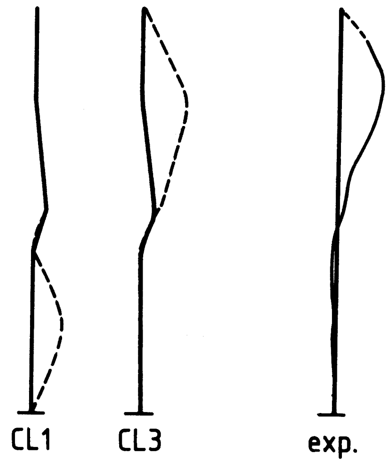

Fig. 4 : Theoretical experimental comparison

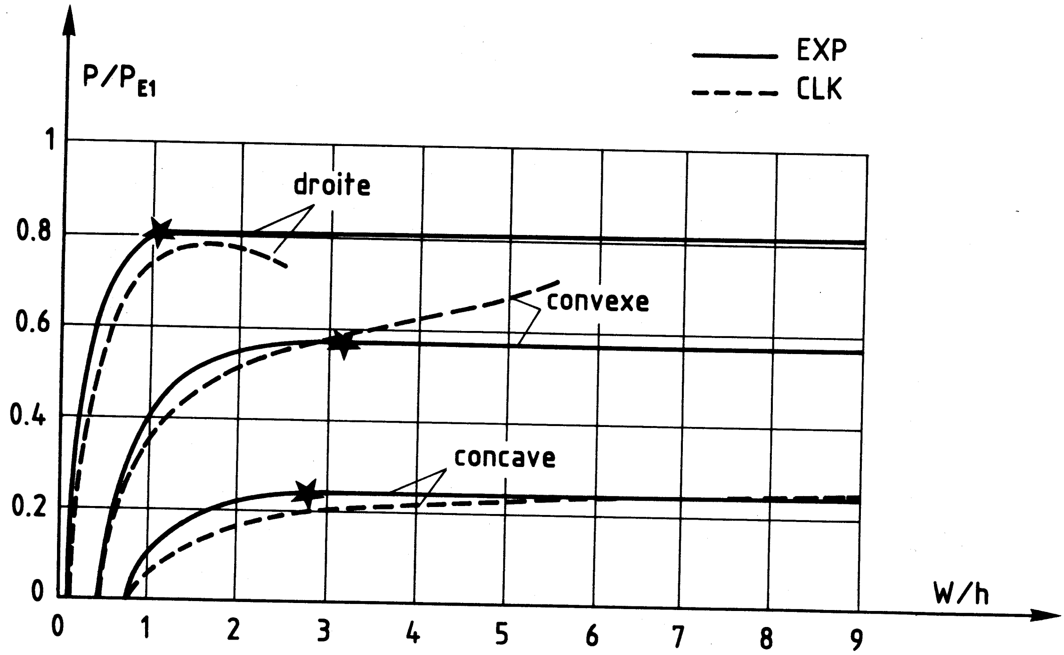

Fig. 5 : Non linear prediction comparison with experiment

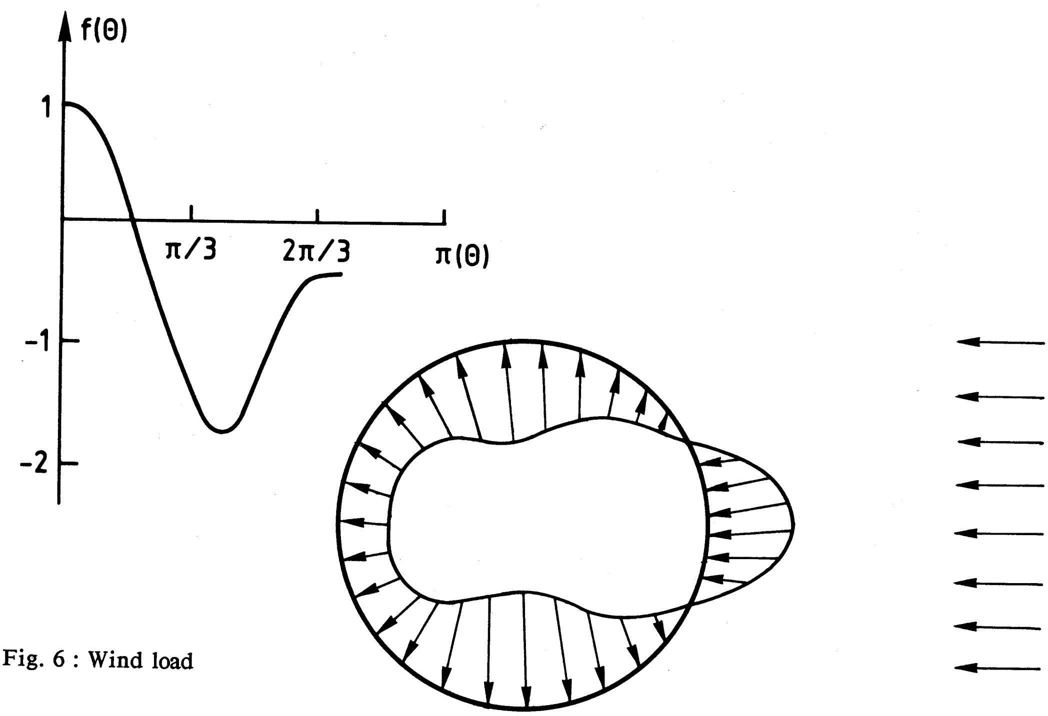

Fig. 6 : Wind load

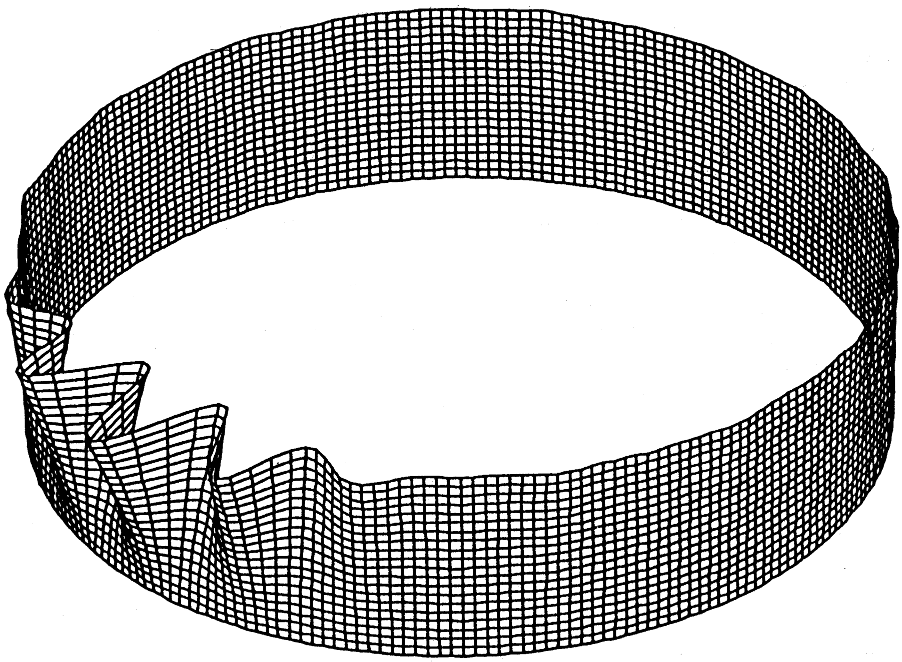

Fig. 7 : Clamped buckling mode

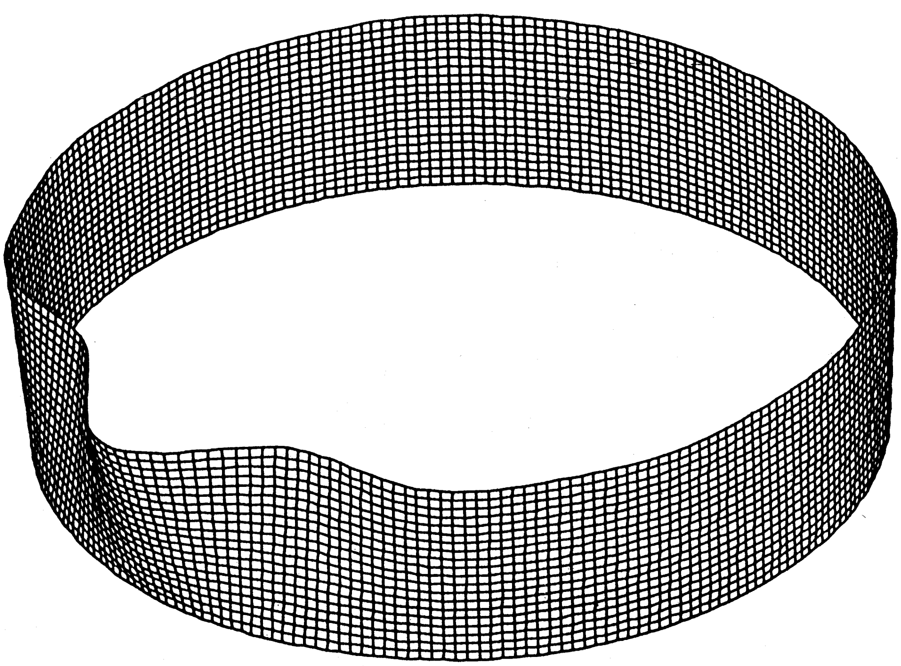

Fig. 8 : Free end buckling mode