Design Rules for Piping: Experimental Validation of Flexibility and Elastic Stress Indices for Elbows Under Bending

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ABSTRACT

Design rules for class 1 piping components are based on stress indices (B, C, K) and flexibility factors (k). For elbows, adjacent straight parts and internal pressure inhibit ovalization of the cross-section, so reducing the sub-mentioned indices.

Published theoretical works and experimental results allow for improvement of coded values.

End effect may be represented by a suitable function of the elbow angle.

The favourable effect of pressure on C_2, for fatigue damage evaluation, can be taken into account.

NOTATIONS

\[ e = \text{thickness} \]
\[ R = \text{mean radius of curvature} \]
\[ D = \text{external diameter } r = D/2 \]
\[ r_m = \text{mean radius of the pipe} \]
\[ h = \text{elbow characteristic parameter} \]
\[ h = (eR)/r_m^2 \]
\[ M = \text{applied moment} \]
\[ M_1 = \text{torsion} \]
\[ M_2 = \text{in-plane bending} \]
\[ M_3 = \text{out of plane bending} \]
\[ \alpha_0 = \text{elbow angle} \]

INTRODUCTION

Design rules for class 1 piping components are based on stress index method. Stress indices (B,C,K) and flexibility factors are used to correlate actual behaviour of a piping system to a basic beam method analysis.

For Fast Breeder Reactors code (RCCMR, 1985) indices defined in ASME Section III article NB3682 (ASME, 1983) have been considered as well, both for class 1 and class 2 components. In the framework of the R & D European working group for Fast Breeder Reactor Structural Integrity (ACT9B), we aim to validate, and if necessary improve, indices used for creep-fatigue damage analysis: the flexibility factor k, which allows for the elastic representative computation of the
system, and the stress index $C_2$ which characterizes the maximum elastic stress in a component. As they have been originally defined for pressurized water reactors, attention will be particularly paid to thin walled components specific of PFR pipings.

This paper concerns curved pipes or elbows. The most important of the above mentioned factors is the influence of ovalization upon both rotations and stress distribution. The geometrical parameter $h$ characterizes the ability to ovalize of an elbow. But end effect may inhibit ovalization of the cross-section. Moreover, pressure, by inducing a stabilizing term in the rigidity matrix decreases both flexibility and maximal stress of the bend. These two effects will be the principal matter of study.

Analysis will be based on both theoretical works and experimental results.

**DESIGN CODE RULES PRESENTATION**

Design codes used in the nuclear industry (ASME, 1983) (RCC-M, 1985), (RCC-MR, 1985) give all the same formulations for the concerned indices:

**flexibility factor $k$:**

For bending: 

$$k_2 = k_3 = \frac{1.65}{h} \times \frac{1}{1 + \frac{r_m}{eE}}$$

$$X = 6 \left( \frac{r_m}{e} \right)^{4/3} \left( \frac{R}{r_m} \right)^{1/3}$$

For torsion: 

$$k_1 = 1.$$ 

**stress index $C_2$:**

$$C_2 = \frac{\sigma}{S} \quad \sigma = \text{maximum stress intensity}$$

$$S = \text{nominal stress} = M/Z$$

$$C_2 = \frac{1.95}{h^{2/3}} \quad C_2 > 1.5.$$ 

$k$ expression is derived from (Rodabaugh et al., 1957). $C_2$ formulation has been given in (Rodabaugh et al., 1978), considering a combination of $C_{22}$ and $C_{23}$, and resulting from a theoretical analysis using Von Karman formulation for ovalization:

$$C_{22} = \frac{1.8}{h^{2/3}} \quad C_{23} = \frac{1.5}{h^{2/3}}$$

with $C_{2M} = [(C_{21} M_1)^2 + (C_{22} M_2)^2 + (C_{23} M_3)^2]^{1/2}$

Code CASE (ASME-CASE, 1983) proposes alternative values for $k$ and $C_2$ resulting from a recent work (Rodabaugh and Moore, 1981). Proposed factors take into account the effect of the straight parts welded to the elbow upon the ovalization, by mean of a function of the elbow angle ($\alpha_0$). The specific effect of the three direction of load are considered.

**LITERATURE REVIEW**

$h$ effects: It is admitted by all authors that the flexibility factor is in inverse ratio to the geometrical parameter $h.$
For $C_2$ index of $90^\circ$ elbows, a $h^{-2/3}$ proportionality first proposed by (Beskin, 1945) has been confirmed by (Rodabaugh and Moore, 1981) which precised the index for out of plane bending ($-0.67$ is replaced by $-0.53$). (Rodabaugh and Moore, 1981), (Thomas, 1981) and (Thomson and Spence, 1983) propose a value of:

$$k = 1.3/h$$

for $90^\circ$ elbows.

**Ends effect:** As the propagation of ovalization is limited in the straight parts, an actual elbow is more rigid than a fictitious complete one ($\alpha_0 = 360^\circ$). Authors have pointed out that major parameters for these effects are distance between straight parts and the mean sections ($\alpha_0/2$) and the relative curvature ($R/r$) (Rodabaugh and Moore, 1978 and 1981) have carried out three dimensional elastic finite elements computations. (Thomson and Spence, 1983) have presented analytical solutions. Both results are presented in form of $k = f(\alpha_0)/h$, $C_2 = g(\alpha_0)h^{-1}(\alpha_0)$, where $f$, $g$ and $l$ are continuous or discrete functions.

Results ($k\times h = f(\alpha_0)$) are compared on figure 1 for in-plane bending flexibility factors: conclusion are very close (gap limited to 10%).

Comparison for $C_2$ depends of $h$ level. But angle effect is always greater or similar in (Thomson and Spence, 1983) than in (Rodabaugh and Moore, 1981): on figure 2, results are compared for $h$ value corresponding to available tests.

**Pressure effect:** Concerned parameters for pressure effect appear to be the nominal pressure stress ($P/r$), the geometrical parameter ($h$) and the relative curvature ($R/r$) [(Rodabaugh et al., 1957), (Dodge and Moore, 1972)].

Formulae for flexibility factors are compared on figure 3.

(Rodabaugh et al., 1957) correspond to $k$ coded value and can be written:

$$k(p) = \frac{1.65}{h} \frac{1}{1 + 6 \frac{S}{E} h^{-4/3} \left(\frac{R}{r}\right)^{5/3}}$$

(Dodge and Moore, 1972) propose results of analytical computations:

$$k(p) = \frac{1.65}{h} \frac{1}{1 + 1.75 h^{-4/3} \exp(-1.15 \psi^{-1/4})}$$

Results of the two formulae do not differ from more than 5% for $R/r$ in the useful domain ($2 < R/r < 3$).

Pressure, by reducing ovalization, reduces flexibility factors. The effect is all the more important as the elbow has a low characteristic parameter and as its relative curvature ($R/r$) is great. The second effect is quantitatively greater than the first one.

Expressions for $C_2$ index take similar forms:

(Rodabaugh et al., 1957)

$$C_2(p) = C_2(0) \frac{1}{1 + 3.25 \frac{S}{E} h^{-3/2} \left(\frac{R}{r}\right)^{13/6}}$$

(Dodge and Moore, 1972)

$$C_2(p) = C_2(0) \frac{1}{1 + h^{-4/3} \exp(-\psi^{-1/4})}$$

with $h' = h\sqrt{1-v^2}$

$$\psi = \frac{S}{E} \left(\frac{R}{r}\right)^2$$

$s = \frac{P}{r}$

$0.05 < h < 1$

$0 < \psi < 0.1$

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Comparison on figure 4 shows similar conclusions for the two formulae in the useful domain. Pressure effect which is not taken into account by codes, may reach 40% of the initial $C_2$ value.

**EXPERIMENTS**

Tests have been gathered up from different laboratories: tests performed at CEA/DEMT have been completed by those performed by (Teidoguchi, 1973), (Boyle and Spence, 1984), and (Rodabaugh et al., 1957). Flexibility factors have been determined on 55 tests. 30 tests were sufficiently instrumented to determine $C_2$ coefficient. Results are available for in-plane bending.

**Right-angle bends without pressure**

Figures 5 and 6 show $k$ and $C_2$, for in-plane bending as a function of the geometric parameter $h$. An upper value of the flexibility should be:

$$k = 2.1/h$$

But experimental results are largely squattered and coded value ($k = 1.65/h$) represents a mean behaviour.

For maximal stresses, a slope of $-0.67$ (on a logarithmic scale) is satisfactory. Stresses are maximum on the inner fiber of the elbow (without pressure) and a conservative value of $C_2$ can be derived from internal results:

$$C_2 = 1.75 \ h^{-2/3}$$

**Elbows without pressure variable $\alpha_0$**

Tests results, on figures 1 and 2, show that experimental influence of $\alpha_0$ is well treated by theory for large angle elbows. For small angle elbows, coefficients are not much lower to $90^\circ$ ones.

For flexibility factors, Rodabaugh proposed function is accurate assuming that flexibility of $30^\circ$ elbow angle may be taken as $45^\circ$ value. Experimental results for $C_2$ can be fitted by the following formula:

$$C_2(\alpha) = C_2(0) \ (\frac{\pi}{2\alpha})^{0.1}$$

**Right-angle bends with pressure**

Experimental results on figures 3 and 4 are close to formulae proposed par (Rodabaugh et al., 1957). But pressure influence on rather thick elbows ($h = .64$) is greater than predicted. Moreover, influence of curvature, represented by $R/r$ parameter, is well treated for component usually used in nuclear industry ($2 < R/r < 3$) but pressure effect on flexibility factors is overestimated for large radii of curvature.

**CONCLUSIONS**

For component geometries used in the nuclear field, literature provides formula for the flexibility factor ($k$) and the elastic stress index ($C_2$) for elbows, that well fit experimental results.
Assuming that facts, design codes (ASME, 1983), (RCC-M, 1985), (RCC-MR, 1985) may be improved concerning the following points:

- angle effect on flexibility factor:
  \[ k = \frac{2}{h}, \quad \alpha = 180^\circ \]
  \[ k = \frac{1.65}{h}, \quad \alpha = 90^\circ \]
  \[ k = \frac{1.5}{h}, \quad \alpha = 45^\circ \]
  \[ k = \frac{1.5}{h}, \quad \alpha = 30^\circ \]

Reduction due to angle is similar to Code Case expressions (for \( \alpha_0 > 45^\circ \)) but absolute values are greater than proposed one.

- angle and pressure effect on \( C_2 \):
  \[ C_2 = 1.75 \, h^{-2/3} \, \frac{1}{1 + A(P,h)} \, f(\alpha_0) \]
  \[ A(P,h) = 1 + 3.25 \, h^{-3/2} \, \frac{S \left( \frac{R}{E} \right)^{13/6}}{r} \]
  with \( S = \frac{Pr}{e} \)
  \[ f(\alpha_0) = \left( \frac{\pi}{2\alpha_0} \right)^{-0.1} \]

Code Case (ASME-CASE, 1983) proposed expressions for angle effect, are more severe for 90° elbows but less conservative for \( \alpha_0 = 45^\circ \) and \( h < 0.5 \).

Pressure effect is significant even for low pressurized pipes, as soon as the elbow is sufficiently thin. For \( h = .15 \, e/r = 0.05 \, R/r = 3 \) \( C_2 \) decreases of a value of 20% as soon as pressure reaches a value of 4 MPa.

The present work concerns in-plane bending (\( C_{22} \)). Further work will be necessary to improve an adequate expression for \( C_{23} \), considering that \( C_{21} \) cannot be differency from \( C_{23} \).

REFERENCES

Fig. 1 - $k$ - Variation with elbow angle, $P=0$

Fig. 2 - $C_2$ - Variation with elbow angle, $P=0$

Fig. 3 - $k$ - Variation with pressure, $\alpha_w=90^\circ$

Fig. 4 - $C_2$ - Variation with pressure, $\alpha_w=90^\circ$

Fig. 5 - $k$ - Variation with $1/h$, $\alpha_w=90^\circ$, $P=0$

Fig. 6 - $C_2$ - Variation with $h$, $\alpha_w=90^\circ$, $P=0$