Analysis of Pump's Shaft Torsional Vibrations in Transient Conditions

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1. INTRODUCTION

When the voltage is applied to an induction motor, the currents in the stator's phases are subject to a transient period. It is consequently also the case for the torques. This period of oscillating torques can creat high vibration stress in the shaftline, at the same frequency as the electrical network. A method to calculate the torque in the case of an induction motor with deep bars is presented in the following paragraphs. A model is proposed to represent the squirrel cage. It allows to take into account the fact the currents are not sinusoïdal and that, in this case, the rotor's winding cannot be represented by only one resistance and one reactance.
For the calculation it is necessary also to introduce the fact that the three phases of the breaker don't close simultaneously.
The electrical model is completed by a mechanical model for the shaftline.
The calculation is realised for the start up of an reactor coolant pump. A comparison is made between the results given by the new model, by the classical model and by tests.

2. CLASSICAL MODEL FOR INDUCTION MOTOR

For a single cage induction motor the model is obtained using by Y.H.KU. transformation and we have the pattern of the figure 1.

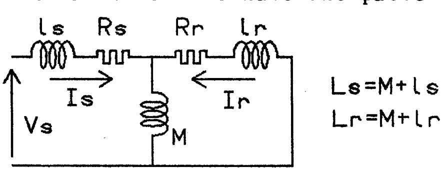


FIG 1

Where ls, R_s are the flux leakage reactance and resistance for one stator's phase, lr and R_r the same values for the rotor, and M the mutual between stator and rotor.

V_s is obtaind by transformation of the stator's voltages.

With this classical model the transient electrical torque for the start up, when the rotor remains stopped is :

$$T_e = Tp\left(1 + e^{-\frac{t}{T1}}\, e^{-\frac{t}{T2}} + A\cos(\omega t - \alpha)\, e^{-\frac{t}{T1}} + B\cos(\omega t - \beta)\, e^{-\frac{t}{T2}}\right),$$

where Tp is the permanent torque, T_1 and T_2 the roots of

$$T^2 - (L_S/R_S + L_R/R_R)\, T + (L_S\, L_R - M^2)/R_R\, R_S = 0$$

If the 3 breaker's phases are not simultaneously closed the maximal torque is multiplied by about $\sqrt{2}$.
In the case of deep bars, in steady state conditions, the values for the rotor's impedances depend on the skin effect. L_r and R_r are not the same for the nominal speed and for starting conditions. In the reality theses values depend on the rotor's speed.

3. MODEL FOR DEEP BARS AT ZERO SPEED

The rotor's winding has one part in the rotor core and one part in the air. With a good approximation we can consider that only the impedances of the parts in the rotor core depend on the rotor's speed, the other values being constant.

3.1. Rotor winding

The dimensions of the rectangular deep bar are given in figure 2 .

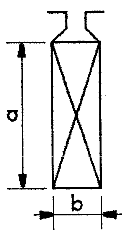


FIG 2

For using the Y.H.KU transformation, one rotor's phase is formed by 2 bars connected by a ring. This ring is the same for all the phases.
The resistance, for one phase, is divided in $R_0 + R_b$. R_0 is the part of the circuit which is in the air, R_b is the part of the bar in the rotor core. This 2 values correspond at the resistance for D.C current.

3.2. Y.H.KU transform

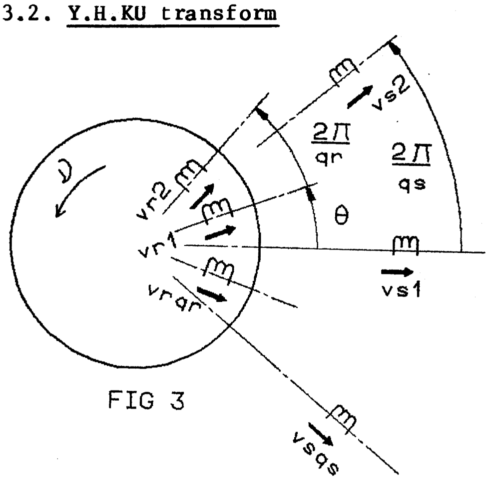


FIG 3

Figure 3 represents an induction motor with q_s phases at the stator and q_r phases at the rotor.
The electrical speed of the rotor is ν.

At zero speed the mutual between the phases of the stator and the phases of the rotor are constant. It is in this case possible to use the Laplace transform, all voltages and currents being null at time $t = 0$.
For the part which is not in the rotor core the leakage flux reactance is l_0.
After calculation we obtain the schema of figure 4

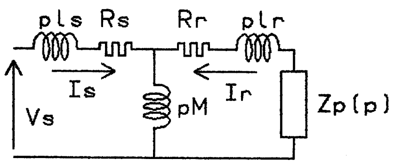


FIG 4

$$L_R = M + l_0, \quad L_s = l_s + M$$

$$Z_p(p) = R_b \frac{x}{th\ x}$$

$$x = \frac{a}{e} \sqrt{\frac{p}{\omega}}$$

$$e = 1/\sqrt{\mu_0 \sigma \omega}$$

The equations are :

$$\begin{cases} (R_s + pL_s)\, I_s + p\, M\, I_R = V_s\,(p) \\ p\, M\, I_s + (R_0 + p\, L_R + Z_b\,(p)\,) = 0 \end{cases} \qquad T_e = \frac{3}{2}\, MP\ \mathrm{Real}\ (j\, i_s^{*}\, i_R)$$

(I_s and I_R and V_s are the Laplace's transforms of i_s, i_r and u_s)

If the 2 first phases of the breaker are closed at $t = 0$ and the third at $\omega t = \psi$ then :

$$V_S\,(p) = E \frac{\omega}{p^2+\omega^2} \left[(p \cos\varphi + \sin\varphi) - j(\omega \cos(\varphi-\psi) - p \sin(\varphi-\psi)) e^{-\frac{p}{\omega}\psi}\right]$$

We can obtain an analytical solution for i_s and i_r from I_s and I_R :

$$\begin{vmatrix} I_s \\ I_R \end{vmatrix} = \frac{V_s}{D\,(p)} \begin{vmatrix} R_0 + p\, L_R + Z_b \\ -\, p\quad M \end{vmatrix}$$

$$D\,(p) = (R_S + p\, L_S)\,(R_0 + pL_R + Z_b\,(p)) - p^2\, M^2$$

The solution is obtained by searching the roots of $(p^2 + \omega^2) D(p) = 0$
All the roots of $D(p)$ are real and negativ.

3.3. Results

The calculations are made to determine the transient electrical torque at zero speed, in three cases.

- The three phases of the breaker are simultaneously closed : PSI=0
- Non simultaneously closing :

PHI = 90 PSI = 90
PHI = 90 PSI = 90

		CLASSICAL		FIG. 4	
PHI	P I	T_{min}	T_{max}	T_{min}	T_{max}
0	0	- 4.54	6.58	- 3.40	5.44
0	90	0	1.84	0	1.44
90	90	- 6.84	8.88	- 5.23	7.27

The torques are given in P.U. The starting torque, in staedy state conditions is $T_s = 1.02$.
It can be observed :
- The transient torques can vary in very different values, depending on the breaker. The variations of the oscillating torque is many times the nominal torque.
- The classical model gives higher values than the model, where the height of the bars is taken into account.

4. GENERAL MODEL

4.1. First model

For using the Y.H.KU transformation at non zero speed we devide every bar in "sub-bar" and, by this way, we have, at the rotor, as many cages as the number of sub-bars.

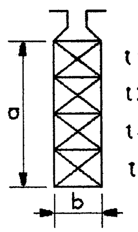
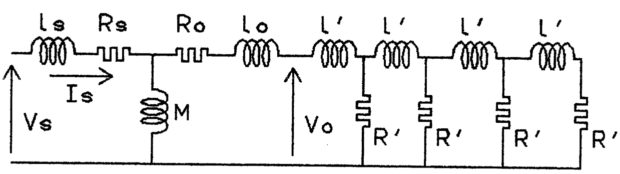


FIG 5

- The height of the bar is devided in n equal parts of height a/n.
- The induction lines are pependicular to the lateral faces of the slot.
- The ring is common to all be "sub-cages".
- The length of the bar in the rotor core is L

The differential equations after transformations are given by the schema of figure 6.



$$l' = (\mu_o \frac{a}{b} L)/n$$

$$R' = nR_b$$

FIG 6

(p is the differential operator $\frac{d}{dt}$ and not (in this case) the variable of the Laplace transform)

The schema must be interpreted, for example, as follow for current i_3 :

$$0 = M\,(j\nu + \frac{d}{dt})(i_s + i_1 + i_2 + i_3 + i_4) + R_o\,(i_1 + i_2 + i_3 + i_4)$$

$$+ (l_o + l')\,(j\nu + \frac{d}{dt})\,(i_1 + i_2 + i_3 + i_4)$$

$$+ l'\,(j\nu + \frac{d}{dt})\,(i_2 + i_3 + i_4) + l'\,(j\nu + \frac{d}{dt})\,(i_3 + i_4) + R' i_3$$

In the right part of the schema there are identical cells with one impedance p l' and one conductance $(R'/(1 + j\frac{\nu}{p}))$.

4.2. Interpretation

At the right of the V_0 voltage there is a circuit which can be represented by a great number of identical cells. It can also be represented by a circuit with distributed impedances. All the cells can be replaced by the single impedance Zb (p) :

$$Z_b\,(p) = \frac{V\,(0)}{I\,(0)} = \frac{1}{Y}\,\frac{\sqrt{YZ}}{th\sqrt{YZ}} \qquad \text{where } Z = (\mu_0\ \frac{a}{p}\ L)p \text{ and}$$

$$Y = (1 + j\,\frac{\nu}{p})/Rb$$

For $\nu = 0$ we find again the result of the § 3.

4.3. New model

According to the fact that the function (z/th z) can be developped in continued fractions.

$$(\,z/th\ \ z) = 1 + \frac{z}{3 + \frac{z}{5 + \frac{z}{7 + \frac{z}{9 + \dots}}}}$$

The model of figure 6 can be transform in model of figure 7.

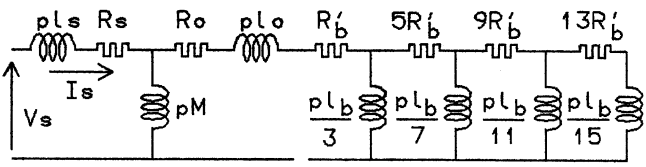
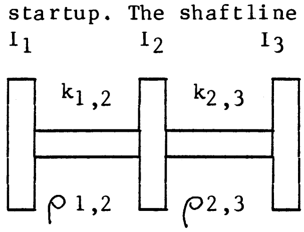


FIG 7 $(R'_b = R_b/(1 + j\ \nu/p))$

Starting from the model of figure 7 it is possible now to write the differential equations.
The advantage of this new model again the model of identical cells is the fact that it needs less cells to obtain the same precision in the calculation. With 4 cells the precision is better than the one which is obtained with more than 20 indentical cells of the figure 6.

5. APPLICATION

The model proposed in figure 7, with 4 cells, has been utilised to calculate the torques in the shaftline of a reactor coolant pump during the startup. The shaftline is represented in figure 8.



$$[I]\ [\ddot{\theta}] + [\rho]\ |\,\dot{\theta}\,| + [K]\ |\,\theta\,| = |\,T\,|$$

FIGURE 8

The two models (fig. 7 and fig. 8) are easy to use in computer.
The results are compared with those obtained with the classical model and with some experimental measurements.

The effect of the water in the impeller is taken into account in the inertia and also in the damping coefficients. The values are adjusted after tests.

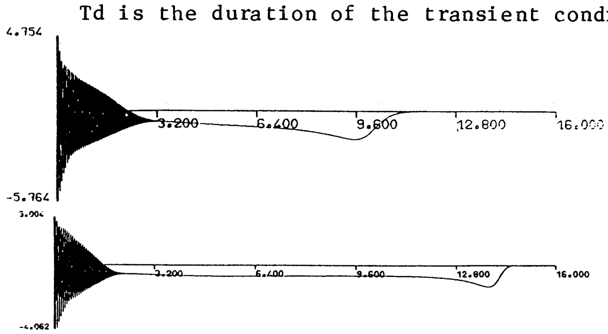
6. RESULTS

The same electrical model allows all the calculation : from the transient period to the nominal speed. The model simulates the non homogenous density in the bars and the skin effect in steady state conditions.
The interesting values are the torques in the various shafts.
They are given by calculus and it is possible to measure them.
The comparisons are summarized in figure 9.

	Shaft 1		Shaft 2		Td
	T_{max}	T_{min}	T_{max}	T_{min}	
Cl. model	2.19	- 1.72	5.76	- 4.75	3.25
New model	1.60	- 1.09	4.08	- 3.0	2.2
Measures	1.62	- 1.17	3.60	- 2.60	2.1

FIGURE 9

Td is the duration of the transient conditions



Classical Model

New Model

FIGURE 10 - TORQUES IN THE SHAFT NUMBER 2

CONCLUSIONS

The model obtained from continued fractions gives a good representation of the reality for the deep bars.
The transient starting up creates stresses in the shafts which can be very high, particularly in the key ways.
Those variating torques contribute to the usage factor of the shaft.
Nearer are the torsional critical speeds to the network's frequency, higher are the torques. In designing the shaftline it is necessary to check this point and to be sure the network's frequence is as far as possible of the torsionnal critical speeds.

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