Crack Nucleation in a Multiaxial Stress State

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INTRODUCTION

During normal working conditions, mechanical parts are often subjected to repeated loads which exceed the yield limits of the material in those areas in which there is geometrical discountinuity. As a result of these cyclic loads, the high strain values and the initial prestrain value cause the nucleation of fatigue cracks which can subsequently propagate and cause component failure. The designer can effect a prediction of their nucleation by means of the stress-strain curves as a function of the life of a given material, obtained from controlled strain tests with an axial load carried out on a smooth specimen in the laboratory.
To obtain a correct prediction of the life of the material a fatigue strength criterion under a multiaxial state of stress must be utilized and the designer must know how this criterion is affected by particular conditions of stress and strain gradient which is present. The approach which is adopted is linked to the definition of the plastic strain energy per cycle ΔW_p, which is linked to the fatigue life of a smooth specimen by the relation (Morrow,1965):

$$\Delta W_p = K \cdot N_f^{\,a} \qquad (1)$$

Some researchers have succeded in finding a link between the value of the characteristic parameter K of the material and the strain condition which is present of the type $K = K(\rho)$, where ρ (Lefebvre,1981) is the strain ratio. The purpose of this paper is to identify the influence of the strain gradient on the nucleation of fatigue cracks. Experimental tests on notched specimens have been conducted and a relationship is proposed to determine the value of the parameters K and a in the eq.(1) versus the strain gradient.The criterion could then be applied in the case of a multiaxial state of stress with the presence of a notch. From (Donzella,1987,1989) multiaxial fatigue tests results are available.

PLASTIC STRAIN ENERGY CRITERION

During the low-cycle fatigue process and in the presence of high strain values, we have energy dissipation caused by the cyclic plastic strain. It is possible to consider that the fatigue damage within the material resulting from the cyclic loading can be related to the amount of the plastic strain energy and the fatigue strenght of a material can be correlate at its capacity to absorb or to dissipate the cyclic plastic strain energy.
The cyclic plastic strain energy per cycle and for unit volume definition is:

$$\Delta W_p = \int \sigma \cdot d\varepsilon_p \qquad (2)$$

Imposing that the half width of the loop can be described by cyclic stress-strain relation: $\sigma'' = k''\cdot\varepsilon_p^{\,n''}$ (3)
for uniaxial load the hysteresis loop area, with extreme values $2\,\sigma_a$ and $2\,\varepsilon_{a,p}$ (fig.1), can be written in the form:

$$\Delta Wp = 4\cdot\sigma_a\cdot\varepsilon_{a,p} - 2\cdot\int_0^{2\sigma_a}\varepsilon_p\cdot d\sigma \qquad (4)$$

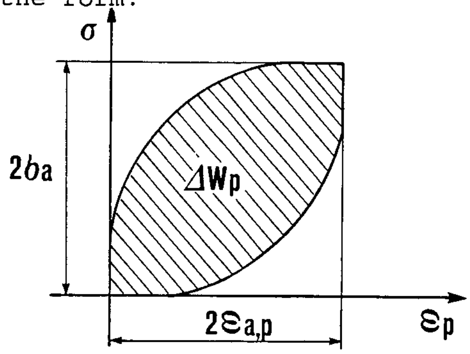

Fig.1 Hysteresis loop shape.

Imposing: $n''=n'$ where n'=cyclic strain hardening exponent and introducing the boundary conditions:

$$k'' = \frac{\sigma_a}{(\varepsilon_{a,p})^{n'}\cdot 2^{n'-1}} \qquad k' = k''\cdot 2^{(n'-1)} \qquad (5)$$

where k' is the cyclic strenght coefficient, we obtained:

$$\Delta W_p = 4\cdot\sigma_a\cdot\varepsilon_{a,p}\,\frac{(1-n')}{(1+n')} \qquad (6)$$

or :

$$\Delta W_p = 2\cdot(2k')^{-1/n'}\cdot\frac{(1-n')}{(1+n')}\cdot(2\,\sigma_a)^{(1+n')/n'} = k\frac{(1-n')}{(1+n')}\cdot(2\,\sigma_a)^{(1+n')/n'}$$

$$k = \frac{2}{(2k')^{1/n'}} \qquad (7)$$

It has been observed that for strain controlled tests, the plastic strain energy per cycle ΔW_p not vary appreciably with cycles, and it is possible to define a ΔW_p value at half-life that have a relationship of the form $\Delta W_p = K\cdot N_f^{\,a}$
and it is possible to define the plastic strain energy absorbed to failure W_f in the form:

$$W_f = \Delta W_p\cdot N_f = K\cdot N_f^{(a+1)} \qquad (8)$$

For multiaxial states of stress and for proportional stressing, the plastic strain according to the Von Mises theory is given by

$$\varepsilon p_{i,j} = \frac{3}{2}\left(\frac{1}{E_s} - \frac{1}{E}\right) s_{i,j} \qquad (9) \quad \text{, where} \quad E_s = \bar{\sigma}/\bar{\varepsilon} \qquad (10)$$

is the Von Mises stress-strain ratio, $s_{i,j} = \sigma_{i,j} - (\delta_{i,j}\cdot\sigma_{k,k})/3$ (11) is the deviatoric stress tensor, and the Von Mises stress and strain expressions are:

$$\bar{\sigma} = \left(\frac{3}{2}\,s_{i,j}\cdot s_{i,j}\right)^{1/2} \qquad (12) \qquad \text{and} \qquad \bar{\varepsilon} = \left(\frac{2}{3}\varepsilon_{i,j}\cdot\varepsilon_{i,j}\right)^{1/2} \qquad (13)$$

The cyclic stress-strain curve for multiaxial and proportional loading can be written as: $\Delta\bar{\ } = k''\cdot\Delta\bar{\varepsilon}_p^{\,n''}$ (14)
We obtain for multiaxial states of stress the relation for the plastic energy dissipated during one cycle:

$$\Delta W_p = k\frac{(1-n')}{(1+n')}\Delta\bar{\sigma}^{(1+n')/n'} \qquad (15)$$

where the $\Delta\bar{\sigma}$ value is defined from the strenght criterion utilized. The total plastic energy to failure under multiaxial states of stress can be approximated by: $W_f = k\frac{(1-n')}{(1+n')}\Delta\bar{\sigma}^{(1+n')/n'}\cdot N_f$ (16)

The conditions under which the tests to define the criterion were carried out (controlled strain tests) were different from those found in the case of mechanical components, where the loads are cyclic and the strain can vary. To provide an approximation of the working conditions of a notched mechanical part, tests of the nucleation of fatigue cracks under controlled stress were conducted with notched specimens.In reality, when the yield limits of the material have been exceeded and a strain gradient is present although the load is pulsating, after the application of the first load cycle, cycles with an average stress equal to zero and an average deformation which is not equal to zero are obtained. In addition, since the stressed area is limited, the surrounding material which still behaves elastically constrains the deformation in the stressed area, approximating the strain-controlled test.

UNIAXIAL LOADING CONDITIONS.

Specimens

The nucleation tests under uniaxial loading conditions were conducted on Keyhole specimens subjected to a pulsating load up to the nucleation of a fatigue crack, visible with penetrating liquids (Vergani,1988). Electrical strain gauges were attached to the specimens ,located in the nucleation area and on the sides and back of the specimens. Three groups of specimens with different notch radiuses were constructed (fig. 2) in order to obtain three notch coefficients Kt, respectively 1.4, 2.3 and 2.9.

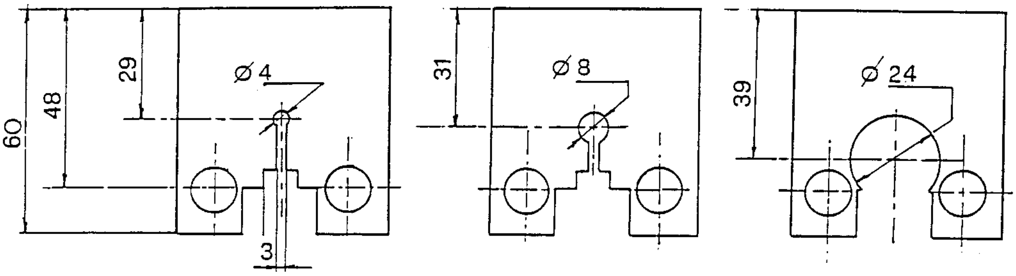

Fig.2 Keyhole specimens.

The steel used is ASTM A 533 B. The cyclic and fatigue characteristics for strain controlled tests were obtained from literature, see table I. In the same table the cyclic coefficients obtained from tests in stress controlled mode are reported.

	K'[MPa]	n'	$\varepsilon'f$	$\sigma'f$[MPa]	b	c
Strain controlled	1038	0.130	0.144	799	-0.062	-0.486
Stress controlled	1777	0.133	0.089	2019	-0.117	-0.556

Tab.I - Cyclic properties of A 533 B.

Experimental results

All the groups of specimens with the same Kt were subjected to pulsating loads of a size which produced the same values of $\Delta\sigma$,in this case $\Delta\sigma$=1050 MPa, $\Delta\sigma$=1120 MPa, $\Delta\sigma$= 1200 MPa. At these values, the yield point of the material is exceeded and, after the first load cycle has been applied , the average stress σm becomes almost zero(f.3), thus creating test conditions similar to those of strain-controlled tests with high average strain εm.Fig.4 provides the values of ΔWp as a function of N in specimens with the same Kt and variations in $\Delta\sigma$. After an initial increase, one can see that there is an area in which ΔW_p is almost constant and the influence of $\Delta\sigma$ on the value of ΔW_p is evident.The values of ΔW_p as a function of N at constant values of $\Delta\sigma$ and variations in K_t are shown in fig.5.

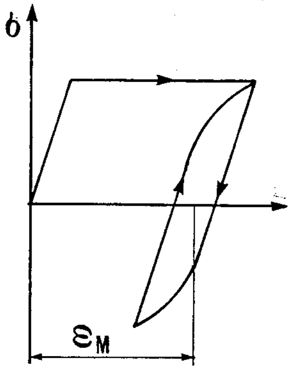

Fig.3 Average strain ε_m.

The difference between the values of ΔW_p of the specimens depends on the different values of the stress concentration factor K_t and thus on the different strain gradients. This parameter determines the extent of the plasticised area on the surface of the specimen and as a consequence the ε_p, and this has a substantial effect on the life of the specimen.The different values of the gradient $\rho s= \Delta\varepsilon / \Delta X$ shown in fig. 6

correspond to different values of the extent of total strain εa as a function of the life of the specimen N_f. Since the value of ΔW_p halfway through the life of the specimen is considered to be significant, these values are shown as a function of N_f in fig.7. The relationΔWp-N_f can be obtained for all the specimens, even though they have a different K_t. The total plastic strain energy can be calculated by integrating the values of ΔW_p as a function of N. In fig.8 the relation Wf - N_f is defined.

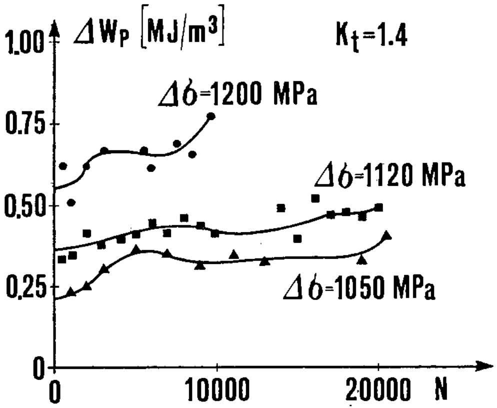

Fig.4 - Plastic strain energy versus N , at K_t=1.4.

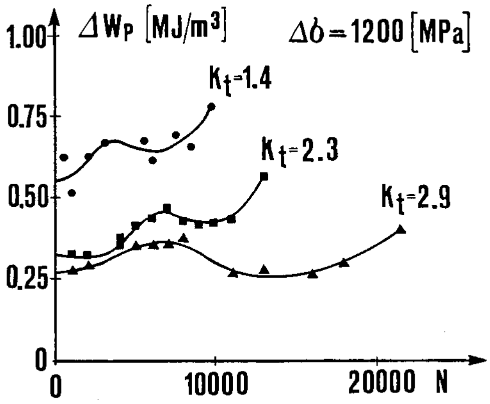

Fig.5 - Plastic strain energy versus N , at $\Delta\sigma$=1200 MPa.

No effect of the gradient on the cyclic plastic strain energy and total plastic strain energy can be identified from these curves.

Comparison with experimental values

The ΔW_p was calculated by means of eq.6 , using experimental values of υ_a and $\vartheta_{a,p}$.It can be seen from Table II that these values are a good approximation of the ΔW_p values obtained from the tests on the basis of the measurement of the hysteresis cycle. This means that (6) reproduces the hysteresis cycle.The forecast value of ΔW_p, calculated by eq.7 with a nominal value of υ_a,is also provided. With equal values of σ_a, the forecast values of ΔW_p are also equal,while there are large variations in the life of the specimen when there are variations in K_t.The values of the forecast ΔW_p versus N_f and the gradient are shown in fig.9. If we assume that the straight lines have the same tangent, we can obtain the relation K = K(ϱ_s).

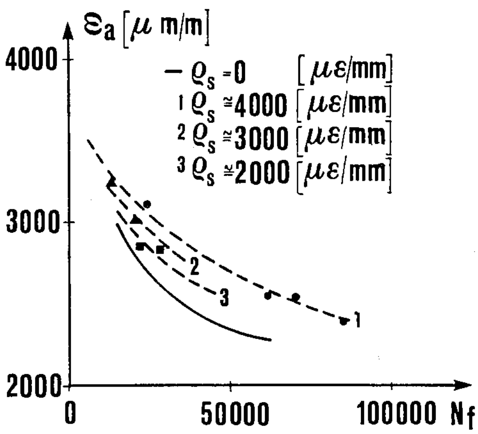

Fig.6 Influence of the strain gradient on the specimens life.

MULTIAXIAL STRESS CONDITIONS

The results obtained from the previous fatigue tests on a pressurised vessel were used to verify the soundness of the criterion with experimental data. The pressurised vessel onto which three nozzles had been welded was subjected to pulsating pressure until trough fracture appeared in the maximum stress areas (nozzle-vessel connections). The tests are described in the

papers: (Donzella,1988,1989).The material used was Fe510BUNI7070/80, the characteristic of which are:
$K'=998$ Mpa, $n'=0.184$, $\sigma'_f=693$ MPa, $\varepsilon'_f=0.077$, $b=-0.080$, $c=-0.377$, $Rm=580$ MPa, $Rel=380$ MPa, $E=220000$ MPa, $\nu=0.28$, $A=25\%$.

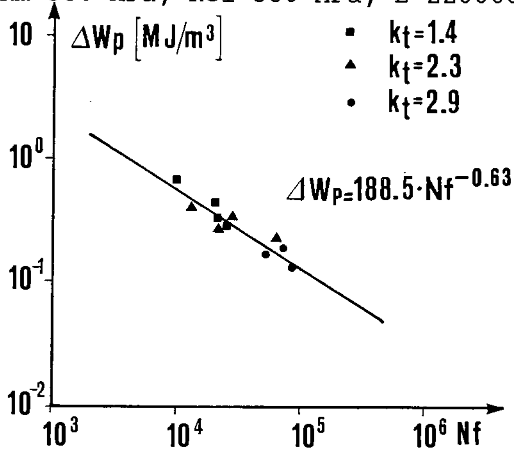

Fig.7 Experimental ΔW_p versus cycles of failure.

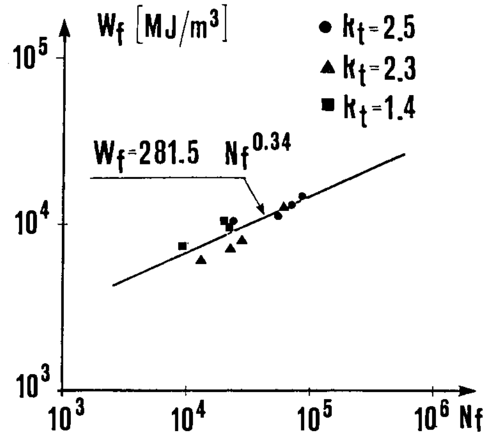

Fig.8 Experimental W_f versus cycles of failure.

Keyhole specimens were constructed with Kt = 1.6 and controlled stress tests were carried out to obtain the fatigue coefficients. The experimental ΔW_p curve versus N_f : $\Delta W_p=109.44\ N_f^{-0.546}$ is shown in fig.10.The results of the experimental tests carried out previously and the (Ellyin,1988) show that the value of K varies as a function of the variation in the strain ratio $\varrho = 2\varepsilon_2/(\varepsilon_1 - \varepsilon_3)$ and the strain gradient ϱ_s. In order to forecast the life of the nozzles we have to consider the characteristic relation of the material $\Delta Wp=KN^a$ and the relationship between $K=K(\varrho)$ and $K=K(\varrho_s)$. The value of K in the monoaxial situation is known: $K=109.44$ MJ/m³.In the situation of compound stress $\varrho = 0.38$ if we assume a dependence similar to that produced by (Ellyin,1988), we obtain K = 58.44 MJ/m³. The relation $K = K(\varrho_s)$ cannot be verified because the gradient of the specimens in this case is similar to that of the vessels.
The strain gauge measurements taken on the nozzles provides the

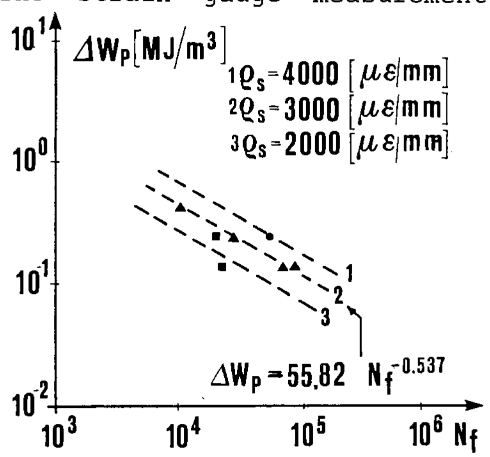

Fig.9 Influence of the gradient on the ΔW_p values from eq.8.

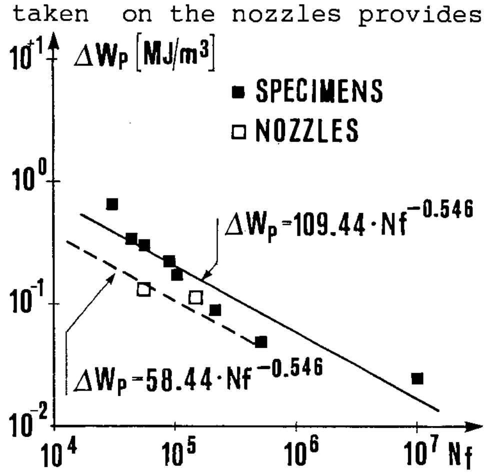

Fig.10 Fe510 specimens and nozzles: ΔW_p values versus N_f.

values of circumferential and axial stresses in the areas under greatest stress. The value of ΔW_p during the life of the gate is almost constant. The values of experimental ΔW_p in correspondance to $N_f/2$ as a function of N_f are shown in fig.10. It is possible to see that they are close to the forecast straight line $\Delta W_p=58,44N_f^{-0.546}$.

CONCLUSIONS

The predictions of an energy based criterion for the multiaxial fatigue failure from notches has been compared with experimental results. -The effect of the strain gradient has been considered to avoid life predictions which are too conservative and a function of $K(\rho_s)$ versus the strain gradient has been proposed.-The effect of the strain ratio has been considered and a function of $K=K(\rho)$ has been obtained from literature.
-The agreement with the experimental results is found to be good.

Specimen	Kt	$\Delta\sigma$ [MPa]	Nf	W f [MJ/m^3]	N. of cycles	ΔWp exp. [MJ/m^3]	ΔWp from eq.6	ΔWp from eq.7
1	2.5	1050	85000	14016	1000	0.0882	0.0796	0.1655
					20000	0.0545	0.0932	
					40000	0.1351	0.1439	
					60000	0.0654	0.1985	
					80000	0.1872	0.2122	
2	2.5	1120	53000	11477	500	0.1323	0.1535	0.2870
					10000	0.1873	0.2529	
					18000	0.1802	0.2221	
					30000	0.2559	0.2898	
					42000	0.1845	0.2306	
3	2.5	1200	24000	10477	1000	0.2740	0.3004	0.5169
					4000	0.3627	0.4400	
					8000	0.3765	0.4529	
					13000	0.2809	0.3776	
					18000	0.3042	0.3959	
4	2.3	1050	61000	12772	4	0.2289	0.2154	0.1655
					22000	0.2234	0.2291	
					30000	0.2318	0.1833	
					40000	0.1637	0.1720	
					55000	0.2227	0.2379	
5	2.3	1120	27000	8844	1000	0.3276	0.2769	0.2870
					6000	0.3140	0.2315	
					11000	0.3493	0.3035	
					18000	0.3953	0.3790	
					24000	0.4026	0.3833	
6	2.3	1200	13000	6444	1000	0.3401	0.3858	0.5169
					3000	0.3300	0.3969	
					6000	0.4374	0.5255	
					9000	0.4219	0.5025	
					11000	0.4340	0.5236	
7	1.4	1050	22000	10174	1000	0.2332	0.2693	0.1655
					3000	0.3079	0.3481	
					7000	0.3501	0.3810	
					13000	0.3299	0.3882	
					19000	0.3309	0.4606	
8	1.4	1120	20000	10611	500	0.3408	0.4004	0.2870
					5000	0.4063	0.4759	
					10000	0.4222	0.5264	
					15000	0.3967	0.5384	
					19000	0.4669	0.5830	
9	1.4	1200	10000	7632	500	0.6257	0.7340	0.5169
					3000	0.6760	0.7781	
					5500	0.6760	0.7763	
					7500	0.6977	0.8066	
					8500	0.6501	0.8286	

Tab.II - Stress controlled tests for A 533 B.

ACKNOWLEDGEMNETS

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