A Special Cracked Pipe Element for Leak Before Break Application

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OBJECTIVE

In order to apply the leak before break concept on pipes, characterization of the stability of circumferential cracks, in case of accidental loadings, has to be done. The questions are the following : can the crack growth be initiated, and if yes how large is the growth. The loading can be static or dynamic, and plastification of the material must be taken into account.
A simple finite element model has been developped and will be an industrial tool in leak before break applications.

DESCRIPTION OF THE MODEL

This model is a generalization of the approach proposed by Tada and Paris (Ref. [1] and [2]). The cracked pipe element (called "hinge element") is a global model which takes into account the flexibility of the pipe due to a circumferential through wall crack (Figure 1).
In finite element computations, a cracked pipe, whose length is L, will be represented by three elements :

- a normal pipe element, length L/2, displacement e_{nc}, ϕ_{nc},
- an hinge element, length null, displacements e_c, ϕ_c,
- a normal pipe element, length L/2, displacements e_{nc}, ϕ_{nc}.

Considered loads are a bending moment M and an axial force P (representing respectively thermal expansion and effect of pressure).

ELASTIC CHARACTERISTICS OF THE CRACKED PIPE ELEMENT

The elastic characteristics of the cracked pipe element are determined using the Zahoor formulae for the geometry dependent factors F_m and F_f.
The expression of the stress intensity factor is the following :

$$K_I = K_f M + K_e P \qquad \text{and } J = \frac{K_I^2}{E} \qquad (E = \text{Young's modulus})$$

$$\text{with } K_f = \frac{\sqrt{\Pi R\Theta}}{\Pi R^2 t} F_f(\Theta/\Pi) \qquad K_e = \frac{\sqrt{\Pi R\Theta}}{2\Pi R t} F_m(\Theta/\Pi)$$

The terms of the flexibility matrix are obtained by derivation of the deformation energy due to the crack :

$$\begin{bmatrix} \phi \\ e \end{bmatrix} = \begin{bmatrix} C_M(\theta) & C_{MP}(\theta) \\ C_{MP}(\theta) & C_M(\theta) \end{bmatrix} \begin{bmatrix} M \\ P \end{bmatrix} \quad \text{with } C_M = \frac{1}{M} \frac{\partial U}{\partial M} \Bigg|_{P=0} \quad .... \quad U = \int_0^A J \, dA$$

U : deformation energy
A : crack area

PLASTIC CHARACTERISTICS OF THE CRACKED PIPE ELEMENT

Plastic characteristics are based on the following assumptions :

- plastic deformations appear just in the cracked section,
- in the cracked section, the stress field is governed by a limit-stress diagram (figure 2).

With these hypothesis, the loading function has the following expression :

$$F(M, P, \Theta, \sigma_0) = \frac{M}{M_0} - \cos\frac{1}{2}(\Theta + \Pi\frac{P}{P_0}) + \frac{1}{2}\sin\Theta \quad \text{with } M_0 = 4\,\sigma_0\,R^2t, \quad P_0 = 2\,\sigma_0\,\Pi\,Rt$$

In case of perfectly plastic behaviour, σ_0 corresponds to the flow stress. The value of the flow stress could be choosen equal to :

$$\sigma_f = \frac{\sigma_y + \sigma_u}{2} \qquad \text{with } \sigma_y : \text{yield stress and } \sigma_u : \text{ultimate stress}$$

Up to now, taking into account hardening is possible only if the (M, ϕ_c) curve for the cracked section is knowed (issued of 3-D computations or of tests). In case of hardening, σ_0 increases from the limit of linearity up to the flow stress ($\sigma_0 = \frac{M}{M_{max}}\sigma_f$) and the loading surfaces growth consequently (Figure 3).
An iterative item is solved to obtain the solution.
As for displacements, J is decomposed in elastic and plastic parts, and J_p is obtained from the plastic displacements increments :

$$J = J_e + J_p \quad \text{and } dJ_p = -\frac{(dW)}{\partial A} = \sigma_0\,(R\cos\Theta\,d\phi p + dep)$$

CRACK GROWTH

For crack growth, the material is characterized by two intrinsic values. J_{1C} which represents the level of energy corresponding to crack initiation and the tearing modulus T which governs the crack growth (Figure 4).
Crack growth is initiated when :

$$J_e + J_p > J_{1C} + TR\,(\Theta - \Theta_0) \qquad \text{with } 2\Theta_0 : \text{initial crack angle}, \quad 2\Theta : \text{present crack angle}$$

Up to now, J_p is supposed constant during the crack growth, and the new crack angle Θ is obtained using an iterative method to solve the equation :

$$J_e(\Theta) - TR\,\Theta + J_p - J_{1C} + TR\Theta_0 = 0$$

DYNAMIC BEHAVIOUR

Modal characterization of a cracked pipe is possible with the hinge element. Figure 5 shows the decrease of pipe frequency as crack angle increases.
For dynamic loads, a contact condition is imposed in case of crack closure. It consists to impose relations between the displacements of the two nodes of the hinge element :

$$u(P_2) - u(P_1) \geq 0 \qquad u : \text{axial displacement} \qquad R : \text{rotation}$$
$$R(P_2) - R(P_1) \geq 0$$

Figures 6 and 7 show responses and K_I values during a sinusoïdal excitation at resonance frequency with and without crack closure condition (damping = 5 %). With crack closure condition, resonance phenomeneon is braked due to the change of modal behaviour during crack closure.

VALIDATIONS

On the other hand, an important program of static tests on in stainless steel cracked straight pipes, with different crack sizes, and subjected to four points bending is actually carried out by C.E.A./D.E.M.T (Ref. [4]). The experimental set-up and test pipe geometry are described on figures 8 and 9. Taking into account a perfectly plastic behaviour for the hinge element (with $\sigma_f = (\sigma_y + \sigma_u)/2$), a good correlation is obtained between the experimental and numerical moment-rotation curves as regards (figure 10) :

- the elastic behaviour (initial slope of the curve),
- the limit behaviour (limit load),

And, for a given (M, ϕ) curve, a good correlation is obtained between the J-integral versus rotation curves determined (figure 11) :

- using a simplified method,
- with the hinge element taking into account hardening.

FUTURE DEVELOPMENTS

The essential problem for the hinge element is how to use the stress-strain curve (σ, ϵ) of the material in terms of global variables (M,ϕ).

One solution is to use scale functions A(Θ), B(Θ) for M and ϕ respectively, so that the (M,ϕ) curve of any cracked pipe can be deduced from that of the perfect pipe. For M, the theoretical expression of A(Θ) is :

$$A(\Theta) = \cos \frac{1}{2} \Theta - \frac{1}{2} \sin \Theta$$

An experimental determination of the scale function B(Θ) is in progress using results of the experimental program previously described.

Another possible solution, to estimate the plastic rotation for a given moment level, is to use the plastic zone correction method (Ref. [3]).

$$\phi_p = \frac{\epsilon_p(\sigma)}{\epsilon_e(\sigma)} \phi_e(\Theta_{eff}) \qquad \text{with } \Theta_{eff} = \Theta_0 + \frac{r_y}{r_m}$$

$$r_y = \frac{1}{\beta \Pi} \left(\frac{K_I}{\sigma_y}\right)^2$$

CONCLUSIONS

The hinge element is a simple one-dimensionnal element to compute the global behaviour of straight pipes with circumferential through wall cracks. Its complete validation and development are in progress. The preliminary results obtained up to now are very encouraging.

Together with tests on straight pipes, tests on elbows with cracks are carried out. The development of the hinge element for the case of cracked elbows has to be studied.

REFERENCES

[1] Paris P.C., Tada H., "The application of fracture proof design methods using tearing instability theory to nuclear piping postulating circumferential through wall cracks" - NUREG/CR-3464 (1983)

[2] Kanninen M.F., "Proceeding of the CSNI/NRC Workshop on ductile piping fracture mechanics" Compilation - Southwest Research Institute, San Antonio (USA) (1984)

[3] Brust F.W., "Approximate methods for fracture analysis of through wall cracked pipes" - NUREG/CR-4853 (1987)

[4] Moulin D., "Experimental determination of J value on circumferentially cracked stainless steel straight pipes under bending" - 2nd Conference on Pipework Engineering and Operation, London (1989)

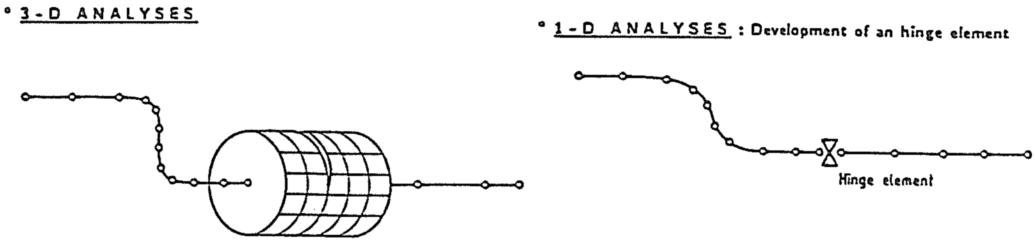

Fig. 1 : Different analyses for cracked pipe lines

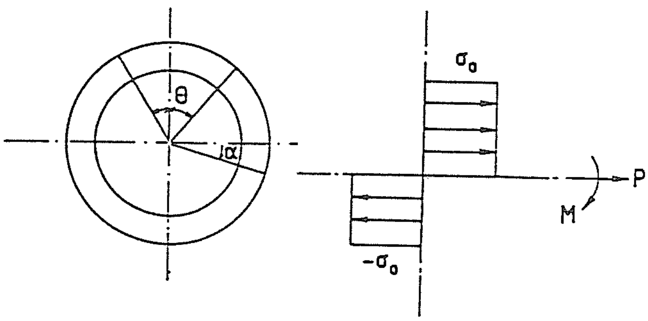

Fig. 2 : Plastic characteristics : limit stress diagram

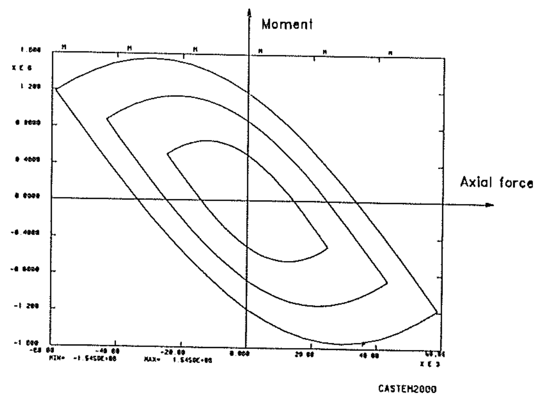

Fig. 3 : Plastic loading surfaces

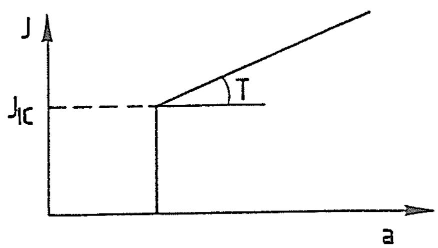

Fig. 4 : Approximate J-R cuve

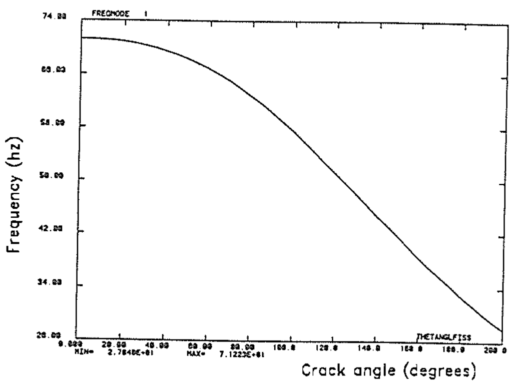

Fig. 5 : Eigen frequencias as a function of the crack angle

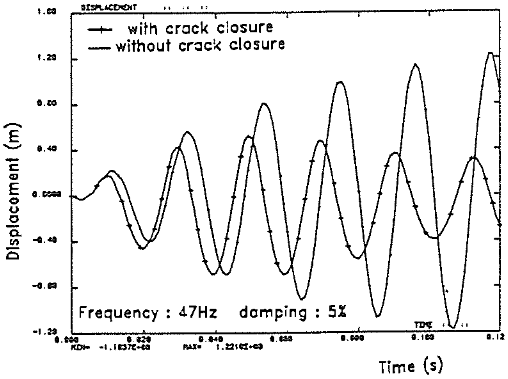

Fig. 6 : Displacement responses during a sinusoïdal excitation

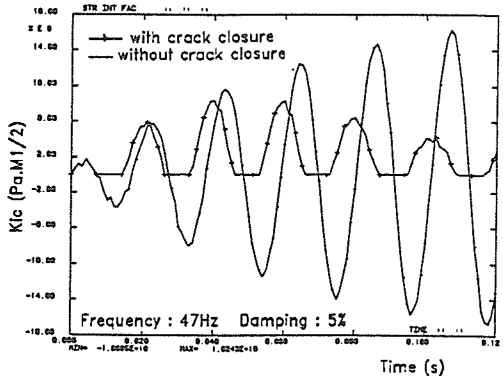

Fig. 7 : K_I values during a sinusoïdal excitation

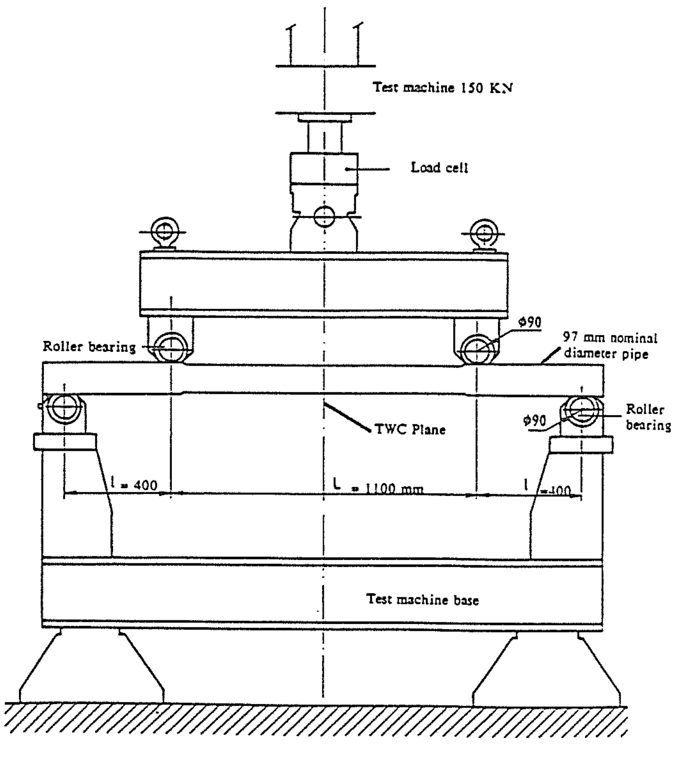

Fig. 8 : Experimental set-up for 4 points bending tests

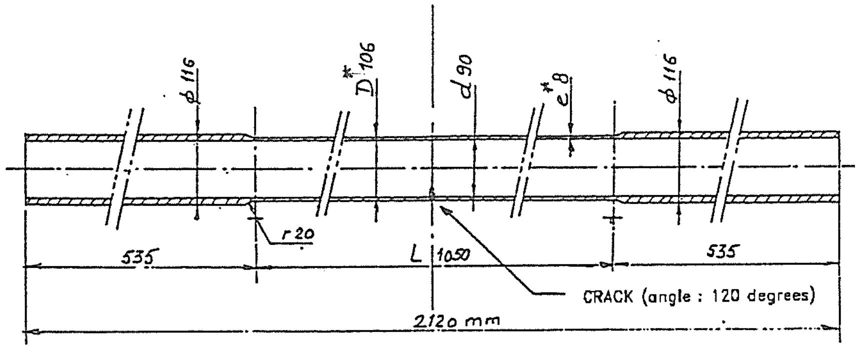

Fig. 9 : Test pipe geometry

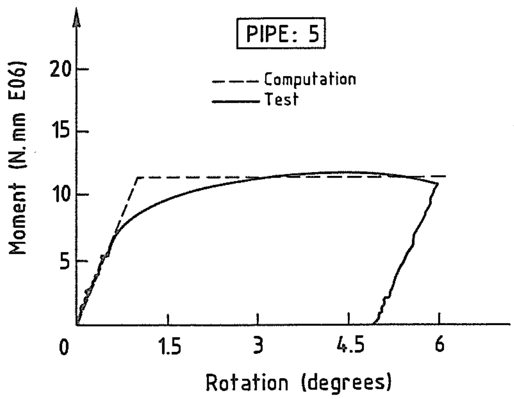

Fig. 10 : Experimental and numerical moment-rotation curves

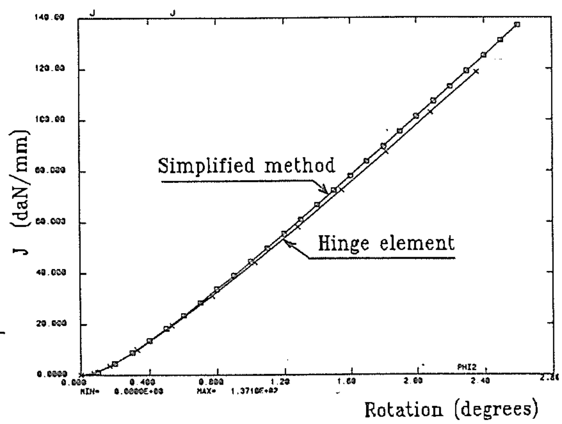

Fig. 11 : J-Integral versus rotation curves