The Pre-Test Analysis of a Containment Vessel A Comparative Study for Over Pressurisation

Yusaf Bangash
Middlesex Polytechnic, London, UK

1. INTRODUCTION

Several finite element studies have been carried out to analyse a 1:6 scale, containment vessel model(1). An attempt has been made in this paper to analyse the same containment using Endochronic concrete failure theory. Another departure is to assume that concrete is represented by a sophisticated 3D-32 noded isoparametric element in which the reinforcement is placed in the body of this element. The reinforcement is represented by 3D-4 noded line element attached to a bond-linkage element (2,3), thus treating the reinforcement fullybonded. The vessel model is pressurised using 20 incremental steps. Displacements, strains, stresses, concrete cracking and reinforcement yielding are obtained at every step where appropriate. A final post-mortem is presented.

2. THEORETICAL ANALYSIS USING FINITE ELEMENT

A three-dimensional incremental stresses are written for up to plastic conditions as

$$\underset{\sim}{\Delta\sigma}^{*} + \underset{\sim}{\Delta\sigma}^{P*} = \underset{\sim}{D}_{T}^{*} \cdot \underset{\sim}{\Delta\varepsilon}^{*} \qquad (1)$$

which can be interpreted in a matrix form as

$$\begin{Bmatrix} \Delta\sigma_x + \Delta\sigma_x^{P} \\ \Delta\sigma_y + \Delta\sigma_y^{P} \\ \Delta\sigma_z + \Delta\sigma_z^{P} \\ \Delta\tau_{xy} + \Delta\tau_{xy}^{P} \\ \Delta\tau_{yz} + \Delta\tau_{yz}^{P} \\ \Delta\tau_{zx} + \Delta\tau_{zx}^{P} \end{Bmatrix} = \begin{bmatrix} (K + \frac{4}{3}G) & (K - \frac{2G}{3}) & (K - \frac{2G}{3}) & 0 & 0 & 0 \\ & (K + \frac{4}{3}G) & (K - \frac{2G}{3}) & 0 & 0 & 0 \\ & & (K + \frac{4G}{3}) & 0 & 0 & 0 \\ & & & \beta' G & 0 & 0 \\ & & & & \beta' G & 0 \\ & & & & & \beta' G \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\varepsilon_z \\ \Delta\gamma_{xy} \\ \Delta\gamma_{yz} \\ \Delta\gamma_{zx} \end{bmatrix} \qquad (2)$$

Where K, G and β' are moduli and aggregate interlocking parameters. The material matrix $\underset{\sim}{D}_{T}^{*}$ is constantly modified to reflect the reduced stiffness(4) across the crack, for example, where one crack normal to X* direction occurs, the concrete is assumed not to resist any tensile stress in that direction. A brief outline is presented later on for the non-linear steps taken in the finite element analysis. The non-linear bond-linkage element is included to assume that the 1:6 model is fully bonded. Since it is a reinforced concrete model it is imperative not to ignore the bond between the reinforcement and model concrete. The stiffness matrix [K] is evaluated as

$$[K]=\int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1}[B]^T[D_T^*][B]d[J]d\xi d\eta d\xi \quad (3)$$

3. STEP-BY-STEP NON-LINEAR FINITE ELEMENT

1. Apply a load increment ΔP_n where n is the load increment. Accumulate total load $P_n= P_{n-1}+\Delta P_n$ and $R=\Delta P_n$, where R is the residual load vector

$$\Delta P_n = K_B\Delta U=(\pi dL\ T^TE_BT)\Delta U_n \quad (4)$$

2. Solve $\Delta U_i=K^{-1}R$, where i is the iteration number and K_T is the stiffness matrix of the structure. (5)
Accumulate total displacements:

$$U_i = U_{i-1} + \Delta U_i \quad \text{where } \Delta U_i= \Delta S_i \cdot T^{-1} \quad (6)$$

Total slip 'i', S_i becomes

$$S_i = S_{i-1} + \Delta S_i \quad (7)$$

3. For each element type calculate strain increments

$$\Delta\epsilon_i = B\Delta U_i \quad (8)$$

and strains $\epsilon_i = \epsilon_{i-1} + \Delta\epsilon_i$ (9)

4. For each element type

$$\Delta\sigma_i = f(\sigma)\Delta E \quad (10)$$

$$\Delta\sigma_{bi} = E_b(\sigma_{bi}-1)\ \Delta S_i$$

where $E_b = \begin{bmatrix} E_h & 0 & 0 \\ 0 & E_v & 0 \\ 0 & 0 & E_\ell \end{bmatrix}$ (11)

Accumulate stresses

$$\sigma_{bi} = \sigma_{bi-1} + \Delta\sigma_{bi} \quad (12)$$

5. Check the state of bond
If $|S_i|> S_{max}$ Bond is broken
If $|S_i|< S_{max}$ Bond stresses are computed
The correct stress is $\sigma_{bi} = \sigma_{bi} - \Delta\sigma_D$ (13)

6. The total stresses are converted into equivalent loads as

$$\int_v B^T\sigma_i dvol = \int B^T\sigma d[J]d\xi d\eta d\xi \quad (14)$$

7. The total internal equivalent loads and residuals are written as

$$P_{int} = \pi dL\ T^T\sigma_{bi} \quad (15)$$

$$R = P_n - \int_v B^T\sigma d|J|d\xi d\eta d\xi \quad (16)$$

For cracking Table 1 is referred to.

The Bond-linkage stiffness matrix is written as:
The explicit form of the bond-linkage stiffness is given as

$$\underset{6\times 6}{K_b} = \begin{bmatrix} K_{b_{11}} K_{b_{12}} \\ K_{b_{21}} K_{b_{22}} \end{bmatrix}, \quad K_{b_{11}} =\pi dL(\ell^2E_h+p^2E_v+r^2E_i), \quad K_{b_{22}} =\pi dL(m^2E_h+q^2E_v+s^2E_i) \quad (17)$$

where $K_{b_{33}} =\pi dL(n^2E_h+t^2E_v)$

$$K_{b_{12}} =K_{b_{21}} = -K_{b_{11}};\ K_{b_{12}} =\pi dL(\ell mE_h+pqE_v+rsE_\ell) \quad (18)$$

$K_{b22} = K_{b11}$ $\quad K_{b13} = \pi dL(\ell nE_h + rtE_v)$

$$K_{b11_{3\times3}} = \begin{bmatrix} K_{b11} & K_{b12} & K_{b13} \\ & K_{b22} & K_{b22} \\ \text{Symmetry} & & K_{b33} \end{bmatrix} \quad K_{b23} = \pi dL(mnE_h + stE_\ell)$$

l,m,n)
p,q,r) = direction cosines
s,t)

πdL = perimeter of the steel

Based on the crack model given (4) for Endochronic failure model, Table 1 gives a step-by-step layout of the crack propagation under incremental pressure.

4. MODEL CONTAINMENT 1:6 SCALE

Figures 1 and 2 give the model parameters and the finite element mesh. The results from this analysis are given briefly in Figs. 3 and 4 and are plotted along with the results obtained by other researchers. (1) The safety factor against the design computed to be 2.75 based on the loss of bond, excessive cracking and the rupture of reinforcement.

5. CONCLUSIONS

The Endochronic Theory adopted in the over pressurisation of the 1:6 containment model give results which are in agreement and disagreement in certain areas with other approaches. It is now wide open to experimental tests to see how the test results are compared. Since the Endochronic model has been tested in other cases, the author is convinced that this analysis is more valid owing to its true representation of constitutive elements.

6. REFERENCES

1. 1:6 Scaled Model of R.C. Containment - A Pre-test Analysis (1987) Report: N/CR 4913 Sand 87-0891, Sandia Laboratories.

2. Bangash, Y. Aircraft Crash Analysis of the Proposed Sizewell-B Containment Vessel (1987). 9th Int. Conf., SMIRT, Vol.J, pp 307-314, Laussane.

3. Bangash, Y. (April 1988). The Rupture Analysis of the Liner Anchored to Concrete of Pressure and Containment Vessel. J. Fracture Mechanics April. Pergamon Press.

4. Bangash, Y. (1987). The Simulation of Endochronic Model in the Cracking Analysis of PCPV. Trans. SMIRT 9th Conf. pp 333-340. Vol.H.

5. Ahmad, M., Bangash, Y. (1987). A Three-Dimensional Bond Analysis Using Finite Element. Computers and Structures, V.25, No.2, pp 281-296.

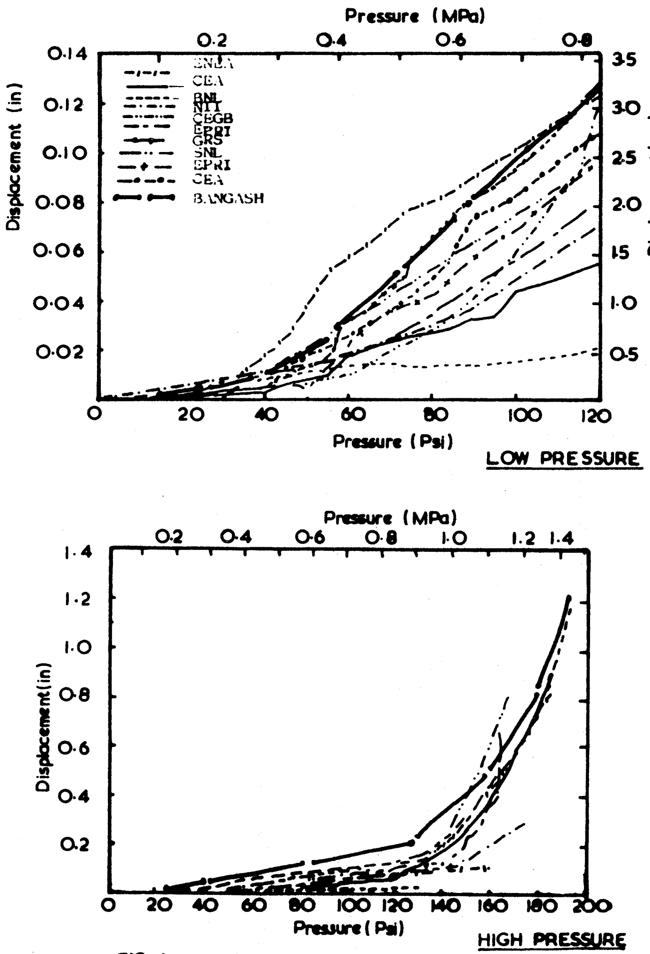


FIG 4 VERTICAL DISPLACEMENTS AT MID HEIGHT OF THE WALL

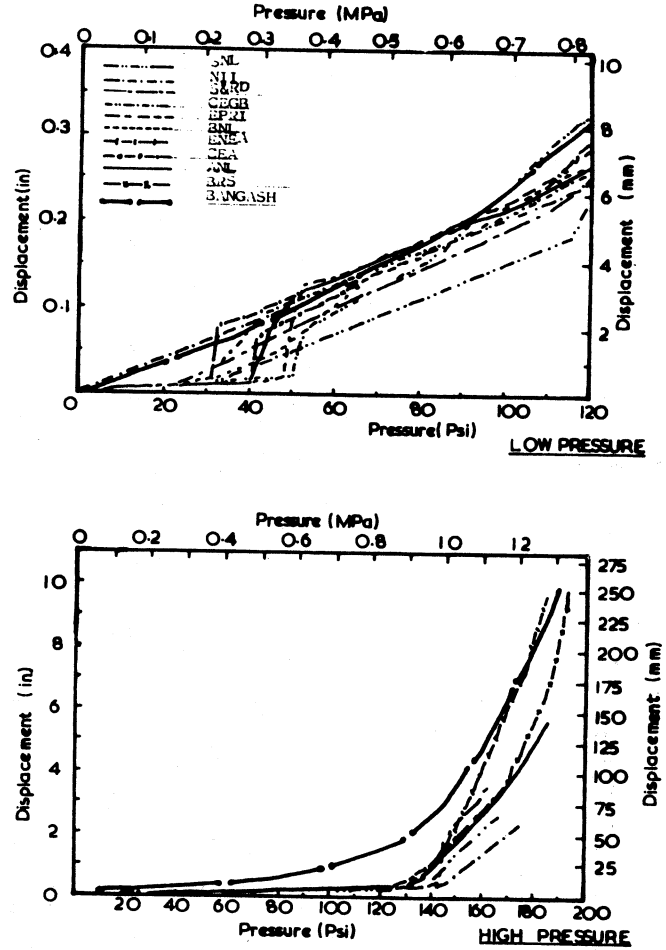


FIG3 RADIAL DISPLACEMENTS AT MID HEIGHT OF THE WALL

TABLE 1 CRACK INVESTIGATION

Crack indicators NCK (1), NCK (2), NCK (3)

NCK (1) — crack normal to the principal stress '1'

NCK (2) — crack normal to the principal stress '2'

NCK (3) — crack normal to the principal stress '3'

NCK (1) = 0, NCK (2) = 0, NCK (3) = 0 } no cracks

NCK (1) = 1, NCK (2) = 1, NCK (3) = 1 } cracks open.

NCK (1) = 2, NCK (2) = 2, NCK (3) = 2 } cracked closed.

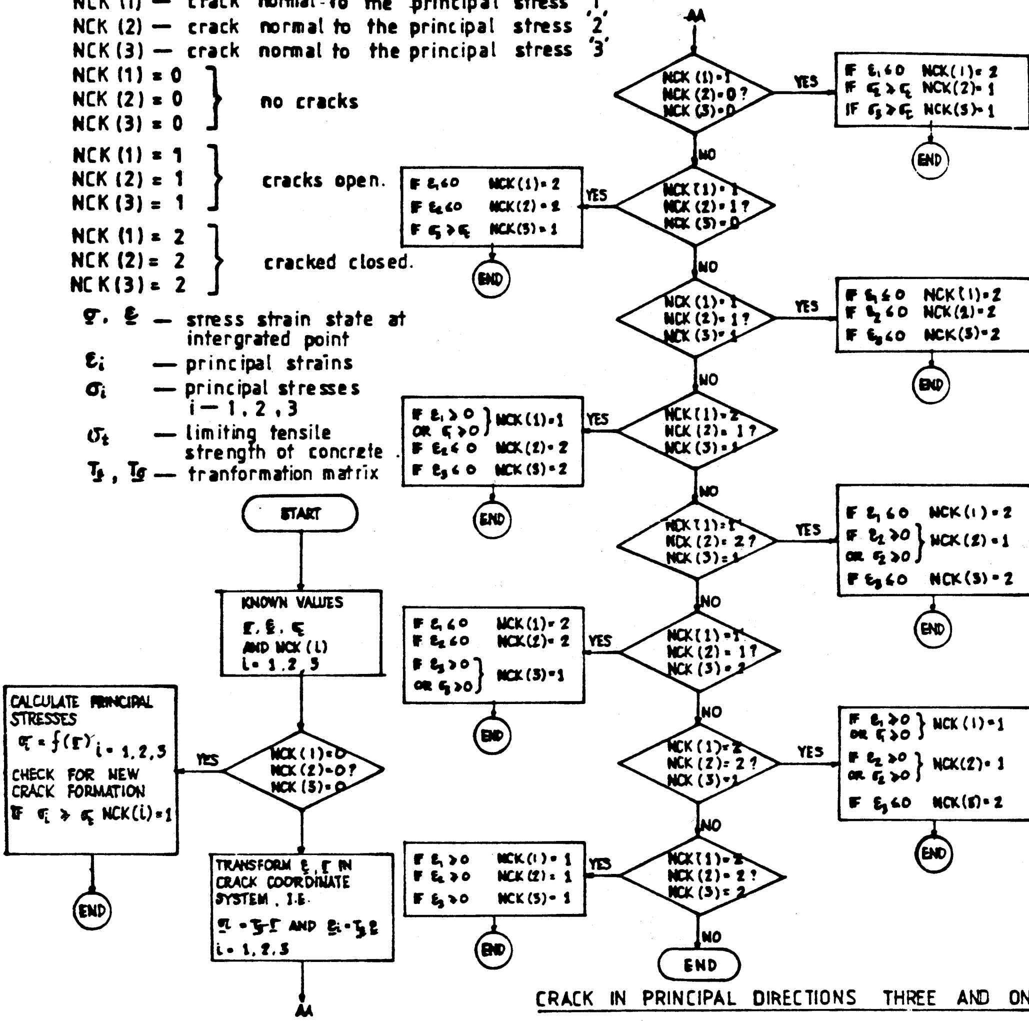
σ, ε — stress strain state at intergrated point

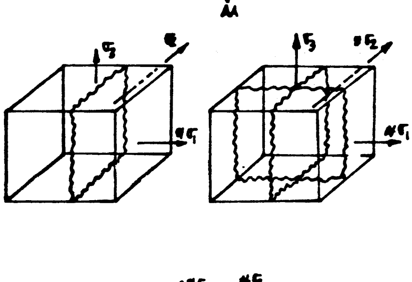
ε_i — principal strains

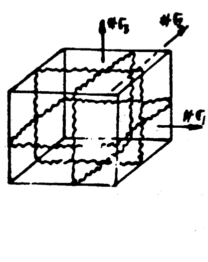
σ_i — principal stresses i — 1, 2, 3

σ_t — limiting tensile strength of concrete

T_ε, T_σ — tranformation matrix







CRACK IN PRINCIPAL DIRECTIONS THREE AND ONE

$$D_{11}^{*} = D_{33}^{*} = D_{12}^{*} = D_{21}^{*} = 0$$

$$D_{13}^{*} = D_{31}^{*} = D_{23}^{*} = D_{31}^{*} = 0$$

$$D_{22}^{*} \quad D_{22} - D_{12}\frac{D_{12}}{D_{11}} - D_{23}\frac{D_{23}}{D_{33}}$$

$$D_{44}^{*} = \beta D_{44}$$

$$D_{55}^{*} = \beta D_{55}$$

$$D_{66}^{*} = \beta D_{66}$$

CRACKS IN ALL THREE PRINCIPAL DIRECTIONS

$$[D^{*}] = [0]$$

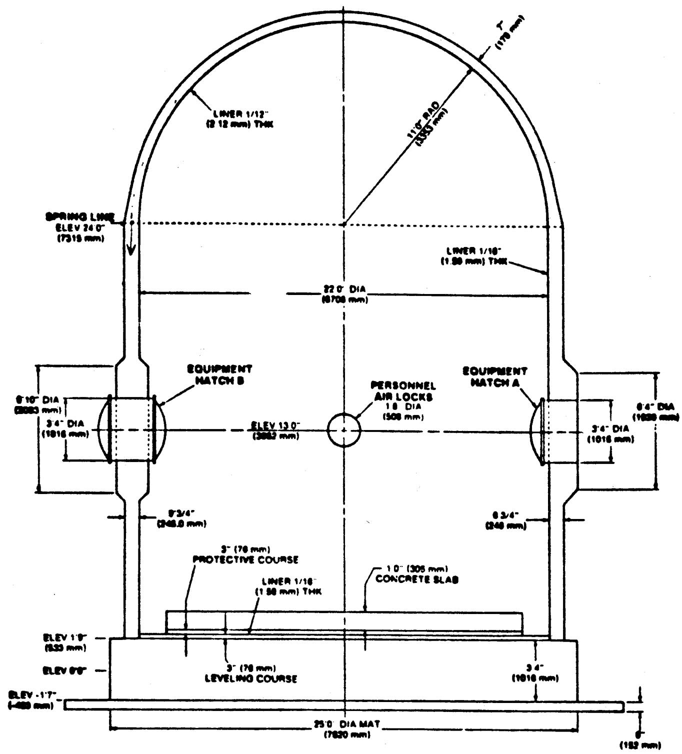


Figure 1. SCHEMATIC OF THE 1:6 SCALE REINFORCED CONCRETE CONTAINMENT MODEL - ELEVATION VIEW

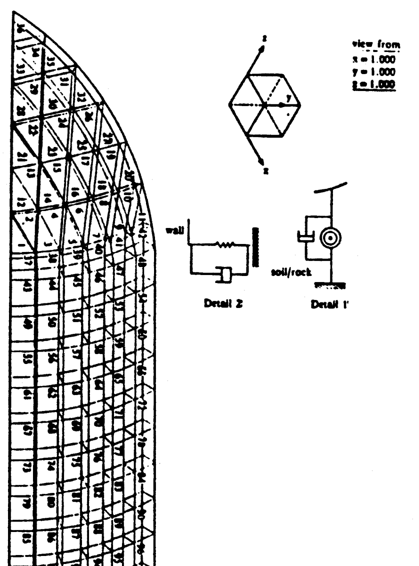


Fig. 2. 1:6 MODEL FINITE ELEMENT MESH