Prediction of Seismic Movement of a Site – Statistical Approach

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SUMMARY

The main objective of these studies is definition of the expected level of future earthquake ground motion for a considered site, applying the probability approach, and based on data from records of past earthquakes.

Determined were the empirical relationship functions of the peak horizontal ground acceleration and the magnitude and hypocentral distance. Also determined were the empirical relationships of dynamic response of linear models of a single-degree-of-freedom system with the same earthquake parameters. Twenty three single-degree-of-freedom systems have been chosen with natural periods in the range of 0.05 s and 5.0 s and damping of 5% of the critical.

The obtained empirical relationship functions have been applied in seismic hazard studies for determination of maximum expected values of peak horizontal ground acceleration, as well as response spectra of the pseudo-relative velocity for different levels of annual overriding probability for the considered site. So determined characteristics of ground motion define the design earthquake motions of the considered site.

INTRODUCTION

Ground motion due to strong earthquake effects is of a probability nature. Because of the random character of earthquakes, ground motion characteristics, such as amplitude, frequency content, time history, duration and other, cannot be determined uniquely, even if the parameters of expected future earthquake are known, as are the magnitude and the hypocentral distance. Accordingly, seismic excitation, i.e., the characteristics of future earthquake motion of the considered site, contain more random, variable and interdependent values, as a result of which in this study, the probability approach has been adopted for their definition, and that is why the studies are based on data from records of past earthquakes.

SELECTION OF STRONG MOTION RECORDS

Selection of 130 three-component records has been made, obtained on 74 instrument sites, from 53 past earthquakes with a magnitude of 3 to 8 on the Richter's Scale and hypocentral distance from 10 to 500 km. The earthquakes took place in Yugoslavia, North Italy, North Greece, Romania, Mexico and USA.

When making the selection it was strived at including earthquakes of different magnitudes and source depths, so that in addition to the available data, data from other seismological stations in Yugoslavia, Italy, Greece, Romania, Mexico and USA have been used, as well as from other international seismological stations.

In the studies, only the horizontal components of the records have been used, so that a data bank has been established consisting of 259 horizontal acceleration time histories. They have been applied as corrected acceleration time histories. During this, the records available in the form of uncorrected data (North Italy) have been processed according to the standard procedure for processing of strong motion records (Petrovski, 1979; Naumovski, 1982). They are grouped in five magnitude classes, and most of them belong to class four, or $5.5 < M < 6.5$. Each record enters the analysis with the corresponding magnitude and hypocentral distance.

RELATIONSHIP FUNCTIONS OF PEAK HORIZONTAL GROUND ACCELERATION WITH MAGNITUDE AND HYPOCENTRAL DISTANCE

For the studies in the field of seismic hazard it is necessary to determine the relationship functions of peak ground acceleration and earthquake characteristics. Empirical study has been applied which gave the empirical relationship functions of peak horizontal ground acceleration with the magnitude and hypocentral distance. For the mathematical model of relationship functions, applied was the model which had been used by several researchers (Schnabel and Seed, 1972; Esteva, 1964, 1970; Donovan, 1973; Esteva and Villavarde, 1973; McGuire, 1974; Ophal and Lahoud, 1974; Goto, Kameda and Sugito, 1981; Naumovski, 1984) and which has the following form

$$Acc = b_1.\exp(b_2M)(Rh+c)^{b_3} \qquad (1)$$

This mathematical model has been applied in the regression analysis. For this purpose, the model was transformed through logarithm of an e base, i.e.:

$$\ln Acc = b_1' + b_2M + b_3.\ln(Rh+20.); \; b_1' = \ln b_1 \qquad (2)$$

Eq.(2) represents median value of the natural logarithm of peak horizontal acceleration. It is a linear function of one dependent variable and two independent variable values. It is used to perform multilinear regression analysis with constant variation. In this, log-normal function of probability distribution for the recorded values of peak horizontal acceleration is assumed, or normal function of probability distribution for its natural logarithm. Determined were regression coefficients b_1', b_2 and b_3, as well as the conditional standard deviation $\sigma \ln Acc$. The obtained empirical relationship is with the following form:

$$Acc = 299.17 \exp(0.559M)(Rh+20)^{-1.145}; \; \sigma \ln Acc = 0.6981 \qquad (3)$$

It defines the ground motion amplitude of the considered site under earthquake effect with a magnitude M and hypocentral distance Rh.

By means of static tests (test for adoption of an empirical model of distribution and nonparametric tests: Kolgomorov-Smirnov and Pearson HI test), with which the critical value of the test has been determined with a significance level of 5%. The significance of the assumed log-normal function of probability distribution has been proved for the peak ground acceleration.

RELATIONSHIP FUNCTION OF THE DYNAMIC RESPONSE OF A LINEAR MODEL OF A SINGLE-DEGREE-OF-FREEDOM SYSTEM WITH THE MAGNITUDE AND HYPOCENTRAL DISTANCE

For determination of spectral characteristics of ground motion during strong earthquake effects, the distribution of dynamic response of a linear model of a single-degree-of-freedom system was statistically studied. As a parameter of the dynamic response, pseudo-relative velocity has been analyzed. Defined were empirical relationship functions of the dynamic response with the magnitude and hypocentral distance for each of the 23 selected models of a single-degree-of-freedom system, with natural periods of 0.5 s to 5.0 s and damping of 5% of the critical. The response has been obtained by means of numerical solution, and the relationship functions by means of regression analysis.

Regression Analysis

The same mathematical model of relationship functions was adopted as was the model for the peak horizontal ground acceleration, i.e.:

$$S(T;\xi=5\%) = b_1.\exp(b_2M)(Rh+20.)^{b_3} \tag{4}$$

where $S(T;\xi=5\%)$ is spectral value of the pseudo-relative velocity, obtained from the spectral relative displacement for a system with a natural period T in s and damping of 5% of the critical.

Applied was multilinear regression analysis with a constant variation for which the following transformed form of Eq.(4) has been used:

$$\ln S(T;\xi=5\%) = b_1' + b_2M + b_3(Rh+20.);\ b_1' = \ln b_1 \tag{5}$$

Eq.(5) is taken as a median value of the natural logarithms of spectral values of the pseudo-relative velocity for a system with a natural period T and damping of 5% of the critical. By regression analysis determined were the values of regression coefficients b_1', b_2 and b_3, as well as the conditional standard deviation $\sigma \ln S$. In the regression analysis assumed was a log-normal function of the probability distribution of the spectral value of pseudo-relative velocity.

Relationship Functions of the Horizontal Spectrum of Pseudo-Relative Velocity

Each of the 23 selected models of a single-degree-of-freedom system was exposed to the effect of 259 time acceleration histories. Obtained were, for each model, 259 maximum values of relative displacement, relative velocity and absolute acceleration. By using relative displacement, pseudo-relative velocity was obtained for each model and these data were basic for the regression analysis. The obtained results from the performed regression analysis are presented in Tab. 1. The adoption of the assumed log-normal function of the probability distribution for the pseudo-relative velocity has been proved by means of statistical tests made for all 23 models. Fig. 1 shows the attenuation functions obtained by application of relationship functions of the linear model of a system with T = 1.0 s and damping of 5% of the critical, and for different values of the magnitude and the hypocentral distance.

Using the obtained relationship functions, response spectra of the pseudo-relative velocity can be determined for different magnitudes and hypocentral distances as well as for different levels of probability. Here, calculated are the response spectra of the pseudo relative velocity as a medium, i.e., for a probability of 50% for an earthquake occurrence with a magnitude M = 6 and hypocentral distances Rh = 7, 12, 35 and 60 km (Fig. 2).

Obtained Results

From the obtained results for the response spectrum of pseudo-relative velocity, it can be concluded that the spectral value increases with the increase of the magnitude, and it decreases with the decrease of the hypocentral distance. The natural logarithms of the conditional standard deviations vary in the range of 0.68940 (T = 0.05 s) to 1.01813 (T = 5.0 s). These values show the wide dispersion of data of the pseudo-relative velocity around its average values, so that with only one more standard deviation, the value of the pseudo-relative velocity will be 2 to 2.8 median values.

It is important to be noted that the obtained relationship functions refer to average to medium soil conditions, which is of special importance due to the fact that local soil conditions affect the shape of response spectra.

These relationships will be used in seismic hazard methodology for determination of response spectra of the pseudo-relative velocity with the same probability of overriding, for the considered site.

SEISMIC HAZARD METHODOLOGY

In the seismic hazard methodology applied here for modelling of earthquake occurrence, Poisson's model has been applied for assuming the random character of earthquake occurrence in time. Seismic sources in the wider region around the site are modelled by the following geometrical shapes: point, line and area. The activity of each seismic source is presented by logarithmic-linear dependence of the frequency of earthquake occurrence from the magnitude,

$$\log n(M) = a-bM \qquad (6)$$

The empirical relationship functions of ground motion characteristics have been applied in the same form in which they have been here expressed, i.e.:

$$S = b_1.\exp(b_2M)\cdot(Rh+20.)^{-b_3}.\varepsilon \qquad (7)$$

The dispersion of data around the average value has been assumed through the random variable ε, whose natural logarithm has a normal function of probability distribution with an average value zero and standard deviation σ.

Each seismic source is divided in elementary parts. Determined was the probability of occurrence of peak value of the pseudo-relative velocity spectrum in case of an earthquake originating from an elementary part of the seismic source,

$$\hat{P}(S>s) = \Phi^*(x_1/\sigma)+(1-h)\left[\Phi^*(x_2/\sigma)-\Phi^*(x_1/\sigma)\right]+h\,\exp(\beta M_1+\beta^2\sigma^2/2b_2)(s/b_1)^{-\beta/b_2}.$$

$$(Rh+20.)^{-\beta b_3/b_2}\,.\left[\Phi^*(x_2/\sigma-\beta\sigma/b_2)-\Phi^*(x_1/\sigma-\beta\sigma/b_2)\right] \qquad (8)$$

where:

$\alpha = a.\ln 10$; $\beta = b.\ln 10$;

$h = \left[1-\exp(-\beta M_2+\beta M_1)\right]^{-1}$

Φ^* = complementary cumulative distribution of Gauss random variable

The probability of occurrence of peak value of the pseudo-relative velocity spectrum, in a certain time t of exposure of the site to earthquake effects from an elementary part of the seismic source is given with the expression:

$$\hat{P}(S<s,t) = \exp\left[-\vartheta\, t\hat{P}(S>s)\right] \qquad (9)$$

where:

ϑ = medium value of number of earthquakes from an elementary part of the seismic source, in a unit of time

For the case of a complex model of seismic activity of the region around the site, the probability of occurrence of peak values of the pseudo-relative velocity spectrum of the site, exposed t years to seismic activity, is:

$$P(S<s,t) = \Pi\,\hat{P}(S<s,t) \qquad (10)$$

Eq.(10), for the complex model of seismic activity received a form suitable for numerical processing:

$$P(S<s,t) = \exp\left[-t \sum_{k=1}^{N} \sum_{j=1}^{N_k} \nu_{j,k} . P_{j,k}\right] \quad (11)$$

where:

k index - seismic source (k=1,N)
j index - elementary part of the k-th seismic source ($j = 1,N_k$)

By application of Eq.(11) calculated were the functions of distribution of the pseudo-relative velocity spectrum for each model of a single-degree-of-freedom system with a period T and damping of 5%, and for a certain time t of exposure of the site. In this way, response spectra of the pseudo-relative velocity can be determined with an equal probability of overriding. They define the earthquake motion of the site.

The response spectra of the pseudo-relative velocity determined for one hypothetical site, and the level of annual probability of overriding of 0.01, 0.002 and 0.001 are shown in Fig. 3.

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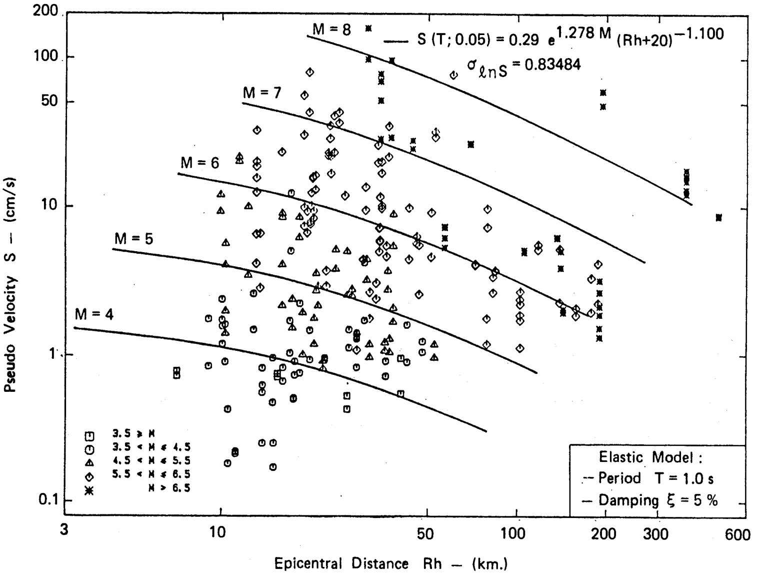

Fig. 1. Attenuation Functions of Horizontal Pseudo - Relative Velocity Spectrum for Elastic Model with T = 1.0 s and $\xi = 5\%$

Table 1. Regression Coefficients and Standard Deviations of Horizontal Pseudo - Relative Velocity Spectrum for Elastic Model of a Single - Degree - of - Freedom System

– Mathematical Model : $S(T, 0.05) = e^{b'_1} e^{b_2 \cdot M} (Rh + 20)^{b_3}$

Period T(s)	b_1'	b_2	b_3	$\sigma_{\ell n}S$
0.05	1.55060	0.46627	−1.14060	0.68940
0.06	2.06064	0.44960	−1.16851	0.68614
0.08	2.81686	0.44858	−1.22640	0.66217
0.10	3.19637	0.48109	−1.27355	0.67132
0.13	3.00453	0.57970	−1.28538	0.70538
0.17	2.56301	0.65999	−1.21027	0.73582
0.20	2.38686	0.69428	−1.18261	0.75610
0.24	2.07009	0.74736	−1.14805	0.78853
0.30	1.67097	0.85502	−1.18191	0.83314
0.34	1.54557	0.90272	−1.21327	0.83518
0.40	1.29096	0.96834	−1.23067	0.85477
0.50	0.68650	1.11738	−1.28964	0.87421
0.60	0.09217	1.20169	−1.27313	0.86627
0.80	−0.68993	1.27222	−1.20655	0.86076
1.00	−1.24456	1.27829	−1.09987	0.83484
1.30	−1.92281	1.26555	−0.94633	0.86774
1.70	−2.59764	1.28818	−0.84612	0.91437
2.00	−3.24904	1.31100	−0.74266	0.95875
2.40	−3.48052	1.35898	−0.79485	0.97767
3.00	−3.94842	1.36958	−0.76042	0.99298
3.40	−4.03050	1.35567	−0.75911	1.01001
4.00	−4.04107	1.33222	−0.77695	1.02317
5.00	−4.14524	1.28245	−0.74127	1.01813

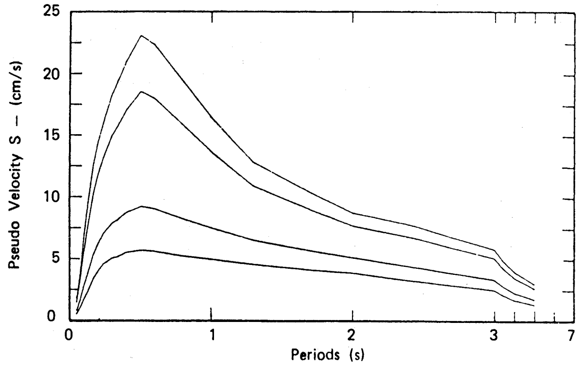

Fig. 2. Median of Horizontal Pseudo - Relative Velocity Response Spectra for $\xi = 5\%$, M = 6 and Rh = 7, 12, 35 and 60 km.

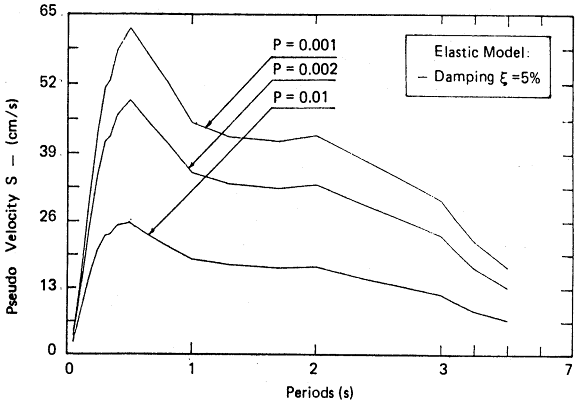

Fig. 3. Horizontal Pseudo - Relative Velocity Response Spectra with Equal Annual Probability of Overriding of 0.01, 0.002 and 0.001