Determination of Waveform Similarity from Seismic Response Spectra

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ABSTRACT

Qualification of equipment by experience often includes generation of a composite response spectrum from available data. Since the nature of a given waveform from which a spectrum is computed can have a significant bearing on the outcome of a qualification, consideration must be given to the similarity of the various waveforms to assure compatibility of the composite with the constituent spectra. In other words, a waveform which corresponds to the composite spectrum must be similar to the waveforms which correspond to the respective constituent spectra. Therefore, a method is presented whereby similarity of excitation waveforms can be established approximately by a comparison of numerical parameters obtained directly from the corresponding response spectra. Sample results are computed for several different typical spectra.

1. INTRODUCTION

Various waveforms have been used to represent seismic excitations for qualification of nuclear plant equipment. However, documentation of the qualification usually has been given in terms of the response spectrum computed from the test or analysis waveform, rather than a time history of the waveform itself. Nevertheless, similarity of the waveforms (and therefore of the corresponding response spectra) must be established in order to assure a valid qualification. Furthermore, methods of quantifying waveform similarities have become important because of a trend toward use of composite spectra in qualifications based on existing experience data [IEEE-344, 1987]. Suitability of waveforms for excitation time histories used in seismic qualifications has been explored previously by Kana and Pomerening [1984]. They have also shown [1988] that several factors are important for complete excitation similarity to exist. However, a convenient method for measurement of waveform and corresponding response spectrum similarity has not been determined. Therefore, the development herein is intended to provide a ready means of comparing waveform similarities without having to generate actual time histories.

2. WAVEFORM PARAMETERS

Generally, several parameters are necessary to characterize the dynamic properties of a waveform which describe its frequency content, stationarity, and amplitude distribution. For typical signals (including sine dwells, sine beats and random signals of various bandwidths and duration), parameters such as power spectral density, probability density, and Peak/RMS ratios can be used to characterize the signals and the response of linear systems to which they may be applied [Bendat and Piersol, 1980]. However, the response spectrum, rather than any of the above, is the typical parameter available from qualification reports. It is possible to estimate the former parameters which would correspond to a given response spectrum, but the methods are relatively elaborate [Unruh and Kana, 1985], and the accuracy required to show waveform similarity probably does not warrant their use. Therefore, an approximate approach to demonstrate waveform similarity will be based on the ratio of the maximum spectral response of a damped single degree-of-freedom oscillator to the peak acceleration (ZPA) for the waveform used to excite the oscillator (i.e., maximum-output/peak-input); a ratio that can readily be obtained directly from the shape of the response spectrum for a given waveform. Comparisons with Peak/RMS ratios for the same waveforms also will be included to relate the development more directly with stationary random process concepts.

The maximum-output/peak-input ratio, $R_X^*(f)/ZPA$ for a simple oscillator, can be called the maximum response factor. The sensitivity of this factor for a single degree-of-freedom oscillator excited by several typical waveforms has been recognized in the past [Kana and Pomerening, 1987], and is shown for convenience in Figure 1. It will be shown that this response factor also can be related to the shape of the excitation response spectrum in terms of the maximum spectral value, the bandwidth, and the center frequency of the amplified region (i.e., where the spectral values are greater than the ZPA). Similarity of this factor between various response spectra will be shown to be sufficient to establish corresponding waveform similarity. Hence, by inspection of the test response spectra generated during a qualification program, it will be possible to develop the ratios required to show waveform similarity for the corresponding excitations. Subsequently, the same approach can be used to assure waveform similarity for any further developed composite spectrum. This will also assure that the generated composite spectrum can be matched by an appropriate time history, without actually having to produce the time history.

From Figure 1, it is obvious that the maximum response factor for a given waveform depends on both the frequency and the damping of the oscillator. Furthermore, it is obvious that the bandwidth and center frequency for the excitation will influence the results. Therefore, herein it will be understood that the maximum response factor will be constant for a spectrum which is flat in the frequency band of concern, some average value for one which is sloped, and some weighted value for complex spectra that include multiple peaks. Hence, the analysis herein deals only with simple spectra that represent flat random energy excitation in a single frequency band of specified width. All analyses will be based on 5% damping for the various spectra, a value representative of typical equipment. Further information on the effects of more complex types of response spectra, and other values of damping can be obtained from the more detailed development of Kana and Pomerening [1988].

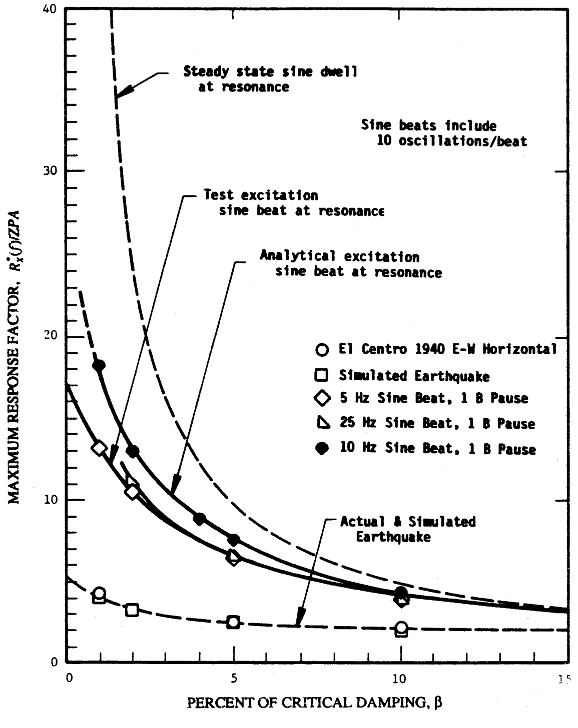
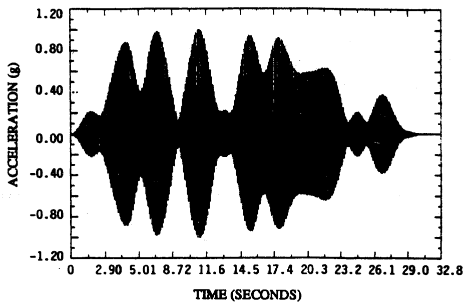
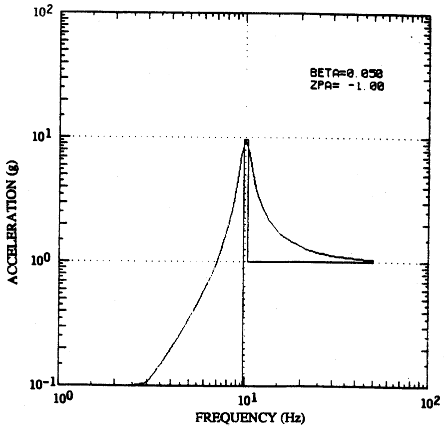


FIGURE 1. MAXIMUM SPECTRA RESPONSE FACTORS FOR A SDOF SYSTEM UNDER VARIOUS EXCITATION CONDITIONS



a) Time history



b) Response spectrum

FIGURE 2. 10 HZ CENTER FREQUENCY WITH A 0.5 HZ BANDWIDTH

For this analysis, it is assumed that the duration of a time history which corresponds to a composite spectrum is similar in length to those of the constituent spectra. In addition, it is assumed that energy in the various amplified regions is present during the entire strong motion portion of the signal. This concept of stationarity is consistent with the requirements for random motion testing defined in IEEE-344 [1987]. Only Gaussian random motion is considered in this analysis, since the data for other types of motion (i.e., sinedwells or sinebeats) are not often presented in terms of a response spectrum. Determination of similarity of these types of waveforms is easier to accomplish directly, rather than using the response spectrum or other parameters mentioned above.

3. SIMPLE FLAT RESPONSE SPECTRA

Maximum response factors were studied for various simple response spectra that are essentially flat in the amplified region of a single frequency band. Corresponding time histories were developed for various spectra having different single bandwidths and center frequencies, but each having a 1.0g ZPA. The corresponding spectra and resulting data were ultimately plotted to show variation of maximum response as a function of bandwidth and center frequency. The corresponding time histories also were used to determine ZPA/RMS ratios for comparison. Up to five corresponding time histories were developed for each response spectrum to indicate statistical influences on the results.

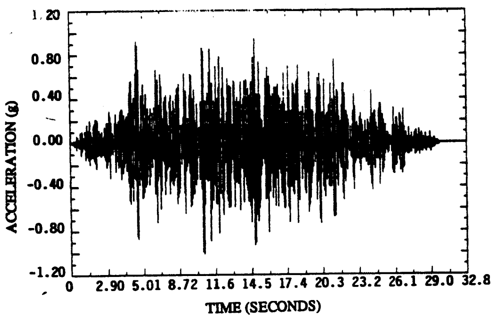
The procedure utilized to generate a time history to match a specified response spectrum is an adaptation of that developed by Unruh [1982]. It is an iterative procedure in which the time history is developed for a weighted linear sum of nonstationary narrowband pseudorandom noise signals. The nonstationarity is associated with the envelope (5 second buildup, 15 second hold, and 10 second decay) used to represent the earthquake signal. For this program, a series of 99 narrowband pseudorandom noise signals were generated at even frequency intervals from 1.0 to 50.0 Hz, i.e., a bandwidth of 0.5 Hz each. An initial guess of the required weighting factors used in the summation was made and the time history generated. The response spectrum for this time history was then calculated and compared to the one required. Adjustments were then made to the weighting factor for each bandwidth, and the procedure repeated until an acceptable match was achieved. At this point, the RMS value for both the entire time history and the strong motion portion (15 second hold) were calculated. In addition, the time history was plotted along with the corresponding response spectrum. Each time history was generated with a sample rate of 512 samples/second over the duration of the signal.

Using the signal generation procedure described above, it was possible to develop spectrally-compatible time histories having energies only in a preselected bandwidth, which could be related to a given spectrum. This was done by forcing to zero all weighting factors for frequency bands outside the preselected bandwidth. This approach is consistent with the requirements of IEEE-344 [1987], which states that energy above the required response spectrum ZPA cutoff frequency should not be included in the time history. However, even with this capability to limit the bandwidth of excitation, it is not physically possible to generate a time history which produces a response spectrum with sharp break-over points. This results from the nature of the response spectrum calculation and bleed-over of values. Therefore, the values for bandwidth presented represent the energy bandwidth as defined above, which corresponds to the bandwidth at about 0.8 times the calculated maximum response spectrum values.

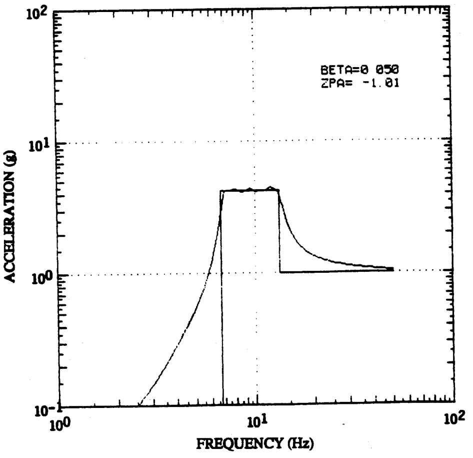
4. TYPICAL RESULTS

Spectrally-compatible time histories were developed and studied for response spectra having center frequencies of 5, 10, and 20 Hz, with bandwidths varying from 0.5 Hz to a maximum, where the lower limit was set at 0.75 Hz and the upper limit was two times the center frequency, minus 0.75 Hz. As noted previously the damping was set at 5%. Figures 2-4 show three examples of a spectrum and its associated time history. In the lower part of the figure, the specified frequency bandwidth is shown by a rectangular curve, along with a response spectrum computed from the time history given in the upper part of the figure. The indicated curve allowed a determination of the 0.8 factor mentioned previously.

The next step of analysis included plotting results from one given set of time history data. Thus, for a specified response spectrum and corresponding time history, both the $R_X^*(f)/ZPA$ and ZPA/RMS ratios were plotted as a function of bandwidth for given center frequencies. However, for each case the results are dependent on different, but statistically similar time histories. Therefore, to obtain some information on such statistical variations, spectrally-compatible transient random time histories were generated from five sets of statistically independent, narrow-band pseudorandom signals. The data were tabulated from each of these time histories, as described above, were analyzed statistically, and

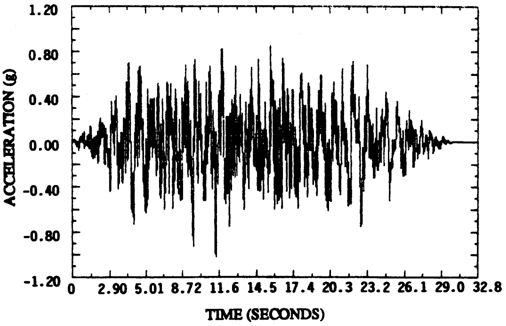


a) Time history

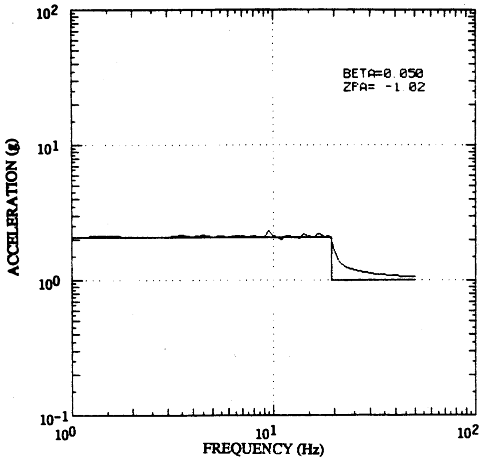


b) Response spectrum

FIGURE 3. 10 HZ CENTER FREQUENCY WITH A 6.5 HZ BANDWIDTH



a) Time history



b) Response spectrum

FIGURE 4. 10 HZ CENTER FREQUENCY WITH AN 18.5 HZ BANDWIDTH

then plotted as shown in Figures 5 and 6. Although the maximum response factor, given in Figure 5 has some significant scatter, there are several observations that can be made:

1) As the bandwidth approaches zero (i.e., sinusoidal excitation) the ratio approaches a value of $1/(2\beta)$, the response of a damped single degree-of-freedom system to steady-state sinusoidal excitation. In this case, the response spectrum reduces to the response of a damped single degree-of-freedom system at the center frequency.
2) Even for a relatively short duration (15 seconds) of the strong motion portion of the signal, the system tends to develop to its full resonance response.
3) At a given bandwidth, the maximum response factor increases with increasing center frequencies. There is a trend to the data in each group and, therefore, it may be possible to develop some analytical expression to match the data.
4) The maximum response factor seems to approach an asymptote. It is felt that the asymptote can be related to the response spectrum for classical shock pulses, and is a function of the center frequency and damping of the system.

Figure 6 gives the corresponding ZPA/RMS ratio. For a stationary random signal with Gaussian distribution, one would expect the value to be around 3.0. Since the signal in question includes an earthquake type envelope, the value is somewhat greater than 3.0 (i.e., approximately 3.3). There is some decrease for the lower bandwidths, but the change occurs only below about 3 Hz. As the bandwidth approaches zero, one would expect the value to approach 1.414, the value for a sinusoidal excitation.

Upon further study of the data given in Figure 5, it was noted that an additional correlation could be made for the mean values of the maximum response factors. By plotting these factors against bandwidth/center frequency ratio, it was found that all three curves in Figure 5 collapse onto the single curve shown in Figure 7. Thus, the values extend from a factor of 10.0 for a sine dwell, to about 1.8 for the broadest possible bandwidth at B = 2.0. Note that this value is the maximum possible, since by definition no bandwidth can be wider than twice the center frequency. Furthermore, a transition region has been estimated, above which the signals are essentially Gaussian random. This transition region is estimated by noting that Figure 7 represents a cross plot of data from Figure 1 at a damping value of $\beta = 5\%$. This result alone can be useful in synthesizing signals to represent given response spectra.

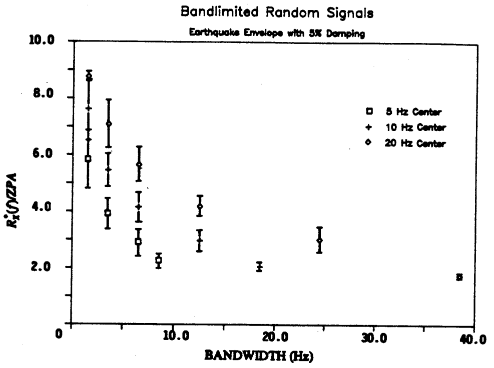


FIGURE 5. MEAN AND 2-SIGMA BANDS FOR $R_X^*(f)/ZPA$ RATIO

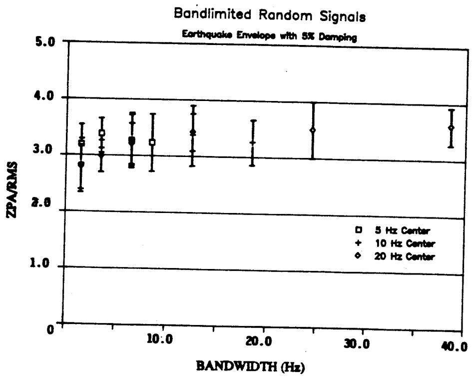


FIGURE 6. MEAN AND 2-SIGMA BANDS FOR ZPA/RMS RATIO

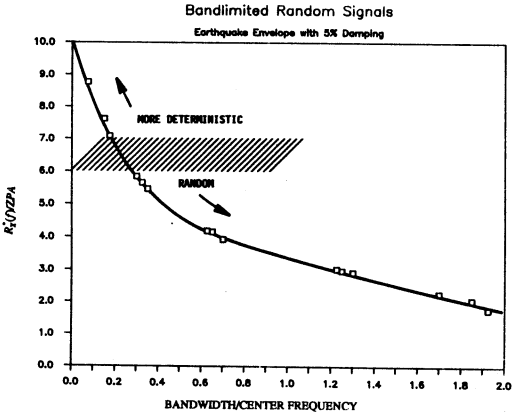


FIGURE 7. MEAN VALUES FOR MAXIMUM RESPONSE FACTOR FOR VARIOUS WAVEFORMS

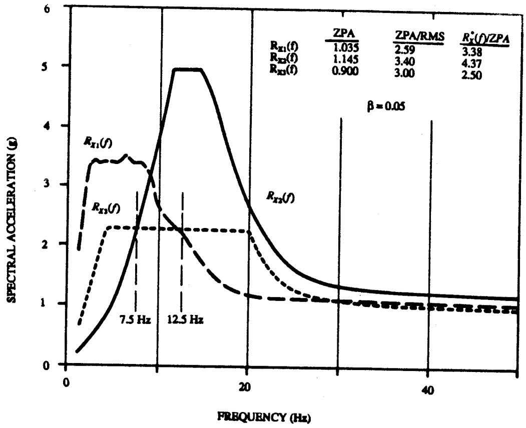


FIGURE 8. DEVELOPMENT OF COMPOSITE SPECTRUM

5. APPLICATION

An application example is given in Figure 8, where $R_{X1}(f)$ and $R_{X2}(f)$ are two constituent spectra for which two different equipment items respectively have been qualified. A composite spectrum is sought for which qualification of both items can be claimed. It has been established that both items have primary structural natural frequencies in the range of 7.5 to 12.5 Hz. The bandwidths and associated center frequencies for each spectrum can be obtained at 0.8 of the peak response values. Thus, for $R_{X1}(f)$ the bandwidth and center frequency are about 8 Hz and 7 Hz respectively, with B=1.4; for $R_{X2}(f)$ they are about 7 Hz and 13 Hz, with B=0.52. By entering Figure 7 with these parameters and the indicated respective response factors, it can be established that both data points fall near the random motion segment of the curve. Thus, both spectra represent typical random waveforms with an earthquake envelope, and with ZPA/RMS ratios of about 3.3 Therefore in order to satisfy waveform similarity, any composite spectrum developed from these constituents should have the same properties (i.e., its parameters also should fit the random motion segment of the curve).

According to Kana and Pomerening [1988], the composite spectrum will be similar to the constituent spectra in frequency content if both $R_{X1}(f)$ and $R_{X2}(f)$ envelope the composite in the critical frequency band of 7.5 to 12.5 Hz. Therefore if its maximum spectral value is chosen to be about 2.3 g, and its ZPA is chosen to be 0.9 g (to be equal or less than the largest ZPA of the two constituents), what choices of bandwidth and center frequency are compatible with the constituent spectra? By entering Figure 7 at a $R_X^*(f)/ZPA$ ratio of 2.3/0.9=2.5, it can be seen that B=1.5 is an appropriate value. Therefore, a composite having a bandwidth from 3 to 21 Hz [at 0.8 $R_X^*(f)$] centered at 12 Hz is appropriate. This composite spectrum is shown as $R_{X3}(f)$ in Figure 8. By satisfying the above procedure it corresponds to a waveform which includes both excitation frequency and waveform similarity, compared with the constituent spectra. Therefore, both equipment items are qualified to the composite spectrum.

6. CONCLUSIONS

The maximum response factor obtained from a given response spectrum can be used to indicate the nature of a time history that is compatible with the response spectrum. Generally, this can be done by determining its location on the curve in Figure 7. Thus, two spectra can be shown to include waveform similarity by demonstrating that they fall on comparable segments of the curve.

Most spectra which have been specified in equipment qualification procedures will fit the random motion region defined in Figure 7. Thus, random waveforms can most appropriately be used for their representation. For this, it should be noted that with a given value of damping, the maximum response factor is a function of both bandwidth and center frequency. Therefore, care must be exercised in estimating enveloping or composite spectra so that this relationship is maintained. Furthermore, since the ZPA/RMS ratio is nearly constant in this range, this part of the curve also approximately represents the ratio $R_X^*(f)/(3.3RMS)$ as well.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

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