On the Amplification Effect of Dipping and Parallel Soil Medium to Seismic Wave

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ABSTRACT

To obtain the amplification spectra due to seismic source for the parallel and dipping layered mediu, we simulate the seismic waves as those emitted from transient SH line source, which is located in the half space overlaid with a single dipping layered medium. Then, from the obtained Fourier spectra, it shows that both the fundamental frequency and Fourier amplification ratio are different for parallel and dipping layered media with smaller amplification for dipping medium, and this phenomenon may be referred to as the concentration of energy in the dipping one. Hence, the reactor erected above sloping foundation must consider this effect.

INTRODUCTION

From the Skopje earthquake(1963), observations of earthquake effects on structures situated above inclined interfaces show an increase of damage against those observed on similar structures with foundation above a parallel layered part (Porceski, 1969), hence, the investigation of seismic wave propagating in dipping layered medium is the main object in our problem. When both the source and receiver are at large distance from the apex, and when the angle of inclination is small, the diffraction and radiation of waves by the apex of a wedge can be omitted. For such cases, an approximate ray theory was first applied by Ishii and Ellis (Ishii, 1970) and Hong and Helmberger (Hong, 1977) to problem of SH waves in a wedge overlaying a half-space (a two layer media). A general formulation of the ray integrals and the inverse transform of the integrals by applying the Cagniart method were made by Pao and Ziegler (Pao, 1982). The generalized ray theory so formulated can be applied to investigate the responses of SH wave propagating in dipping as well as parallel layered media. In this study, we refer to literature (Pao, 1982), of which the slowness formula for upward source segment was corrected (Chen, 1986), and the numerical results for the responses of transient SH wave propagating in parallel and dipping medium, are calculated in this study, then the comparisons and conclusion for the amplification spectra between these two are thus made.

THE GENERALIZED RAY INTEGRAL

The emitted wave without impinging on the boundary is referred to as the source ray. The solution of which is governed by

$$c_s^2 \nabla^2 u + B = \frac{\partial^2 u}{\partial t^2}$$

where $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial z^2$, $c_s$ denotes the wave speed in half space, and $u(x,z,t)$ is the horizontal displacement in $y$-direction.
As shown in Fig. 1, the source is described by the function $B(x, z, t) = \delta(t) \delta(x) \delta(z - z_o)$, where $\delta$ denotes Dirac's delta function. To solve Eq. (1), we define the Laplace transform as

$$
\tilde{u}(x, s) = \int_0^\infty u(x, t) e^{-st} dt,
$$

(2)

where the overhead bar denotes the transform with the parameter $s$, and define the Fourier transform as

$$
\hat{u}(\xi, s) = \int_{-\infty}^{\infty} \tilde{u}(x, s) e^{-is\xi x} dx,
$$

(3)

where the overhead hat denotes the transformation with the parameter $s \xi$, then by applying Eqs. (2) and (3) to (1), we obtain

$$
\mathcal{F} = \mathcal{F}(s) \int_{-\infty}^{\infty} S(\xi) e^{s \xi (x, z ; \xi)} d\xi,
$$

(4)

where

$$
\xi = \frac{1}{c_1} \xi - \xi (z - z_o),
$$

(4.1)

is the phase function for the source, $\mathcal{F}(s) = a_1^2 \mathcal{F}(s)/4 \eta$, $a_2 = 1/c_2$, $\xi^2 = \xi^2 + a_2^2$, $\mathcal{F}(s)$ is the Laplace transform of $f(t)$, and the expression of $S(\xi) = 1/\xi$ is defined as the source function. Furthermore, if the source ray impinges upon the surface and is further transmitted into surface layer as shown in Fig. 1 and 2, then the Laplace transform for the displacement can be written as

$$
\tilde{u}_o = \mathcal{F}(s) \int_{-\infty}^{\infty} S(\xi) T^{(1)}(\xi) e^{s \xi (x, z ; \xi)} d\xi,
$$

(5)

where $u_o$ denotes the displacement in the upper layer, and

$$
T^{(1)}(\xi) = 2 \mu_2 \xi / (\mu_1 + \mu_2 \xi),
$$

is the transmission coefficient for upward wave, $a_1 = c_1$ is the slowness for the transmitted ray, $s^2 = a_2^2 + \xi^2$ and the phase function is

$$
g_o = \frac{1}{c_1} \xi - \xi (z_o - h) - \eta(h - z),
$$

(6)

With reference to literature (Pao, 1977) and Fig. 1, the general expression of displacement for the ray reflected $k$ times in parallel medium can be written as

$$
\tilde{u}_k = \mathcal{F}(s) \int_{-\infty}^{\infty} S(\xi) \Pi(\xi) D e^{s \xi (x, z ; \xi)} d\xi,
$$

(7)

with phase function

$$
g_k(x, z; \xi) = \left\{ \begin{array}{l}
\xi (x - \xi (z_o - h) - \eta(kh + z),
\xi (x - \xi (z_o - h) - \eta[(kh + (h-z)])
\end{array} \right.
$$

(8)

and the product of reflection and transmission coefficients

$$
\Pi(\xi) = T^{(1)}(\xi) R^{(0)}(\xi) R^{(1)}(\xi) \cdots R^{(k)}(\xi),
$$

(9)

on which $R^{(0)}(\xi) = 1$ is the reflection coefficient on free surface, $R^{(1)}(\xi) = (\mu_1 - \mu_2 \xi) / (\mu_1 + \mu_2 \xi)$ is the reflection coefficient on interface, and $D$ is the receiver function, which is

$$
D = \left\{ \begin{array}{ll}
1, & \text{in interior},
2, & \text{on free surface},
2\mu_2 / (\mu_1 + \mu_2 \xi), & \text{on interface for downward incident},
2\mu_1 / (\mu_1 + \mu_2 \xi), & \text{on interface for upward incident}.
\end{array} \right.
$$

As for the solution of the responses in dipping layered medium, we can solve the same problem in an alternative coordinate system of $x', z'$ as shown in Fig. 2, which gives

$$
\tilde{u}(x', z'; s) = \mathcal{F}(s) \int_{-\infty}^{\infty} S(\xi') e^{s \xi (x', z' ; \xi')} d\xi',
$$

(10)
where the prime symbol denotes all the quantities with respect to the dipping coordinate system of \( x' \) and \( z' \), and

\[
g(x', z'; \xi') = \frac{1}{2} (x' - z_0 \sin \alpha - \xi' [z' - z_0 \cos \alpha]),
\]

(10.1)

Then, through the condition of invariance of the phase functions (Pao, 1982), we equate the phase function in Eqs. (4.1) and (10.1), which yield

\[
g(x, z; \xi) = g(x', z'; \xi'),
\]

(11)

for simplicity, we let \( z_0 = 0 \) in Eq. (10.1) and assume the source segment emitted upwards, then Eq. (11) leads to

\[
\zeta x + \zeta z = \zeta' x' + \zeta' z',
\]

(12)

and by substituting the following relationship of

\[
x = x' \cos \alpha - z' \sin \alpha,
\]

(13)

\[
z = x' \sin \alpha + z' \cos \alpha,
\]

(14)

hence, with reference to literature (Chen, 1986) and Fig. 2, we obtain the Laplace transform of displacement and phase function for the wave propagating at interface are

\[
\bar{u}(x', z'; s) = \mathcal{F}(s) \int_{-\infty}^{\infty} S(\xi') \varpi_{g}(x'_d, z'_d; \xi') d\xi',
\]

(15)

\[
g(x', z'; \xi') = \frac{1}{2} (x'_0 - z_0 \sin \alpha) + \xi' (z'_0 - z_0 \cos \alpha),
\]

(16)

whereas those in the surface layer are expressed as

\[
\bar{u}_0(x', z', s) = \mathcal{F}(s) \int_{-\infty}^{\infty} S(\xi') T^{(1)}(\xi') \varpi_{g}(x'_d, z'_d; \xi') d\xi',
\]

(17)

\[
g_0(x', z'; \xi') = g(x'_0, z'_0; \xi') + i \xi'_0 (x' - x'_0) + \eta_0 (z' - z'_0),
\]

(18)

then, by substituting Eq. (16), \( \xi'_0 = \xi' \) and \( z'_0 = h \cos \alpha \) into Eq. (18), we obtain

\[
g_0(x', z'; \xi') = -i \xi'_0 z_0 \sin \alpha - \xi' (z_0 - h \cos \alpha - \eta_0 h \cos \alpha + i \xi'_0 x' + \eta_0 z'),
\]

(19)

furthermore, from the invariance of phase function and Eq. (19), it yields

\[
g_0(x, z; \xi) = -i \xi'_0 z_0 \sin \alpha - \xi' (z_0 - h \cos \alpha - \eta_0 h \cos \alpha + i \xi_0 x + \eta_0 z),
\]

(20)

and if the ray is reflected from point (1), as shown in Fig. 2, of which the ray integral and phase function can be written as

\[
\bar{u}_1(x, z, s) = \mathcal{F}(s) \int_{-\infty}^{\infty} S(\xi) T^{(1)}(\xi') R^{(s)}(\xi) \varpi_{g}(x', z'; \xi') d\xi',
\]

(21)

\[
g_1(x, z; \xi) = g_0(x_1, z_1; \xi_0) + i \xi_1 (x - x_1) - \xi_1 (z - z_1),
\]

(22)

then, from Snell's law, it gives \( \xi_1 = \xi_0 \), \( \eta_1 = \eta_0 \)

and by substituting Eq. (20) into Eq. (22), it yields

\[
g_1(x, z; \xi) = -i \xi'_0 z_0 \sin \alpha - \xi' (z_0 - h \cos \alpha - \eta_0 h \cos \alpha + i \xi_0 x - \eta_0 z),
\]

(23)

If we express Eq. (21) in dipping coordinate system, we obtain

\[
\bar{u}_1(x', z', s) = \mathcal{F}(s) \int_{-\infty}^{\infty} S(\xi') T^{(1)}(\xi') R^{(s)}(\xi) \varpi_{g}(x', z'; \xi') d\xi',
\]

(24)

and from invariance of phase function, Eq. (23) can be rewritten as

\[
g_1(x', z'; \xi') = -i \xi'_0 z_0 \sin \alpha - \xi' (z_0 - h \cos \alpha - \eta_0 h \cos \alpha + i \xi'_0 x' - \eta'_0 z'),
\]

(25)

In the like manner, and by repeatedly applying Snell's law and invariance of phase, we obtain the general expression for the ray integral as
\[
\bar{u}_k(s) = \bar{F}(s) \int_{-\infty}^{\infty} S(\xi') \Pi(\xi') D e^{s \xi'_k} d \xi',
\]
with the product of reflection and transmission coefficient
\[
\Pi(\xi') = T^{(0)}(\xi') R^{(0)}(\xi_1) R^{(1)}(\xi_2) R^{(2)}(\xi_3) \cdots \begin{cases} R^{(0)}(\xi_k), & k=1,3,5, \ldots \vphantom{R^{(0)}(\xi_1)} \\
R^{(1)}(\xi_k), & k=0,2,4, \ldots \vphantom{R^{(0)}(\xi_1)} \end{cases},
\]
the transform formula for slownesses as
\[
\begin{cases}
\xi_k = \xi_{k-1} & = \xi_k \cos(ka) + i \eta_k \sin(ka) \\
\eta_k = \eta_{k-1} & = i \xi_k \sin(ka) + \eta_k \cos(ka), & k = 1,3,5, \ldots \vphantom{\eta_0}
\end{cases}
\]
and the general expression for phase function given as follows
\[
g_k = -i \xi_k \bar{z}_0 \sin a - \xi'(z_0 - h) \cos a - (h \cos a) \eta_0 - 2h \cos a \sum_{j=1}^{m/2} \eta_{(2j-1)} + i \xi_{(2j-1)} + (-1)^k \eta_{mz}
\]
where
\[
m = \begin{cases} k, & k = 0,2,4, \ldots \vphantom{2} \\
2k-1, & k = 1,3,5, \ldots \vphantom{2} \end{cases}
\]

LAPLACE INVERSE TRANSFORM BY CAGNIARD METHOD

So far we will employ the well-known Cagniard method to inverse Eqs. (7) and (26) and for the convenience we let
\[
\bar{I}_k(s) = \int_{-\infty}^{\infty} S(\xi') \Pi(\xi') D e^{s \xi'_k} d \xi',
\]
then, the inverse transform can be obtained by inspection.

With reference to Fig.3, it is desired for physical significance to keep variable t as a real number, which must be greater than zero. Hence the contour ACM in \( \xi' \)-plane is chosen so that Eq. (32) maps it upon t-plane in real t-axis, and by observing Fig. 3, we can find the stationary point M, which is governed by the following equation
\[
\frac{dt}{d\xi'} = 0
\]
then, the inverse transform can be obtained by inspection.

Referring to literature (Chen, 1986), the integral contour in Eq. (31) can be converted into AMC as follows
\[
\bar{I}_k(s) = 2R \int_{AMC} S(\xi') \Pi(\xi') D e^{s \xi'_k} d \xi',
\]
where 'R' denotes 'Real Part of' and as discussed in the same reference, there is no contribution to the response for the contour between A and M, hence the time \( t_M \) corresponding to \( \xi'_M \), obtained from Eq. (34), is referred to as the arrival time, and the inverse transform of Eq. (35) is thus obtained by inspection as follows
\[
I_k(t) = 2H(t-t_M)R[S(\xi'(t)) \Pi(\xi'(t)) D \frac{d \xi'}{dt}] \int_{-\infty}^{\infty} S(\xi') \Pi(\xi') D d \xi' d t.
\]
Thus, by convolving \( I_k(t) \) in the foregoing equation with \( F(t) \), and applying the formula of integration by parts, we obtain the solution of Eq. (26) as follows
\[
u_k(t) = a^2 z_{0} / 2 R \int_{-\infty}^{\infty} e^{j \xi'_k (t-s)} R \int_{t_M}^{t} S(\xi'(\tau)) \Pi(\xi'(\tau)) d \xi' d \tau.
\]
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NUMERICAL RESULTS AND CONCLUSION

The formula for calculating the responses of each individual ray has been derived previously, as for the total response is expressed as

\[ u_T(t) = \sum \frac{h_k}{k} u_k(t), \]  

(38)

In order to obtain the velocity amplification spectrum numerically, the physical parameters from SMART-1 array, such as the mass density, shear modulus, shear wave speed for surface layer and half space are respectively given as \( \rho = 1800 \text{ kg/m}^3 \), \( \mu = 1.023 \times 10^8 \text{ N/m}^2 \), \( c_s = 775.67 \text{ m/s} \), \( \rho_2 = 2700 \text{ kg/m}^3 \), \( \mu_2 = 14.4 \times 10^7 \text{ N/m}^2 \), \( c_2 = 2309.4 \text{ m/s} \), and the distance between the source and the surface is \( z_s = 2.0 \text{ km} \).

To simulate the seismic source, the time function

\[ f(t) = \begin{cases} 
(P/\Delta^2) (-3t^2 + 4\Delta t^3), & \text{if } 0 \leq t \leq \Delta \\
(P/\Delta^2) [-3(2\Delta-t)^2 + 4\Delta(2\Delta-t)^3], & \text{if } \Delta \leq t \leq 2\Delta \\
0, & \text{if } 2\Delta \leq t,
\end{cases} \]  

(39)

is assumed, where \( P \) is the magnitude of the applied force, \( \Delta \) is half of the applied duration, and we assume \( \Delta = 0.2 \text{ sec} \) in this study.

To calculate the total responses, the thickness of surface medium just under receiver is 0.4 km, and the dipping angle is \( \alpha = 10^\circ \). Then, the amplification spectra for parallel and dipping layered medium in location (−2,0), (−1,0), (0,0) and (1,0) are respectively depicted in Fig. (4.a), (4.b), (4.c) and (4.d). From these figures, it shows that the concentration of energy in upper medium is more obvious in the dipping case for their lower amplification effect.

REFERENCES


(a) The Cagniard path in $\xi'$-plane

(b) The Cagniard path in $t$-plane

Fig 3. The mapping of complex $\xi'$ on to real $t$.

(a) $X = -2$ Km

(b) $X = -1$ Km

(c) $X = 0$ Km

(d) $X = 1$ Km

Fig 4. The Fourier amplification of velocity at various locations for parallel and dipping layered medium.