An Efficient Algorithm for Nonlinear Seismic Response of Multiple-Support Excitation

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INTRODUCTION

Due to the significant advancement in analytical capabilities in recent years, nonlinear large deformation analysis of structures is becoming increasingly desirable. Standard design codes, e.g., the Load and Resistance Factored Design (LRFD) codes proposed recently by the American Institute of Steel Construction, Inc., have suggested the use of such procedures. However, it is known by the profession that this type of analysis procedure can be extremely costly, and can even be prohibitive when applied to large real structures, even for static cases. When the dynamic or seismic response of structures is considered, the problem multiplies. Obviously, an economical solution technique for both static and dynamic analyses would be highly desirable.

Most of the currently available nonlinear analysis techniques for frames are based on an assumed displacement field. In this approach, the Hermitian functions are employed to interpolate the transverse as well as axial deformations of an element, and the appropriate nonlinear terms are retained in the strain-displacement relationships. In order to capture the effects of change in the axial length of a beam due to large deformations, several elements are needed to model each beam member. The necessity for a large number of elements coupled with the use of a numerical integration scheme to obtain the tangent stiffness matrix for each element several times during the analysis makes this approach uneconomical. It needs to be pointed out here that a numerical integration scheme to obtain the tangent stiffness matrix is not always necessary (Nedergaard and Pederson, 1985). Alternatively, the assumed stress method (Kondoh and Atluri, 1987; Nee and Haldar, 1988; Shi and Atluri, 1988) can be used to derive an explicit form of the tangent stiffness satisfying joint equilibrium and displacement compatibility. In this approach, the stresses of an element can be obtained directly instead of using the less accurate method of using the derivatives of the displacement functions as in the assumed displacement field approach. Because of this feature, and since no integration is needed in obtaining the tangent stiffness in this approach, the method is very efficient and economical.

The purpose of this paper is to propose a finite element-based procedure considering the assumed stress approach to estimate the nonlinear dynamic responses including the seismic responses of large deformed frame structures. In a seismic analysis, the same ground motion is usually assumed to act simultaneously at all parts of the foundation of the structure. This is a reasonable assumption if the base dimensions of the structure is small compared to the wavelength of the excitement at the base rock (Clough and Penzien, 1975). However, if the structures are long, such as dams and bridges, this assumption may not reflect the worst possible conditions. Therefore, it is important to develop analysis procedures capable of dealing with multiple-support excitation, i.e., different seismic inputs can be applied to different support points. The method proposed in this paper is developed in such a way that multiple-support excitation cases can easily be analyzed.

GOVERNING EQUATIONS

As discussed earlier, the assumed stress method, originally proposed by Kondoh and Atluri (1987) is used to derive the governing equations. The development of the static governing equations cannot be described here due to lack of space. However, they are widely available in the literature. Only the dynamic governing equations will be developed very briefly here.

The dynamic governing equations are developed in such a way that the consideration of uniform as well as multiple-support excitation cases will be facilitated. The equation of motion of a linear system under dynamic and seismic loadings can be expressed as (Clough and Penzien, 1975)

$$\underset{\approx}{M}\,\ddot{\underset{\sim}{D}}_T + \underset{\approx}{M}_{FR}\,\ddot{\underset{\sim}{D}}_R + \underset{\approx}{C}\,\dot{\underset{\sim}{D}}_T + \underset{\approx}{C}_{FR}\,\dot{\underset{\sim}{D}}_R + \underset{\approx}{K}\,\underset{\sim}{D}_T + \underset{\approx}{K}_{FR}\,\underset{\sim}{D}_R = \underset{\sim}{F} \tag{1}$$

$\underset{\approx}{M}, \underset{\approx}{C}, \underset{\approx}{K}$ = mass, damping and stiffness matrices of the free degree of freedom on the system; $\underset{\sim}{D}_T$ = total displacement vector of the free degree of freedom of the system; $\underset{\sim}{D}_R$ = displacement vector of the restrained degree of freedom of the system; $\underset{\approx}{M}_{FR}, \underset{\approx}{C}_{FR}, \underset{\approx}{K}_{FR}$ = mass, damping and stiffness matrices that couple the free and restrained degrees of freedom; and $\underset{\sim}{F}$ = external dynamic force vector of the free degree of freedom.

The nodal displacements of the free degree of freedom can be decomposed into pseudo- (or quasi) static and relative (or vibrational) displacements. Hence, the total displacement of the system can be expressed as:

$$\begin{bmatrix} \underset{\sim}{D}_T \\ \underset{\sim}{D}_R \end{bmatrix} = \begin{bmatrix} \underset{\sim}{D} \\ \underset{\sim}{O} \end{bmatrix} + \begin{bmatrix} \underset{\approx}{A}\,\underset{\sim}{D}_R \\ \underset{\sim}{D}_R \end{bmatrix} \tag{2}$$

where $\underset{\approx}{D}$ = relative displacements of the free degree of freedom; and $\underset{\approx}{A} = -\underset{\approx}{K}^{-1}\,\underset{\approx}{K}_{FR}$. For the rigid-base uniform seismic excitation case, $\underset{\approx}{A}$ will be an identity matrix $\underset{\approx}{I}$.

Substituting Eq. 2 into Eq. 1 and neglecting the contributions of damping to the effective seismic forces since it is expected to be negligible (Clough and Penzien, 1975), the equation of motion for the case of a linear system with multiple-support excitation can be expressed as

$$\underset{\approx}{M}\,\ddot{\underset{\sim}{D}} + \underset{\approx}{C}\,\dot{\underset{\sim}{D}} + \underset{\approx}{K}\,\underset{\sim}{D} = \underset{\sim}{F} - \underset{\approx}{M}\,\underset{\approx}{A}\,\ddot{\underset{\sim}{D}}_R - \underset{\approx}{M}_{FR}\,\ddot{\underset{\sim}{D}}_R \tag{3}$$

For the nonlinear static case, the displacement increment of the free degree of freedom for each iteration at each time step Δt, including the external load vector $\underset{\sim}{F}$, can be expressed as

$$^{t}\underset{\approx}{K}\;^{t+\Delta t}\Delta\underset{\sim}{D}^{(k)} = {}^{t+\Delta t}\underset{\sim}{F}^{(k)} - {}^{t+\Delta t}\underset{\sim}{R}^{(k-1)} \tag{4}$$

where $^{t}\underset{\approx}{K}$ = tangent stiffness of the system at time t ; $^{t+\Delta t}\Delta\underset{\sim}{D}^{(k)}$, $^{t+\Delta t}\underset{\sim}{F}^{(k)}$ = incremental displacement vector and the external load of the kth iteration at time $t+\Delta t$, respectively; and $^{t+\Delta t}\underset{\sim}{R}^{(k-1)}$ = the internal force vector of the (k-1)th iteration at time $t+\Delta t$.

In the dynamic case, the static equation can be modified by adding inertia and damping forces (Bathe, 1982). Considering Eqs. 3 and 4, the equilibrium equation for the nonlinear vibration analysis can be expressed as

$$\begin{aligned} &\underset{\approx}{M}\,^{t+\Delta t}\ddot{\underset{\sim}{D}}^{(k)} + {}^{t}\underset{\approx}{C}\;^{t+\Delta t}\dot{\underset{\sim}{D}}^{(k)} + {}^{t}\underset{\approx}{K}\;^{t+\Delta t}\,\Delta\underset{\sim}{D}^{(k)} = {}^{t+\Delta t}\underset{\sim}{F}^{(k)} \\ &- {}^{t+k\Delta}\underset{\sim}{R}^{(k-1)} - \underset{\approx}{M}\;^{t}\underset{\approx}{A}\;\ddot{\underset{\sim}{D}}_R^{(k)} - \underset{\approx}{M}_{FR}\;^{t+\Delta}\ddot{\underset{\sim}{D}}^{(k)} \end{aligned} \tag{5}$$

Eq. 5 can be solved in a step-by-step numerical procedure using the Newmark β method. The displacement and velocity vectors and damping effects within each time step Δt are assumed as follows (Bathe, 1982):

$$ {}^{t+\Delta t}\dot{\tilde{D}}^{(k)} = {}^{t}\dot{\tilde{D}} + [(1-\eta)\, {}^{t}\ddot{\tilde{D}} + \eta\, {}^{t+\Delta t}\ddot{\tilde{D}}^{(k)}]\Delta t \tag{6} $$

$$ {}^{t+\Delta t}\tilde{D}^{(k)} = {}^{t}\tilde{D} + {}^{t}\dot{\tilde{D}}\Delta t + [(1/2 - \beta)\, {}^{t}\ddot{\tilde{D}} + \beta\, {}^{t+\Delta t}\ddot{\tilde{D}}^{(k)}]\, \Delta t^2 \tag{7} $$

In this paper $\eta = 1/2$ and $\beta = 1/4$ are assumed. For these values, the acceleration is considered to be constant in the interval Δt. The damping is assumed to be proportional to the system's mass and tangent stiffness matrices as

$$ {}^{t}\underset{\approx}{C} = \alpha\, \underset{\approx}{M} + \gamma\, {}^{t}\underset{\approx}{K} \tag{8} $$

It is assumed that the displacement, dynamic force and seismic acceleration vectors of the kth iteration at time $t+\Delta t$ can be expressed in the incremental form as

$$ {}^{t+\Delta t}\tilde{D}^{(k)} = {}^{t+\Delta t}\tilde{D}^{(k-1)} + {}^{t+\Delta t}\Delta\tilde{D}^{(k)} \tag{9} $$

$$ {}^{t+\Delta t}\tilde{F}^{(k)} = {}^{t+\Delta t}\tilde{F}^{(k-1)} + {}^{t+\Delta t}\Delta\tilde{F}^{(k)} \tag{10} $$

and

$$ {}^{t+\Delta t}\ddot{\tilde{D}}_R^{(k)} = {}^{t+\Delta t}\ddot{\tilde{D}}_R^{(k-1)} + {}^{t+\Delta t}\Delta\ddot{\tilde{D}}_R^{(k)} \tag{11} $$

Manipulating and assmbling some terms together, and using Eqs. 9, 10, and 11, Eq. 5 results as

$$ {}^{t}\underset{\approx}{K}_D\, {}^{t+\Delta t}\tilde{D}^{(k)} = {}^{t+\Delta t}\tilde{F}_D^{(k-1)} + {}^{t+\Delta t}\Delta\tilde{F}_D^{(k)} - {}^{t+\Delta t}\tilde{R}^{(k-1)} \tag{12} $$

where ${}^{t+\Delta t}\Delta\tilde{D}^{(k)}$ = the increment of relative displacement vector of the free degree of freedom; ${}^{t}\underset{\approx}{K}_D$ = the modified tangent stiffness matrix and can be shown to be

$$ {}^{t}\underset{\approx}{K}_D = f_1\, \underset{\approx}{M} + f_2\, {}^{t}\underset{\approx}{K} \tag{13} $$

${}^{t+\Delta t}\tilde{F}_D^{(k-1)}$, ${}^{t+\Delta t}\Delta\tilde{F}_D^{(k)}$ = the modified external force and its incremental vectors, respectively. The modified external force vector can be expressed as

$$ \begin{aligned} {}^{t+\Delta t}\tilde{F}_D^{(k-1)} = {} & {}^{t+\Delta t}\tilde{F}^{(k-1)} + {}^{t+\Delta t}\tilde{P}^{(k-1)} - \underset{\approx}{M}\, {}^{t}\underset{\approx}{A}\, {}^{t+\Delta t}\ddot{\tilde{D}}_R^{(k-1)} \\ & - \underset{\approx}{M}_{FR}\, {}^{t+\Delta t}\ddot{\tilde{D}}_R^{(k-1)} \end{aligned} \tag{14} $$

and ${}^{t+\Delta t}\tilde{R}^{(k-1)}$ = the internal force vector of the system as shown by Haldar and Nee (1989).

The term ${}^{t+\Delta t}\tilde{P}^{(k-1)}$ in Eq. 14 is the modified force vector contributed by discplacement, velocity and acceleration at time t and displacement at time $t+\Delta t$, and can written as

$$ \begin{aligned} {}^{t+\Delta t}\tilde{P}^{(k-1)} = {} & \underset{\approx}{M}\, [f_1\, {}^{t}\tilde{D} + f_3\, {}^{t}\dot{\tilde{D}} + f_4\, {}^{t}\ddot{\tilde{D}} - f_1\, {}^{t+\Delta t}\tilde{D}^{(k-1)}] \\ & + {}^{t}\underset{\approx}{K}\, [f_5\, {}^{t}\tilde{D} + f_6\, {}^{t}\dot{\tilde{D}} + f_7\, {}^{t}\ddot{\tilde{D}} - f_5\, {}^{t+\Delta t}\tilde{D}^{(k-1)}] \end{aligned} \tag{15} $$

The incremental external force term ${}^{t+\Delta t}\Delta\tilde{F}_D^{(k)}$ can be shown to be

$$ {}^{t+\Delta t}\Delta\tilde{F}_D^{(k)} = {}^{t+\Delta t}\Delta\tilde{F}^{(k)} - \underset{\approx}{M}\, {}^{t}\underset{\approx}{A}\, {}^{t+\Delta t}\Delta\ddot{\tilde{D}}_R^{(k)} - \underset{\approx}{M}_{FR}\, {}^{t+\Delta t}\Delta\ddot{\tilde{D}}_R^{(k)} \tag{16} $$

The coefficients f_i's are constants and can be evaluated in terms of η, β, α, γ, and Δt as

$$f_1 = \frac{1}{\beta \Delta t^2} + \frac{\eta \alpha}{\beta \Delta t}; \quad f_2 = \frac{\eta \gamma}{\beta \Delta t} + 1; \quad f_3 \quad \frac{1}{\beta \Delta t} + \frac{\eta \alpha}{\beta} - \alpha;$$

$$f_4 = (\frac{1}{2\beta} - 1) + \eta \alpha (\frac{1}{2\beta} - \frac{1}{\eta}) \Delta t; \quad f_5 = \frac{\eta \alpha}{\beta \Delta t}; \quad f_6 \frac{\eta \gamma}{\beta} - \gamma; \quad f_7 = (\frac{\eta \gamma}{2\beta} - \gamma) \Delta t \tag{17}$$

Equation 12 now can be solved using the procedure proposed by Haldar and Nee (1989).

EXAMPLES

A one-bay three-story frame, as shown in Fig. 1, is considered in this example. The frame is subjected to a uniformly distributed gravity load of 260 lb/in (46.37 kg/cm) and a lateral load due to the seismic loading. The members' sizes and their cross-sectional properties are also shown in Fig. 1.

The frame is subjected to the 1940 N-S component of the El Centro earthquake. For illustrative and verification purposes, the damping is assumed to be proportional to the mass only, i.e., $\underset{\approx}{C} = 0.65 \underset{\approx}{M}$ ($\approx$ 4% of critical damping in the first node). This linear case is analyzed by GTSTRUDL for verification purposes. The results are quite consistent.

The responses of the same frame are then evaluated using the proposed nonlinear analysis technique. Both the rigid base and multiple-support excitation cases are considered. Responses at the same three node locations for both cases are shown in Figs. 2 and 3. From Fig. 2, the peak displacements at node 4 are found to be + 4.21 in. and - 3.71 in. (1 in. = 2.54 cm). Fig. 3 represents the same responses for the multiple-support excitation case. In this case, the input time history of support 2 (node 8) is assumed to have a 0.1 second time delay after support 1 (node 1). Comparing Fig. 2 (rigid base excitation) and Fig. 3 (multiple-support excitation) reveals that the peak responses occur at different locations (a slight shift to the left in Fig. 3), and the peak displacements are also different (+ 3.945 in. and - 3.35 in.). This shows the importance of the consideration of multiple-support excitation.

CONCLUSION

A method is proposed here to estimate the nonlinear response of large deformed plane frame structures under dynamic and seismic loading. The method is capable of considering multiple-support seismic excitation loading. The proposed finite element-based method is developed using the assumed stress method. The proposed method is extremely efficient since the tangent stiffness can be expressed in the explicit form in the finite element algorithm. The method is verified using the numerical results available in the literature. It is observed that the proposed method is very reliable and accurate in studying the nonlinear dynamic and seismic analysis of plane frames.

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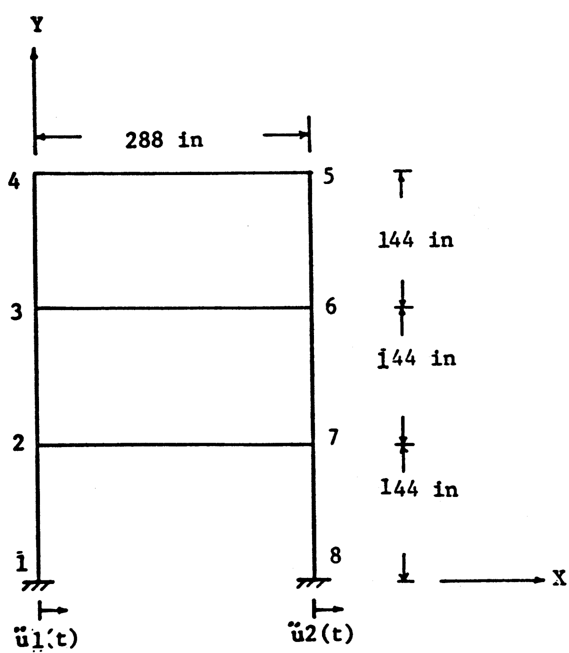

		AREA (in^2)	E (psi)	I (in^4)
Column	W21x50	14.7	29×10^6	984.0
Girder	W16x67	19.7	29×10^6	954.0

Fig. 1 Geometry and Material Properties for Example 1

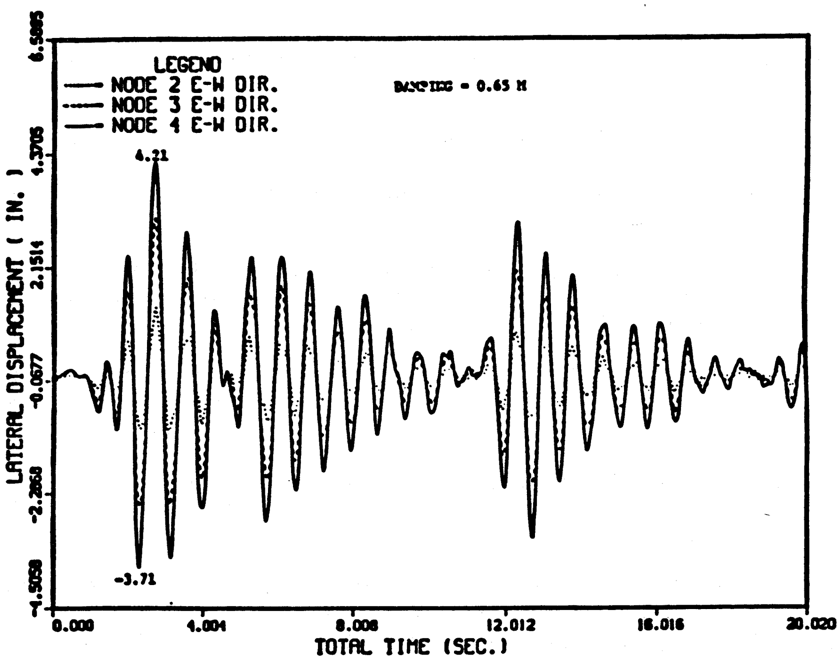

Fig. 2 Rigid Base Excitations

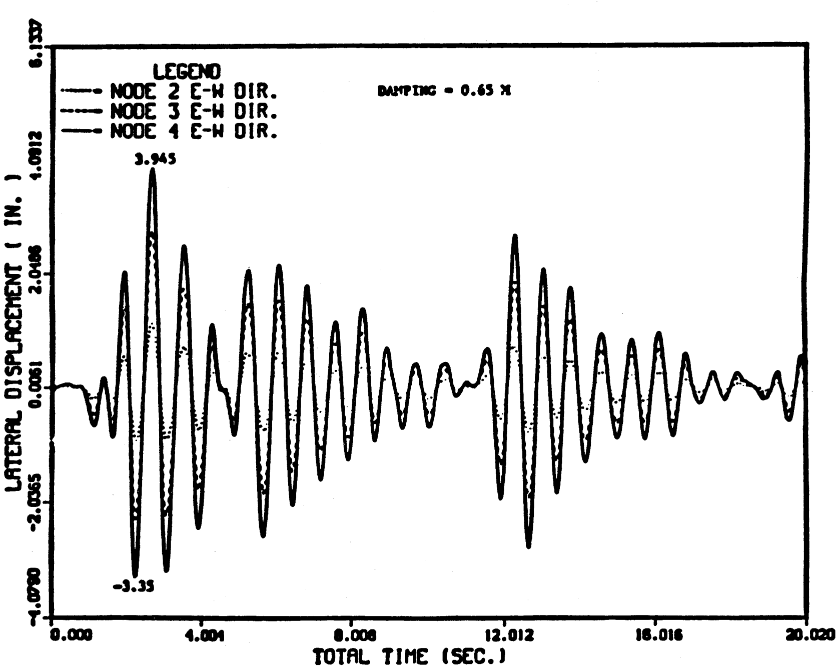

Fig. 3 Multiple-Support Excitations