Viscoplastic Behaviour of AISI 316H – Connection of Constitutive Equations to Multiaxial Experiments

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INTRODUCTION

The aim of this paper is to highlight some fundamental features of existing viscoplasticity theories in relation to various different experiments with finite inelastic strains and strain rates. Such theories, which are essentially phenomenological, have to obey general constitutive principles of continuum mechanics such as objectivity, material symmetry, determinism and local action. Moreover, some additional features of inelastic rate-dependent behaviour such as (a) material symmetry change ("deformation-induced anisotropy"), (b) directionality effects, (c) yield surface existence, (d) isotropic as well as anisotropic damage, creep, cycling, recovery and softening, have to be taken into account in order to meet cumbersome uniaxial and limited multiaxial experimental data. In this paper, aspects (a), (b) and (c) are being looked for in the examined theories while aspect (d) is not considered here. Especially aspect (b), which is very often forgotten or misunderstood, is stressed here. Finally, the role of a correct deformation geometry as well as irreversible thermodynamics are of great importance for inelastic deformation processes and must be properly treated. In the next sections a comparison of some existing viscoplasticicity models is given and a programme of biaxial experiments on cruciform specimens is briefly highlighted.

SOME VISCOPLASTICITY MODELS

In order to make a uniform notation for the various different constitutive models we list the following relations (which hold for finite strains):

$$F = F_E . F_p, \quad (F:(\kappa_o) \to (\chi_t), \; F_p:(\kappa_o) \to (\kappa_t), \; F_E:(\kappa_t) \to (\chi_t)), \tag{1}$$

$$2E_E = F_E^T . F_E - 1, \quad 2e_p = 1 - F_p^{-T} . F_p^{-1}, \quad e_p + E_E = F_p^{-T} . E . F_p^{-1}, \tag{2}$$

$$\dot{p} \equiv (2/3 \; \dot{e}_p : \dot{e}_p)^{\frac{1}{2}}, \quad p(t) = \int_o^t \dot{p}(\tau) d\tau, \tag{3}$$

$$S = (\det F_E) F_E^{-1} . T . F_E^{-T}, \quad \Delta S = S - S(\dot{p}=o) \equiv S - S^*, \tag{4}$$

$$\hat{d} W_p / dt = S : [(1 + 2E_E) . L_p], \tag{5}$$

$$L_p \equiv \dot{F}_p . F_p^{-1}, \quad 2\dot{e}_p = (1-2e_p).L_p + L_p^T.(1-2e_p), \tag{6}$$

$$f = \phi(S,\theta,e_p,p), \quad f^* = \phi(S^*,\theta,e_p,p), \tag{7}$$

where $\mathbf{F}, \mathbf{F}_E$ and $\mathbf{F}_p$ are total, thermoelastic and plastic deformation gradient tensors (all configurations (κ_0), (κ_t) and (χ_t) are continuous and with residual stresses in general), $\mathbf{E}_E$ and $\mathbf{E}$ are Lagrangian thermoelastic and total strain tensors, $\mathbf{e}_p$ is the Eulerian plastic strain tensor, $\dot{p}$ is the plastic strain rate intensity, p is the accumulated plastic strain, $\mathbf{S}, \mathbf{S}^*$ and $\Delta\mathbf{S}$ are the Piola-Kirchhoff dynamic, static stress and overstress tensors, $\mathbf{T}$ is the Cauchy stress tensor, $\hat{d}W_p/dt$ is the plastic power, $\mathbf{L}_p$ is the plastic velocity gradient tensor (whose symmetric part $\mathbf{D}_p$ is plastic stretching and its antisymmetric part $\mathbf{W}_p$ is a plastic spin tensor), $\dot{\mathbf{e}}_p$ is the time rate of the plastic strain tensor, f and f^* are dynamic and static yield functions characterizing the behaviour of the considered body B as follows: $f > 0$, $\dot{p} > 0$, $f^* = 0$ (viscoplastic behaviour), $f = f^* = 0$, $\dot{p} = 0$ (elastoplastic yield limit) and $f = f^* < 0$, $\dot{p} = 0$ (elastic interior of the yield surface).

1. In viscoplasticity theories with yield surface and the associated flow rule (Perzyna, 1988; Chaboche and Rousselier, 1983) the evolution equation for plastic strain rate reads:

$$\eta\dot{\mathbf{e}}_p = \langle f\rangle\psi(f)\partial_{\mathbf{S}} f \equiv \zeta(\mathbf{S}_d - \mathbf{B}), \quad (\mathbf{S}_d \equiv \mathbf{S} - \frac{1}{3}\mathbf{1}\,\mathrm{tr}\mathbf{S}), \tag{8}$$

with Huber-Mises yield function:

$$f = \frac{(S-B)_{eq}}{\kappa(p)} - 1 \equiv -1 + \frac{1}{\kappa}\left[\frac{3}{2}(S_d - B):(S_d - B)\right]^{\frac{1}{2}}, \tag{9}$$

where $\mathbf{B}(p)$ is called the back stress representing kinematic hardening, ψ is a function of f and $\langle x\rangle = 1$ for $x > 0$ and $\langle x\rangle = 0$ otherwise. Chaboche and Rousselier assume additivity $\mathbf{e}_p + \mathbf{E}_E \approx \mathbf{E}$ which holds for small strains only while Perzyna in some recent papers takes into account finite plastic strain obeying (2) . It should be noted here that evolution equation (8) is in fact intrinsically uniaxial since it is characterized by the same scalar evolution equation

$$p = \langle f\rangle\,\psi\left[\frac{(\sigma-B)_{eq}}{\kappa(p)} - 1\right] \tag{10}$$

for various different strain paths. However, experimentally it has already been noted that curves (p, S_{eq}) are different for tension and torsion (Gil Sevillano et al., 1982; Young et al., 1974). Such directionality of inelastic behaviour has been discussed by Weertman and Hecker, 1983, in their dislocation cell wall formation theory, but there is a need to predict such a discrepancy in behaviour by some phenomenological theory as well.

2. The existence of plastic strains from the very beginning of a deformation process has been postulated by Bodner (1987), admitting in that way that plastic straining exists even during unloading. His evolution equation reads (under the assumption of small strains):

$$\dot{\mathbf{e}}_p = \Lambda\mathbf{S}_d, \tag{11}$$

whose corresponding generating scalar evolution equation

$$\dot{p} = S_{eq}\, g\left[\frac{S_{eq}}{Z(W_p)}\right] , \qquad (12)$$

(W_p is the plastic work acting here as a plastic deformation history parameter) is the same for all proprotional stress paths. Thus, it again does not distinguish between tension and torsion; it is in such a way insensitive to directionality.

A similar situation appeared for the no-yield-surface theories of Miller and Krieg (cf. Abdel-Kader et al., 1988) whose evolution equations introduce back stress **B** in (11) in the way similar to (8).

It should be mentioned that all these models may be considered as special cases of a theory with yield surface when it shrinks to a point.

3. Cernocky and Krempl (1980), for the case of small (i.e. infinitesimal) strains, assumed a non-associate flow rule:

$$\eta\dot{\mathbf{e}}_p = \mathcal{D}_0^{-1} :(\mathbf{S}-\mathbf{S}^*),\ \mathbf{S}^* = \xi\mathcal{D}_0:\mathbf{E},\ \mathcal{T}_0 \equiv \frac{1}{\eta}\mathcal{D}_0^{-1}, \qquad (13)$$

where ξ,η are scalar functions of $(\mathbf{S},\mathbf{E})$ and $\mathcal{D}_0$ is the fourth order tensor of elastic constants. Its directionality can be seen from the examples of uniaxial tension:

$$(\eta\dot{e}_{p33}+\xi e_{p33})\begin{Bmatrix} -\frac{1}{2} & 0 & 0\\ 0 & -\frac{1}{2} & 0\\ 0 & 0 & 1\end{Bmatrix} = (1-\xi)\,\frac{S_{33}}{E}\begin{Bmatrix} -\nu & 0 & 0\\ 0 & -\nu & 0\\ 0 & 0 & 1\end{Bmatrix} , \qquad (14)$$

(E is Young modulus and ν is the Poisson coefficient) and torsion:

$$\begin{Bmatrix} 0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{Bmatrix}\left[\eta\dot{e}_{p23}+\xi e_{p23}-(1-\xi)\,\frac{S_{23}}{\mu}\right] = 0, \qquad (15)$$

under the assumption that $|e_{p33}| \gg e^2_{p33}$, $|e_{p23}| \gg e^2_{p23}$.

Obviously, (13) accounts for directionality but $\mathcal{T}_0$ does not properly predict the direction of $\dot{\mathbf{e}}_p$. This is clearly seen from (14) which can be satisfied only if $\nu = \frac{1}{2}$, which does not hold for most metals.

4. In a fourth viscoplastic model with yield surface and non-associate flow rule for the case of finite strains (cf. Micunovic, 1987; Maruszewski and Micunovic) the evolution equation for the plastic velocity gradient tensor is obtained by isotropic tensor functions (Boehler, 1987) and thermodynamic restrictions imposed by the second law:

$$\begin{aligned}\eta\mathbf{L}_p = {} & \langle f\rangle[d_0\mathbf{1} + d_1(1-k)\mathbf{S} + d_2(1-k^2)\mathbf{S}^2 + d_3(1-k)(\mathbf{S}.\mathbf{e}_p+\mathbf{e}_p.\mathbf{S}) +\\ & d_4(1-k^2)(\mathbf{S}^2.\mathbf{e}_p+\mathbf{e}_p.\mathbf{S}^2) + d_5(1-k)(\mathbf{S}.\mathbf{e}^2_p+\mathbf{e}^2_p.\mathbf{S}) +\\ & d_6(1-k^2)(\mathbf{S}^2.\mathbf{e}^2_p+\mathbf{e}^2_p.\mathbf{S}^2_p)] + \langle f\rangle(1-k)[w_1(\mathbf{S}.\mathbf{e}_p-\mathbf{e}_p.\mathbf{S}) +\\ & w_2(\mathbf{S}^2.\mathbf{e}_p-\mathbf{e}_p.\mathbf{S}^2)(1+k) + w_3(\mathbf{S}.\mathbf{e}^2_p-\mathbf{e}^2_p.\mathbf{S}) + w_4(\mathbf{S}^2.\mathbf{e}^2_p-\mathbf{e}^2_p.\mathbf{S}^2)(1+k)], \qquad (16)\end{aligned}$$

where terms with coefficients $d_0,\ldots,d_6$ determine the plastic stretching tensor whereas terms with coefficients $w_1,\ldots,w_4$ define the plastic spin tensor and d is found from the condition $\mathrm{tr}\mathbf{L}_p = 0$, while the static stress tensor is naturally assumed to have the same direction as the dynamic stress tensor, i.e. $\mathbf{S}^* = k\mathbf{S}$. Here coefficients $d_0,\ldots,d_6$, $w_1,\ldots,w_4$, η depend on invariants $\{\theta,\pi_1,\pi_2,\pi_3,\sigma_1,\sigma_2,\sigma_3,\mu_1,\mu_2,\mu_3,\mu_4\} \equiv \{\theta,\mathrm{tr}\mathbf{e}_p,\mathrm{tr}\mathbf{e}^2_p,\mathrm{tr}^3_p,\mathrm{tr}\mathbf{S},\mathrm{tr}\mathbf{S}^2,\mathrm{tr}\mathbf{S}^3,\mathbf{S}:\mathbf{e}_p,\mathbf{S}^2:\mathbf{e}_p,\mathbf{S}:\mathbf{e}^2_p,\mathbf{S}^2:\mathbf{e}^2_p\}$ with θ as the temperature.

Consider the special case of an initial yield surface with $\pi_2 \sim 0$. In this case (16) reduces into (with $\mathbf{D}_p \approx \dot{\mathbf{e}}_p$ from (6)):

$$\dot{\mathbf{e}}_p \approx \alpha_1(\mathbf{S} - 1/3\,\mathbf{1}\mathrm{tr}\mathbf{S}) + \alpha_2(\mathbf{S}^2 - 1/3\ \mathbf{1}\ \mathrm{tr}\mathbf{S}^2),\ \mathbf{W}_p \approx 0, \tag{17}$$

where $\alpha_m = (1-k)d_m/\eta (m = 1,2)$. This equation is a good approximation of (16) for small plastic and elastic strains.
Obviously, (16) is able to take into account the directionality of an inelastic deformation process. For instance, the example of uniaxial tension is given by the evolution equation:

$$\frac{3}{2}\eta(1+2e_{p33})\dot{e}_{p33} = \langle f\rangle\{[S_{33}-S^{*}_{33}(e_{p33})](d_1+2d_3\ e_{p33}+2d_5\ e^2_{p33}) + + [S^2_{33}-S^{*2}_{33}(e_{p33})](d_2+2d_4\ e_{p33}+2d_6\ e^2_{p33})\}, \tag{18}$$

while in the case of torsion we would have:

$$\eta\dot{e}_{p23} = \langle f\rangle [S_{23}-S^{*}_{23}(e_{p23})]\ [d_1-2d_3e^2_{p23}+2d_5e^2_{p23}(1+2e^2_{p23})]. \tag{19}$$

Here, finite plastic strains (without necking) and small elastic strains are assumed. It should be noted, however, that $d_1,\dots,d_6$ in (18) depend on $\{\theta,S_{33},e_{p33}\}$ whereas in (19) d_1,d_3,d_5 depend on $\{\theta,S_{23},e_{p23}\}$ and could be determined from experimental results by making use of (18) and (19).

5. At the end of this section a few remarks about material symmetry deserve our attention. In the theories of Perzyna, Chaboche and Rousselier, Bodner, Miller, Krieg, Cernocky and Krempl for an initially isotropic body B, stress is the linear function of elastic strain (α is the coefficient of thermal expansion):

$$\mathbf{S} = \mathcal{D}_0:(\mathbf{E}_E-\alpha\Delta\theta\mathbf{1}), \tag{20}$$

with elastic constants given by

$$\mathcal{D}_0^{\alpha\beta\gamma\delta} = g^{\alpha\beta}g^{\gamma\delta} + \mu(g^{\alpha\gamma}g^{\beta\delta}+g^{\alpha\delta}g^{\beta\gamma}), \tag{21}$$

$g^{\alpha\beta}$ being a fundamental tensor of the (κ_t) configuration (for small strains spatial coordinates in (χ_t) may be employed as well).
On the other hand, anisotropy caused by plastic deformation enters also elastic constants in the model of Micunovic (1987) (cf. also Micunovic et al., 1988) where:

$$\begin{aligned}\mathcal{D}_{\alpha\beta\gamma\delta} = {} & g_{\alpha\beta}(\lambda g_{\gamma\delta}+c_5e_{p\gamma\delta}+c_7e^2_{p\gamma\delta})+\mu(g_{\alpha\gamma}g_{\beta\delta}+g_{\alpha\delta}g_{\beta\gamma}) + \\ & + c_2(g_{\alpha\delta}e_{p\beta\gamma}+g_{\beta\gamma}e_{p\alpha\delta})+c_4(g_{\alpha\delta}e^2_{p\beta\gamma}+g_{\beta\gamma}e^2_{p\alpha\delta}) + \\ & + (c_5g_{\gamma\delta}+c_1e_{p\gamma\delta}+c_6e^2_{p\gamma\delta})e_{p\alpha\beta}+(c_7g_{\gamma\delta}+c_6e_{p\gamma\delta}+c_3e^2_{p\gamma\delta})e^2_{p\alpha\beta},\end{aligned} \tag{22}$$

under the assumption that thermoelastic strains are small whereas plastic strains finite. Again, if plastic strains are also small we should have $\mathcal{D} \approx \mathcal{D}_0$.

BIAXIAL EXPERIMENTS ON CRUCIFORM SPECIMENS

Having in mind the brief review of the viscoplasticity models listed in the preceding section, an experimental programme of the kind given in Fig.1 becomes logical. For a cruciform specimen engineering (i.e. Piola-Kirchhoff) stress, temperature and Lagrangian total strains are measured as functions of time. Then plastic strains are either calculated or measured by unloading. From the determined plastic strain tensor as a function of time we calculate its time rate, plastic strain rate intensity and accumulated plastic strain. Either by p = 0.2% or by temperature measurements initial yield surface is

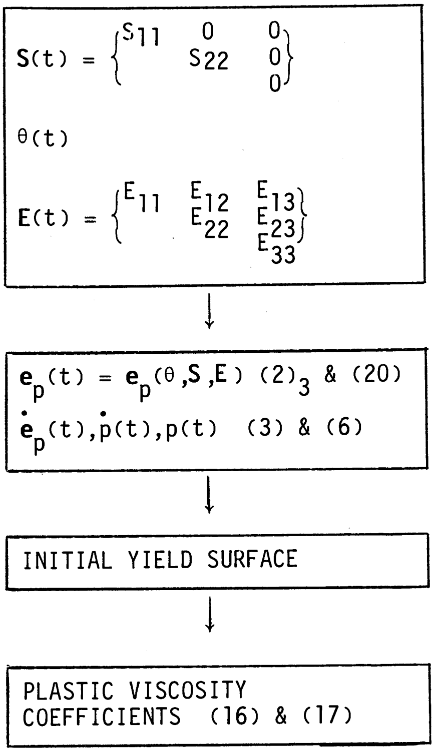

Figure 1: Experimental programme

detected. Finally, evolution equations (16) or (17) are applied and for the initial yield surface associativity of the flow rule is investigated. Further experimental details and results are given in our next paper at this conference.

CONCLUDING REMARKS

Concerning the viscoplasticity models considerd, the following conclusions may be drawn:

a) In the theory of Perzyna and Chaboche and Rousselier (with Huber-Mises yield surface and the associated flow rule) the accepted scalar evolution equation for the plastic strain rate intensity does not recognize the directions of strain paths being the same for tension and torsion, for instance. Plastic strain-induced anisotropy is taken into account only by kinematic hardening, i.e. the back stress.

b) The theories of Bodner, Krieg and Miller (without yield surface) incorporate the same directionality, as in the theories of Perzyna and Chaboche and Rousselier. Again, their evolution equations essentially do not recognize diverse strain path directions and the back stress is taken into account in the same way as above.

c) The overstress concept and a non-associate flow rule are essential ingredients of the theory of Cernocky and Krempl. It is sensitive to directionality effects but colinearity between the plastic viscosity fourth rank tensor $\mathcal{T}_0$ and the fourth rank tensor of elastic constants $\mathcal{D}_0$ fails for some cases of deformation such as tension.

d) The non-associate theory of Micunovic (1987) allows for directionality effects and introduces plastic strain-induced anisotropy even in the fourth rank tensor or elastic "constants".

A multiaxial experimental programme is planned so as to be able to decide among the different theories based on a reliable judgement of the associativity or non-associativity of experimentally obtained flow rules.

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