Determination by the Homogenization Technique of the Overall Behaviour of Steam Generator Tubes Weakened by Micro–Cracks Arrays

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INTRODUCTION

During a steam generator tubes examination, axially orientated and regular microcracks arrays on the inner wall were discovered. In order to suggest a leak-before-break analysis of a long axial through-wall crack located among microcracks arrays, we propose to evaluate the influence of microcracks on the elastic stiffness of the tube and its bursting pressure by means of an overall homogenized behaviour. Within the framework of a heterogeneous plates theory [3], the macroscopic behaviour is obtained by solving elementary problems on three-dimensional cracked unit-cells.

The macroscopic elastic membrane stiffness for arrays of any depth is estimated by bounds. These bounds are analytically determined from plane elasticity solutions obtained by finite element discretization.

In the same way, we give bounds of the "overall flow stress", obtained with kinematic and static approaches of Limit Analysis [4].

PERIODIC HOMOGENIZATION PROCEDURE APPLIED TO HETEROGENEOUS PLATES

The effect of the curvature radius is neglected, so that the cracked tube domain is approximated by a plate. The periodic distribution of the microcracks (see Fig. 1), in the θ-z directions, leads to the same periodicity of the microscopic strain $\varepsilon(V)$, and stress σ fields (far from the ends of the tube). At the scale of the structure (loaded by pressure), we define the following macroscopic quantities : membrane effort tensor N, membrane strain work density W_{def} . These quantities are averaged values of the microscopic fields on a representative unit-cell $\mathcal{Y}$ (spatial period), the volume of which is denoted by $|Y|$, and its thickness by t (Fig. 1). Thus we have the following results :

(1) There exists $\mathcal{E} = \begin{pmatrix} \mathcal{E}_{zz} & \mathcal{E}_{\theta z} \\ \mathcal{E}_{\theta z} & \mathcal{E}_{\theta\theta} \end{pmatrix}$, an uniform membrane strain tensor on the cell $\mathcal{Y}$, and ψ a periodic (on the $z \mathbin{/\!/} y_1$, $\theta \mathbin{/\!/} y_2$ directions) displacement field, so that the microscopic displacement V on $\mathcal{Y}$ reads : $V(y) = \mathcal{E}.y + \psi(y)$, $\forall\, y \in \mathcal{Y}$; the space of such periodic fields ψ is denoted by $\mathcal{V}(\mathcal{Y})$;

(2) The macroscopic membrane effort tensor N is :

$$N \equiv t\,\langle\sigma\rangle = \frac{t}{|\mathcal{Y}|} \cdot \int_{\mathcal{Y}} \sigma(y).dy \quad ;$$

(3) The strain work density, for a microscopic displacement $\mathbf{V}$, in a σ field, reads :

$$\mathcal{E}_{def}(\sigma,V) \equiv t\,\langle \sigma.\varepsilon(V) \rangle = \frac{t}{|\mathcal{Y}|} \cdot \int_{\mathcal{Y}} \sigma(y).\varepsilon(V(y)).dy$$

As there is neither loading on the crack surface nor on the inner and outer cell surface, the equilibrium equation : $\int_{\mathcal{Y}} \sigma.\varepsilon(\psi) = 0\ ,\ \forall\ \psi \in \mathcal{V}(\mathcal{Y})$ leads to :

(4) $$\mathcal{E}_{def}(\sigma(U),U) = W_{def}(U) = t\,\langle \sigma.\varepsilon(U) \rangle = N\ .\ \mathcal{E}$$

so as the $\mathcal{E}$ tensor appears to be the macroscopic membrane strain tensor. Our aim is to search for a relation between $\mathcal{E}$ and N, i.e. we have to calculate W_{def} for any N (or $\mathcal{E}$) prescribed to the cell $\mathcal{Y}$.

MEMBRANE ELASTIC STIFFNESS

Let's assume a linear relation between microscopic fields : $\sigma = \Lambda.\varepsilon(U)$, on $\mathcal{Y}$. This assumption implies a linear response U, σ, N, of the cell subjected to a given macroscopic strain $\mathcal{E}$: we can define the homogenized membrane elastic tensor : $N = \Lambda^{hom}\ .\ \mathcal{E}$.

The calculation of the Λ^{hom} components can be done by computing the strain energy $\frac{1}{2} W_{def}(U)$, for three elementary prescribed strains $\mathcal{E}$ or by computing the complementary energy $W^{*}_{comp}(\sigma)$, for three elementary prescribed efforts N.

But the calculation of Λ^{hom} for any geometry and depth of microcracks array would be rather expensive. So we propose to compute approximations of the microscopic response U (for a given strain $\mathcal{E}$) (resp. σ, for a given membrane effort N). The resolution of the elastic problem on the cell $\mathcal{Y}$, under the prescribed strain $\mathcal{E}$, leads to a minimization problem of the potential energy on the space $\mathcal{V}(\mathcal{Y})$ of the admissible displacement fields. Choosing a subspace $\tilde{\mathcal{V}}(\mathcal{Y})$ for the minimization, we get a solution $\tilde{U}$, approximation of U, such as :

(5) $W_{def}(U) \leqslant W_{def}(\tilde{U})$. With the tensor $\tilde{\Lambda}^{hom}$ corresponding to $W_{def}(\tilde{U})$, it reads : $\mathcal{E}\ .\ \Lambda^{hom}\ .\ \mathcal{E} \leqslant \mathcal{E}\ .\ \tilde{\Lambda}^{hom}\ .\ \mathcal{E},\ \forall\ \mathcal{E}$.

This method produces upper bounds of elastic constants. In the same way, lower bounds can be obtained. If we search for an approximated solution of the stress elastic problem, we have to minimize the complementary energy : $W^{*}_{comp}(\tau) = 1/2\,\langle \tau.\ \Lambda^{-1}.\tau \rangle$ on the space $\mathcal{S}^{N}(\mathcal{Y})$ of the statically admissible (with the membrane effort N) stress fields. Choosing a subspace $\hat{\mathcal{S}}^{N}(\mathcal{Y})$ for the minimization, we get a solution $\hat{\sigma}$, approximation of σ, such as :

(6) $$-\,W^{*}_{comp}(\hat{\sigma}) \leqslant -\,W^{*}_{comp}(\sigma)$$

Since the solution σ corresponds to the displacement solution U, defined above, (for macroscopic fields N and $\mathcal{E}$ related by $N = \Lambda^{hom}\ .\ \mathcal{E}$), we have :

(7) $$-\,W^{*}_{comp}(\sigma) = W_{pot}(U) = \frac{1}{2} W_{déf}(U)$$

As in (5), we define the tensor $(\hat{\Lambda}^{hom})^{-1}$ corresponding to $W^{*}_{comp}(\hat{\sigma})$; then the inequality (6) reads :

(8) $$-\,N\ .\ (\hat{\Lambda}^{hom})^{-1}\ .\ N \leqslant .\ N\ .\ (\Lambda^{hom})^{-1}\ .\ N\ ,\ \forall\ N$$

By inverting (8), we get the desired bounds :

$$(9) \qquad \mathscr{E} \cdot \hat{\Lambda}^{hom} \cdot \mathscr{E} \leq \mathscr{E} \cdot \Lambda^{hom} \cdot \mathscr{E} \leq \mathscr{E} \cdot \tilde{\Lambda}^{hom} \cdot \mathscr{E}, \; \forall \mathscr{E}$$

We shall consider first the case of through-wall arrays, the results of which will be usefull for arrays of any depth.

<u>Through-wall microcracks arrays</u>

Since the geometry of cracks remains invariable through the thickness, we choose the plane elasticity assumptions. Under plane strain conditions (resp. plane stress), the elasticity problem reduces to a two-dimensional one, on a section Y (see Fig. 2) of the 3D cell $\mathscr{Y}$.

For a given macroscopic strain $\mathscr{E}$, the solution U_D, under plane strain conditions, provides an upper bound $\frac{1}{2} W_{def}(U_D)$ of the strain energy. In plane stress, we can associate a displacement solution U_c (for the same problem as in plane strain, but with an other Lamé modulus $\lambda_c = \frac{\nu . E}{1-\nu^2}$). This solution gives a lower bound $1/2 \; W_{def}(U_c)$. The corresponding elastic tensors Λ_D^{hom} and Λ_c^{hom} are orthotropic, with 4 moduli : E, ν, E_D^*, μ_D^* (resp. E_c^*, μ_c^*). There is the same compliance 1/E in the microcracks direction, while in the perpendicular direction we get a weakening with $E^* < E$. This reads :

$$(10) \quad (\Lambda^{hom})^{-1} = \frac{1}{t}\begin{pmatrix} 1/E & -\nu/E & 0 \\ -\nu/E & 1/E^* & 0 \\ 0 & 0 & 1/2\mu^* \end{pmatrix}, \text{ denoting } N \text{ by : } \begin{pmatrix} N_{zz} \\ N_{\theta\theta} \\ N_{\theta z} \end{pmatrix}, \; \mathscr{E} \text{ by : } \begin{pmatrix} \mathscr{E}_{zz} \\ \mathscr{E}_{\theta\theta} \\ \mathscr{E}_{\theta z} \end{pmatrix}$$

The elastic problem on the plane cell Y, under prescribed N, can be solved in terms of an Airy function. Then we prove this Airy function doesn't depend on elastic moduli (E, ν). This property leads to a relation between the homogenized moduli (E^*, μ^*) calculated for a given (E, ν) and those obtained for an other (E', ν'). In the same way, homogenized moduli in plane strain are related to those in plane stress.

As we can find in literature elastic moduli for a rectangular distribution of microcraks [2], we chose to study a hexagonal one (this is close to real arrays in tubes). For a wide range of parameters (β_x: microcrack length , $|Y|$: area of the 2D-cell) we solved the elastic periodic problem (in plane stress) by using a finite element discretization (with our own code ALI-BABA).

The Fig. 3 shows homogenized elastic moduli E_c^*, μ_c^*, for several values of the geometrical parameters β_x, $|Y|$, under plane stress conditions with $\nu = 0{,}3$. We have also computed the stress intensity factors at the microcrack tip.

<u>Arrays of any depth</u>

In this case, we choose simple approximations of the displacement : $\tilde{U}$, to get an upper bound of W_{def} (resp. of the stress : $\hat{\sigma}$, to a lower bound). The field $\tilde{U}$ is built with the 2D solution for the cracked cell Y in the cracked part of the 3D cell $\mathscr{Y}$, and an uniform field in the uncracked part :

$$(11) \quad \begin{cases} \tilde{U}_\alpha(y_1, y_2, y_3) = (\mathscr{E}\, y)_\alpha + f(y_3) . \chi_{D\alpha}(y_1, y_2) & \alpha = 1, 2 \\ \\ \tilde{U}_3(y_1, y_2, y_3) = -\frac{\nu}{1-\nu}(\mathrm{tr}\mathscr{E}) . y_3 \end{cases}$$

with f a regular function, vanishing on the uncracked part of $\mathcal{Y}$. By minimizing $W_{def}(\tilde{U}(f))$, which depends on the "unique degree of freedom" f, we get the corresponding elastic tensor : Λ_{+}^{hom}. The components of Λ_{+}^{hom} are explicit functions of the depth parameter t_f/t, and also of the 2D elastic homogenized moduli E^*, μ^*, for the corresponding plane parameters β_x and $|Y|$. In the same manner, we use the 2D stress solution σ_c, to build a 3D approximation $\hat{\sigma}$ section by section. The corresponding expression of the complementary energy $W^*_{comp}(\hat{\sigma})$ leads to define a tensor Λ_{-}^{hom} which depends explicitly on t_f/t, E^*, μ^*.

For a particular plane hexagonal distribution of cracks (β_x, $|Y|$), the Fig. 4 shows the values of $\Lambda^{hom}_{+\,2222}$ and $\Lambda^{hom}_{-\,2222}$ (stiffness perpendicular to the cracks), which bound the real values, for any t_f/t.

OVERALL FLOW STRESS AND LIMIT PRESSURE

It is difficult to find the homogenized law between N and $\mathcal{E}$ in the case of an elastoplastic constitutive relation in the cell. Conversely, the determination of the homogenized yield surface $f^{hom}(N)$ (which corresponds to a macroscopic strength criterion) leads to the solving of a limit analysis problem on the cell $\mathcal{Y}$, [4]. We define here the "overall flow stress" σ_f^{hom} as the limit membrane load : $\lambda_{lim} N_o$, with $N_o = \begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix}$, corresponding to the pressure loading on the tube. This overall flow stress depends only on the geometry and is proportional to the material flow stress σ_f (Von Mises microscopic strength criterion reads here : $\sigma_{eq} \leqslant \sigma_f$). The problem posed on the cell $\mathcal{Y}$ reads :

$$(12) \qquad \sigma_f^{hom} = \mathrm{Sup}\left\{ \lambda, \;\middle|\; \begin{array}{c} \exists\, \sigma \text{ , statically admissible in } \mathcal{Y} \text{ with :} \\ t\,\langle\sigma\rangle = \lambda . N_o \text{ and such as :} \\ \sigma_{eq}(y) \leqslant \sigma_f ,\ \forall\, y \in \mathcal{Y} \end{array} \right\}$$

This static approach is equivalent with the associated kinematic one :

$$(13) \quad \sigma_f^{hom} = \mathrm{Inf}\left\{ \mathcal{P}(V), \;\middle|\; \begin{array}{c} \mathcal{P}(V) \text{ : power of plastic dissipation in a velocity} \\ \text{field V, kinematically admissible on } \mathcal{Y}\text{, associated} \\ \text{with the macroscopic strain rate } \mathcal{E} \text{ such as :} \\ |\mathcal{Y}| .\, N_o .\, \mathcal{E} = 1 \end{array} \right\}$$

We can determine bounds of the limit loading by choosing particular fields σ (resp. V). We consider first the through-wall arrays : their failure modes are rather simple, so that we may propose analytical bounds.

Through-wall microcracks

We choose in the characterization (12) plane stress fields, uniform on strips or lozenges inscribed on the cell Y. We get analytical expressions for lower bounds, which are plotted on the Fig. 5, for a particular hexagonal array. In the other hand, we build 2D admissible velocity fields with rigid blocks, separated by shear bands or slip lines, and we compute the associated dissipation power. For higher β_x, size of microcrack, Prandtl-Hill's slip lines mechanism gives a better result. The obtained upper bounds are plotted on the Fig. 5, and show a rather tight domain of dimensionless overall flow stress. Some elastoplastic finite element calculations were carried out (with ALI-BABA), for several sizes β_x, in order to have more accurate results. Moreover, shear-bands failure modes are obtained : see Fig. 6.

Arrays of any depth. Comparison with experimental data

We restrict ourselves to a simple statical approach, by placing in parallel the uncracked part with the flow stress σ_f and the cracked one with the homogenized flow stress $\sigma_{f\,2D}^{hom}$, obtained for through-wall arrays. It leads to :

$$\sigma_f^{hom} = \sigma_f \left(1 - \frac{t_f}{t} \cdot \left(1 - \sigma_{f\,2D}^{hom} / \sigma_f\right)\right) \quad (14)$$

Now, we shall consider tubes with periodic microcracks arrays, tested during an experimental program devoted to the crack stability criteria in tubes [1]. The dimensionless bursting pressures (its value is 1 for an unmicrocracked tube) are reported on the Table 1, for several microcrack depth. The comparison with our prediction (14) is satisfactory enough. In the case of a long through- wall crack surrounded in a microcracks array, our prediction is based on the bulging M factor criterion $P_{lim} = \sigma_f \cdot \frac{1}{M} \cdot \frac{t}{R}$. The M factor is calculated, taking into account the elastic homogenized orthotropy, induced by the microcrack array. The comparison experiment-prediction, reported on the Table 2, seems to show that the failure, due to the macrocrack, is rather governed by the local yielding property (flow stress σ_f) and not by the macroscopic one $\left(\sigma_f^{hom}\right)$.

t_f/t	Experiment	Prediction with σ_f^{hom}
0	1,00	1,00
48 %	/	0,72
71 %	0,71	0,60
84 %	0,54	0,52
100 %	0,52	0,46

Table 1. Dimensionless bursting pressures of tubes weakened by microcracks arrays of various depth t_f/t.

t_f/t	Experiment	Prediction with σ_f^{hom}	Prediction with σ_f
0	1,00	1,00	1,00
48 %	0,96	0,79	0,92
71 %	0,94	0,68	0,88

Table 2. Dimensionless bursting pressures of tubes with a long crack surrounded by microcracks arrays of various depth t_f/t.

CONCLUSIONS

This study has shown the ability of a homogenization method to determine the effect of microcracks arrays on the elastic and collapse behaviour of pressurized tubes. In a leak-before-break analysis of a cracked tube, we can use the overall characteristics as calculated by our method, to link crack opening area and critical sizes.

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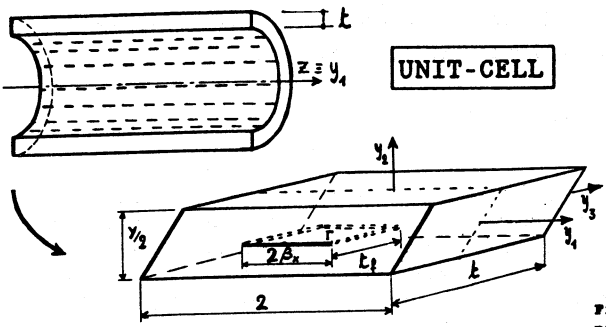

Fig 1. Unit-cell 𝒴 and geometrical parameters.

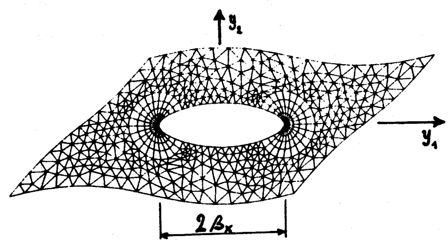

Fig 2. Elastic deformed mesh of the 2D-cell Y under prescribed N^{22} (ALIBABA F.E.M. code)

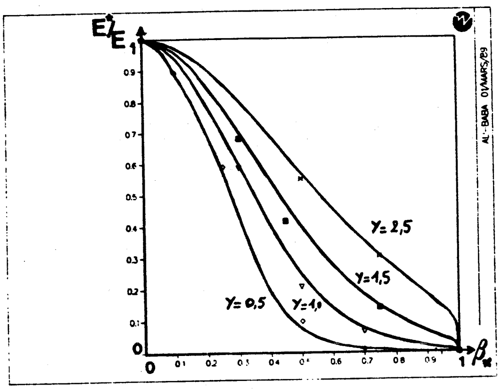

Fig 3. 2D plane stress homogenized modulus E^*/E, for several cell sizes $|Y|$, vs. microcrack size β_x.

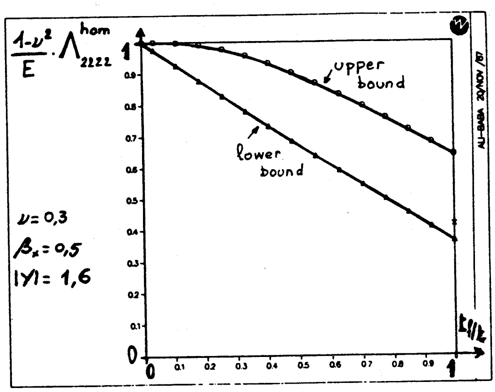

Fig 4. Bounds of dimensionless elastic stiffness coefficient Λ^{hom}_{2222} vs. microcrack depth t_f/t (with $|Y|=1.6$, $\beta_x=0.5$).

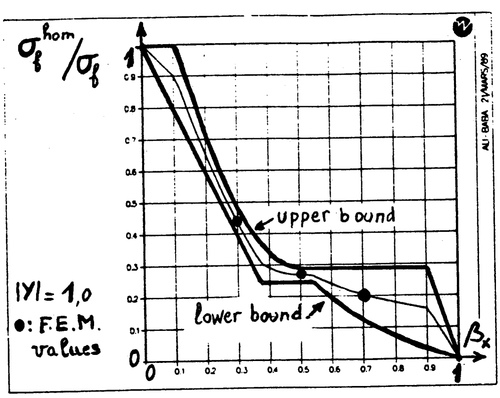

Fig 5. Overall flow stress σ_f^{hom}/σ_f bounds vs. microcrack size β_x ($|Y|=1.0$). Dots stand for numerical ALIBABA values.

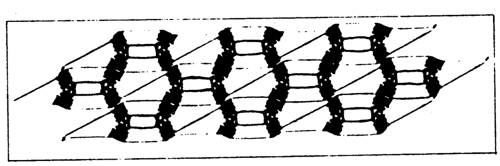

Fig 6. Numerical shear-bands at the limit loading on several 2D cells ($|Y|=1.0$, $\beta_x=0.3$).