Reliability Analysis of Nonlinear MDOF Dynamic Systems

Tsuyoshi Takada
Shimizu Corporation, Tokyo, Japan
Masanobu Shinozuka
Princeton University, Princeton, NJ USA

1. INTRODUCTION

When one evaluates structural integrity of nulcear power plant systems, a number of limit states associated with different ranges of structural response parameters must be considered. Of particular importance is the limit state that generally describes state of excursion of the structural response into severely nonlinear range. Unfortunately, reliable response analysis of nonlinear structures is achievable only by means of simulation methods particularly when these structures are subjected to ground acceleration idealized as a stochastic process (Shinozuka,1972; Vaicaitis, 1972). Although some approximate methods have been proposed to accomplish such analyses, these can neither deal with severely nonlinear response nor with structures having a large number of degrees of freedom.

One of the more practical approximations that are currently used by the profession for the purpose of design as well as analysis consists of utilizing the response modification factor (RMF). This factor was originally developed for single-degree-of-freedom systems for their nonlinear response analysis and design (Newmark, 1973). Use of RMF is not effective, however, for multiple-degree-of-freedom (MDOF) systems because the nonlinear deformation usually concentrates at certain part of the systems. In this context, the present authors recently developed a method of optimum design which minimizes the spatial concentration of nonlinear deformation within each building, regardless of its specific dynamic characteristics (Takada, 1988a,b). Indeed, the objective of the present paper is to demonstrate a method of reliability analysis for MDOF buildings thus optimally designed.

In this paper, nonlinear MDOF shear-type systems designed optimally are subjected to seismic motions idealized as a nonstationary stochastic process. With the aids of the established RMF-ductility factor relationship for the optimum system, RMF can be used to develop the limit state in the equivalent linear systems. Then, the linear random vibration theory can be used to evaluate the limit state probability as a solution to the classical first excursion probability problem at least in approximation. As a numerical example, a reliability of a three-story building is discussed and the results are compared by using a simulation method.

2. STRUCTURAL RANDOM RESPONSE

2.1 Idealization of Earthquake Motions

An input earthquake acceleration $\ddot{z}(t)$ to be used for the ensuing response analysis is assumed as a nonstationary stochastic process.

$$\ddot{z}(t) = f(t)\,g(t) \tag{1}$$

in which $g(t)$ denotes the stationary stochastic process and $f(t)$ a deterministic envelop function representing the time-varying intensity of the input earthquake motions.

In order to find the auto-correlation function associated with the stationary process $g(t)$, the well-known power spectrum proposed by Kanai and Tajimi is used (Tajimi, 1960).

2.2 Nonstationary Linear Random Response Analysis

The motions of masses subjected to the input acceleration may be expressed in terms of the following equilibrium equation:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{I}\ddot{z} \tag{2}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, Rayleigh damping and stiffness matrices of the system, respectively, and $\mathbf{I}$ an influence matrix. $\mathbf{x}$ denotes the motion vector of masses.

The variance of the interstory displacement response, the interstory velocity and the covariance of the above two quantities are calculated because these are needed in the computation of the probability of failure.

3. EQUIVALENT LINEAR CRITERIA OF OPTIMUM SYSTEMS

3.1 Response Modification Factor of Optimum Systems

RMF originally represents the ratio of a maximum linear response to the maximum nonlinear response of systems subjected to the identical earthquake motion. The RMF R is often described in terms of the ductility factor μ. Therefore, the R-μ relationship indeed involves various kinds of effects associated with the structural nonlinearity. If a system is an SDOF system, the following simple formula is well known (Newmark, 1973).

$$R = \frac{Q_l}{Q_n} = \sqrt{2\mu - 1} \tag{3}$$

In case of an MDOF system, the application of the above formula is not straightforward. Many researchers point out that there exists a concentration of nonlinearity on certain stories and that only a few stories may be subjected to serious damage (Akiyama, 1980). Recently, the present authors studied on the R-μ relationship of MDOF systems and concluded that in case that MDOF systems have such an optimum stiffness distribution that can minimize the spatial variation of maximum linear interstory displacement response of every story, Eq. (3) is practically applicable for each and every story (Takada, 1988a,b).

The R-μ relationship associated with the i-th story of such optimum systems may be written in the following statistical form:

$$R_i = \frac{Q_{li}}{Q_{ni}} = \epsilon\sqrt{2\mu_i - 1} \tag{4}$$

in which ϵ denotes the deviation from Eq. (3) and is assumed a log-normal random variable whose mean $\hat{\epsilon}$ and variance β_ϵ^2 are roughly summarized as follows:

$$\hat{\epsilon} = 1.2$$

$$\beta_\epsilon^2 = s(\mu_i - 1)^t \tag{5}$$

where β_ϵ, representing scatter around the median relationship, is considered as a function of a ductility factor. The larger ductility level yields a larger scatter. s and t are empirical constants which are around 0.04 and 0.4, respectively.

3.2 Equivalent Linear Criteria

Utilizing the above R_i-μ_i relationship of the systems designed optimally, the ultimate failure point where a failure occurs may be translated into the corresponding linear point on the force-displacement diagram as shown in Fig. 1. This translated point is called "an equivalent linear criterion" since this point is corresponding to the specified ultimate failure point.

4. DYNAMIC RELIABILITY ANALYSIS

Once a response (load effect) and capacity (resistant effect) are determined as mentioned above, a reliability analysis of the optimum systems may be performed easily. The reliability analysis here is to estimate the probability that a linear random response exceeds the equivalent linear criterion during a certain time duration (Shinozuka, 1968).

4.1 Nonstationary First Excursion Probability

To compute the following probability that the linear random shear force response of the i-th story $Q_{li}(t)$ exceeds the equivalent linear criterion of the same story Q_{eqi} in the time interval $(0, T_d)$ under the zero initial condition,

$$P_{fi} = prob\{|Q_{li}(t)| > Q_{eqi} \ : \ 0 < t < T_d\} \tag{6}$$

the following formula is utilized (Yang, 1972):

$$P_{fi} = prob\{Q_{li}(t), Q_{eqi}\} = prob\{U_{li}(t), R_i U_{yi}\} = 1 - \int_0^{\infty} \exp\left\{-\int_0^{T_d} \nu(t, U_{li}(t))\, dt\right\} f_\epsilon(\epsilon)\, d\epsilon \tag{7}$$

with an expected crossing rate ν and $f_\epsilon(\epsilon)$ a probabilistic density function of the log-normal random variable ϵ. The above semi-infinte integration takes account of the variability of the R_i-μ_i relationship of optimum systems.

5. NUMERICAL EXAMPLE AND DISCUSSION

For the input earthquake motion, assuming the relatively soft soil condition, the Kanai-Tajimi parameters are selected as written on Fig. 2. Figure 3 shows the envelop function to be used in the following response analyses (Shinozuka, 1967).

Let us take two nonlinear three-story structures which have different mechanical properties as shown in Fig. 4. Both structures have uniform distribution of yielding displacement of each story. Structure A is an optimum structure having identical standard deviation of the linear interstory displacement response under the stochastic excitation as shown in Fig. 5. Structure B has the same stiffness along the height, so no optimization is employed in this structure. Standard deviation of interstory displacement response of structure B is plotted in Fig. 6. The mechanical properties of both structures are listed on Table 1. The ratio of the stiffness and masses for both structures are selected so that the fundamental natural period of the linear structures becomes 0.5 seconds. The damping ratio is assumed 5 percent for the first and second modes.

In the determination of the equivalent linear criteria of each story of structure A, the ultimate ductility factor μ_{cri} is considered as a parameter ranging from 1.0 to 10.0. For the optimum structure A, the equivalent linear criteria for the i-th story Q_{eqi} can be computed by using Eq. (4). Note that the ultimate strength Q_{cri} is equal to the yielding strength Q_{yi} since the perfect elasto-plastic nonlinearity is assumed for each interstory force-displacement relationship of both structures.

At the same time, a Monte Carlo simulation analysis is performed for both structures in order to obtain the probability of failure, in which step-by-step time integration response analyses are carried out by using one hundred different artificial earthquake motions (Shinozuka, 1974).

Figure 7 shows the probability of failure of each story of structure A as a function of ductility factor criteria from both methods, provided that the yielding displacement level is selected as the same amount of the maximum standard deviation of each interstory displacement response. Figure 8 is the same plot when the yielding displacement level is reduced to two-third of the maximum standard deviation of interstory displacement response. From these figures, it can be observed that the proposed method provides the results similar to those from the simulation method although the proposed method shows a slightly different tendency in the large range of ductility criteria. In addition, it is seen that the nonlinear response tends to concentrate on the first story despite of the optimization employed.

Figure 9 shows the same plot for the structure B only from the simulation method. It can be observed that only the first story is subjected to serious damage, whereas the other stories remain either in the elastic or low nonlinear region. This clearly states that the optimum structure shows excellent performance even in the nonlinear region from the seismic damage point of view. It should be noted that the structures are unrealistically vulnerable to the intensity of the seismic input in order to obtain the high nonlinear range of structural response.

If a ductility criterion is fixed and the intensity of the input earthquake motion varies, the probability of failure can be expressed as a function of the intensity of the input motion. This relationship is so-called a fragility curve in the current PRA study (Kennedy, 1984; Lai, 1983). Figure 10 indicates the fragility curves based on the proposed method, in which fixed ductility criteria are taken as 1.0, 2.0, 5.0 and 10.0. To evaluate an annual probability of failure, these curves will be combined with a hazard curve representing the annual occurrence rate associated with the intensity of earthquake motions.

6. CONCLUSION

This paper presents a simplified method to estimate the probability of failure for nonlinear MDOF shear-type systems designed optimally due to seismic stochastic excitations. Utilizing the RMF, this method facilitates an assessment of probability of failure that the nonlinear response exceeds the nonlinear criteria of the systems.

7. ACKNOWLEDGEMENTS

This research was supported by National Science Foundation under Grant No. ECE 86-07591 through the National Center for Earthquake Engineering Research under Grant Nos. SUNYRF NCEER-86-3031, 86-3033 and 87-1006.

8. REFERENCES

Akiyama, H. (1980), A Seismic Ultimate Design of Building Structures, Tokyo University Press, Tokyo, Japan

Kennedy, R.P., Ravindra, M.K. (1984), "Seismic Fragilities for Nuclear Power Plant Risk Studies," Nuclear Engineering and Design, Vol. 79

Lai, S.P. (1983), "Seismic Safety: 10-story UBC Designed Steel Building," Journal of Engineering Mechanics, ASCE, Vol. 109, No.2

Newmark, N.M., Hall, W.J. (1973), "Procedure and Criteria for Earthquake Resistant Design," Building Practices for Disaster Mitigation, NBS, Building Science Series No.46

Shinozuka, M., Sato, Y. (1967), "Simulation of Nonstationary Random Process," Journal of Engineering Mechanics Division, ASCE, Vol. 93

Shinozuka, M. (1974), "Digital Simulation of Random Processes and Its Applications," Journal of Sound and Vibration, Vol. 25, No.1, pp. 111-128

Shinozuka, M. and Wen, Y.K. (1972), "Monte Carlo Solution of Nonlinear Vibrations," AIAA Journal, Vol. 10, No.1, pp. 37-40

Shinozuka, M., Itagaki, H. and Hakuno, M. (1968), "Dynamic Safety Analysis of Multistory Buildings," Journal of Structural Division, ASCE, Vol.94

Tajimi, H. (1960), "A statistical Method of Determining the Maximum Response of a Building Structure During an Earthquake," Proceedings of 2nd WCEE

Takada, T., Hwang, H.H.M. and Shinozuka, M. (1988a), "Response Modification Factor for MDOF systems," Proceedings of 9th WCEE, 1988 (in print)

Takada, T. and Shinozuka, M. (1988b), "Response Modification Factor of Optimum Structures Subjected to Various Ground Motions," NCEER technical report in preparation

Vaicaitis, R., Jan, C.M. and Shinozuka, M. (1972), "Nonlinear Panel Response from a Turbulant Boundary Layer," AIAA Journal, Vol.10, No.7, pp. 895-899

Yang, J.N. (1972), "Nonstationary Envelop Process and First Excursion Probability," Journal of Structural Mechanics, Vol. 1, No.2, pp. 231-248

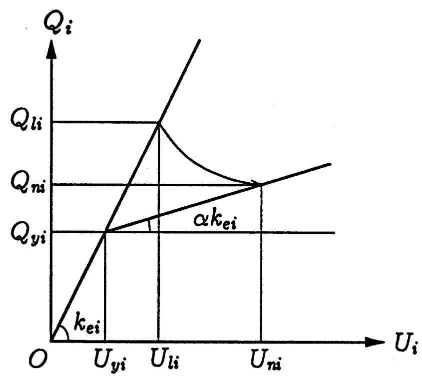

Fig. 1 Concept of equivalent linear criteria

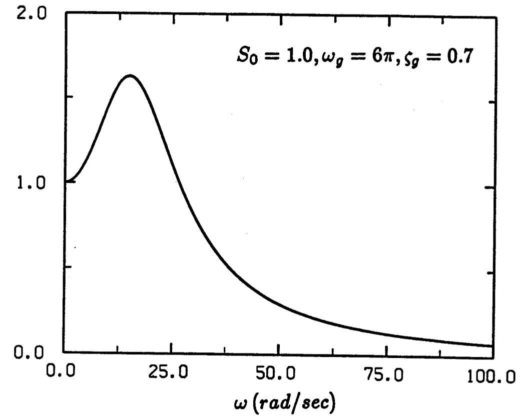

Fig. 2 Kanai-Tajimi power spectrum (Tajimi, 1960)

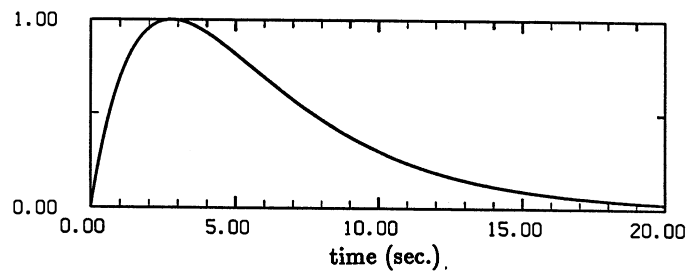

Fig. 3 Deterministic envelop function (Shinozuka, 1968)

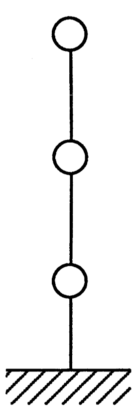

Fig. 4 Three-story structure

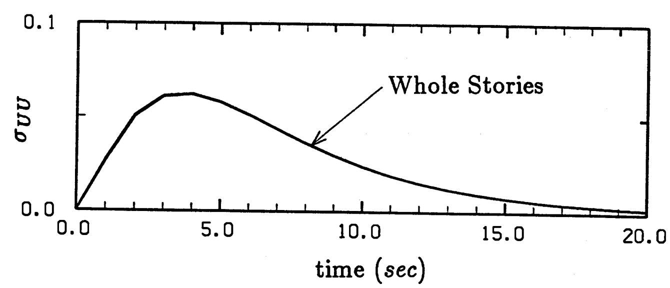

Fig. 5 Interstory displacement response histories (Structure A)

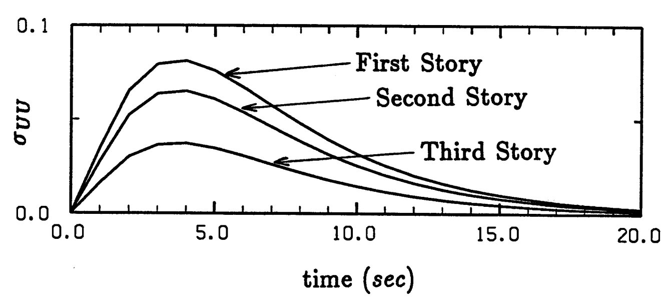

Fig. 6 Interstory displacement response histories (Structure B)

Table 1 Structural properties

	Structure A (Optimum Structure)			Structure B		
Story	Mass m_i/m_1	Shear Stiffness k_i/k_1	Yielding Disp. U_{yi}/U_{y1}	Mass m_i/m_1	Shear Stiffness k_i/k_1	Yielding Disp. U_{yi}/U_{y1}
3	1.0	0.51	1.0	1.0	1.0	1.0
2	1.0	0.82	1.0	1.0	1.0	1.0
1	1.0	1.00	1.0	1.0	1.0	1.0
	m_1=1.0	k_1=1013	U_{y1}=0.06,0.04	m_1=1.0	k_1=800	U_{y1}=0.06

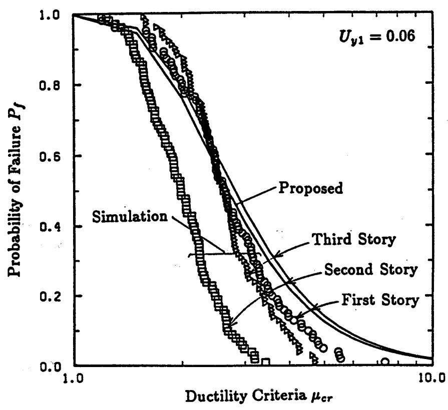

Fig. 7 Probability of failure versus ductility criteria (Structure A, $U_y = 0.06$)

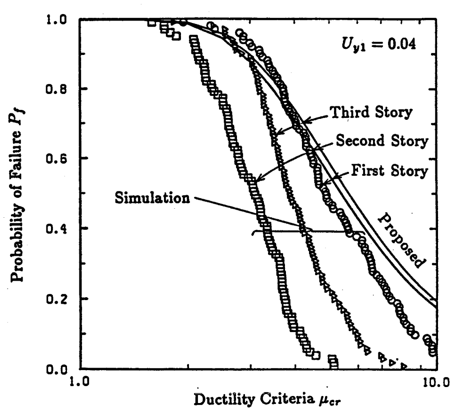

Fig. 8 Probability of failure versus ductility criteria (Structure A, $U_y = 0.04$)

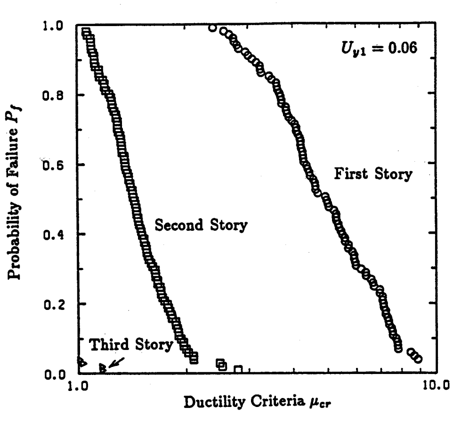

Fig. 9 Probability of failure versus ductility criteria (Structure B, $U_y = 0.06$)

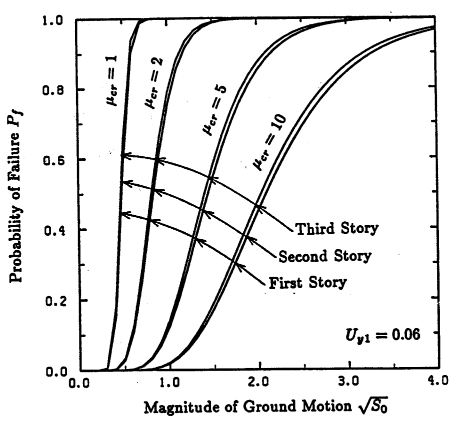

Fig. 10 Fragility curves (Structure A, $U_y = 0.06$)