Probabilistic Seismic Collapse of SDOF Elasto-Plastic System

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Abstract

This paper presents an original method to evaluate the collapse probability of a Single DOF elasto-plastic system seismically excited by a gaussian white noise. The crossing by the plastic drift D(t) of a fixed double symmetrical barrier defines the collapse of the system. The plastic drift D(t) is modelled as a Markov process, whose basic parameters are the average frequency and the variance of schematized plastic jumps (respectively ν_P and $E[\delta_P^2]$ which can be numerically approximated). Then the derived formulation of collapse probability is compared and validated by results of extensive time history simulations (Monté-Carlo method).

INTRODUCTION

This paper deals with the probabilistic dynamic behavior of SDOF elasto-plastic systems excited by gaussian white noises. At first sight, such a study could appear only of an academic interest: few actual structures and excitations can be modelled so simply in engineering fields. Nevertheless it can help the understanding of real dynamic behaviors. For example, and this has motivated the present work, it recovers three main aspects of earthquake engineering:

- the dynamic nature of seismic responses,
- the stochastic nature of earthquake excitations,
- the nonlinear nature of load-displacement curves for ductile structures which can support large plastic deformations.

Therefore one considers in this paper a single oscillator elastic-perfectly plastic, whose basic parameters are the mass m, the viscous dashpot c, the elastic stiffness k and the symmetrical elastic limits $\pm X_e$ (see figure 1). This SDOF system, initially at rest, is subjected to an anchor acceleration, which is modelled by a stochastic stationary gaussian white noise with zero mean value and G_0 Power Spectral Density. An Associated Linear System (ALS) is constructed by extending to infinity the elastic slope of the elasto-plastic system and is subjected to the same excitation. Especially it permits to express the excitation intensity (PSD level G_0) by the standard deviation of the ALS displacement response σ_{XL}.

The relative displacement response of the elasto-plastic oscillator constitutes a stochastic process of which a time history sample Y(t) is shown figure 2_a. Classically Y(t) can be partitionned in 2 parts:

(1) $$Y(t) = D(t) + X(t)$$

where D(t) represents the plastic deformation accumulated at time t resulting from all crossings of elastic limits during [0,t]. By construction the remaining part X(t) represents the elastic deformation of Y(t) and is always restricted to the range $[-X_e, +X_e]$ (figures 2_b and 2_c).

This paper deals with oscillator "collapses" similar to plastic instabilities: the oscillator is made collapsed when the relative displacement Y(t) exceeds an "*à priori*" fixed limit Y_{Ma}. The ratio of this maximum permitted limit Y_{Ma} to the elastic limit X_e defines the allowable ductility μ_a ($\mu_a = Y_{Ma}/X_e$).

Mathematically the response Y(t) is calculated or estimated from 0 to time T without taking into account such limits, then the absolute maximum of the relative displacement Y(t) is extracted ($Y_{MAX} = \mathrm{Max}\,|Y(t)|$, $t \in [0.,T]$) and compared to the permitted limit:

* $Y_{MAX} > Y_{Ma} = \mu_a.X_e \longrightarrow$ SDOF collapse during [0,T]
* $Y_{MAX} \le Y_{Ma} = \mu_a.X_e \longrightarrow$ the SDOF is "saved".

By using (1) and the relation $-X_e \le X(t) \le X_e$, one notes: $Y_{MAX} \le D_{MAX} + X_e$ $\Leftrightarrow D_{MAX} \ge Y_{MAX} - X_e$. Thus within the framework of this study, it is sufficient to characterize the plastic deformation process D(t) and then to solve a so-called "first passage problem" related to the first crossing by D(t) of a double symmetrical barrier $\pm D_U$ deduced from a fictitious ductility μ_a-1 : $D_U = (\mu_a - 1).X_e$

MARKOVIAN MODEL OF THE PLASTIC DRIFT D(t)

Let $d_1 = D(t_1)$ the plastic deformation accumulated at time t_1. The plastic deformation D(t) does not change ($D(t) = d_1$) as long as the system remains in the elastic domain: $d_1 - X_e < Y(t) < d_1 + X_e \quad \Leftrightarrow \quad -X_e < X(t) < +X_e$

During a single plastic excursion (one has at this time $|X(t)| = X_e$), the plastic deformation D(t) varies from d_1 to d_2 as shown on figure 3_a. After that D(t) becomes constant ($D(t) = d_2$, new elastic phase) up to the following plastic excursion. The plastic deformation D(t) is so formed by straight horizontal lines (corresponding to elastic phases) connected by transient portions (corresponding to plastic excursions).

But often the plastic excursions occur consecutively in clumps, as illustrated figure 3_b: a first "positive" plastic excursion ($X(t) = +X_e$) is followed, one half-cycle later, by a "negative" plastic excursion ($X(t) = -X_e$), which, itself, may be followed, one half-cycle later, by an other "positive" excursion, and so on. By simplification the SDOF response is considered "plastic" as long as the clump of plastifications is going - this corresponds to a transient part of D(t) (see Fig.3_c) - and elastic in the other cases - this corresponds to "long" straight parts of D(t) ("long" means greater than an half-period of the ALS).

During the whole length of the excitation, it seems reasonable to assume that the SDOF elastic phases are longer than the plastic ones. Consequently the transient portions of D(t) can be drastically simplified and considered instantaneous as seen figure 3_c. By this way a cluster of <u>N</u> successive plastifications is reduced to <u>one</u> schematic plastification. Then the simplified plastic process D(t) appears like a succession of irregular steps with unequal heights - the positive or negative amplitude δ_P of each schematized plastic jump - and unequal lengths - the duration T_P between two successive plastic jumps.

As the elasto-plastic model has no strain hardening, and as the excitation is stationary, it can be assumed, by neglecting the SDOF response transient part due to the initial rest, that the statistical caracteristics of plastic jumps are time independent. Especially (E[] being the mathematical expectation):

$E[T_P] = 1/\nu_P$; $E[\delta_P^2] =$ constant ; $E[\delta_P] = 0.$ for symmetry reasons.

In addition the schematized plastic jumps may be considered to be independent. This condition of independence is not rigorous, but constitutes a good approximation due to the regrouping of successive plastifications. Consequently the plastic process D(t) can be modelled as a markovian process, i.e. similar to a brownian motion. It is written:

(2) $$D(t) = \sum_{J=1}^{N(t)} \delta_P(t_J)\ \mathbb{1}(t-t_J)$$

where N(t) is the number of plastic jumps during [0,t] following a Poisson law; $\mathbb{1}(t-t_J)$ is the Heaviside function, $= 0$ for $t < t_J$, $= 1$ for $t \geq t_J$; $\delta_P(t_J)$ is the amplitude of the schematic plastic jump which occurs at time t_J. Such a process is easy to manipulate [1]. Particularly one has at time t:

(3) $$E[D(t)] = E[N(t)]\ .\ E[\delta_P] = 0.$$

(4) $$E[D^2(t)] = E[N(t)]\ .\ E[\delta_P^2] = \nu_P t.\ E[\delta_P^2] = \sigma_D^2(t)$$

For "large" values of N(t), the Central Limit Theorem may be invoked in order to approximate the distribution of plastic deformations at time t, law $f_t(D=\alpha)$, by a gaussian law with a zero mean value and the standard deviation $\sigma_D(t)$:

(5) $$f_t(\alpha) = \frac{1}{(2\pi)^{1/2}\sigma_D(t)} \exp\left[-\alpha^2/2\sigma_D^2(t)\right]$$

The first passage problem is quickly solved for brownian motions [3]. Let the collapse defined by the level $\pm D_U$ and consider the time t_1; obviously any trajectory which leads to a level greater than $+D_U$ at time t_1, is a collapse trajectory for the oscillator during $[0,t_1]$ (see fig.4). The probability to have such trajectories is:

(6) $$P'(D_U,t_1) = \int_{D_U}^{\infty} f_{t1}(\alpha)\ .\ d\alpha$$

Let $D_1(t)$ such a trajectory. The trajectory $D_2(t)$, constructed from $D_1(t)$ as shown figure 4 (identical from 0 up to D_u, symmetrical with respect to D_U afterwards), is also a collaspse trajectory. Both trajectories $D_1(t)$ and $D_2(t)$ have the same probability of occurence: for markovian processes the future states are independent from anterior ones. The probability for crossing (at least one time) the barrier $+D_U$ by D(t) during the period $[0,t_1]$ is thus:

(7) $$P(D_U,t_1) = 2\ .\ P'(D_U,t_1)$$

The probability for not crossing $+D_U$ is $[1-P(D_U,t_1)]$, the one for not crossing $-D_U$ is identical. The product of both of them gives the probability to get a "survival" trajectory. Consequently the collapse probability is:

(8) $$R(D_U,t_1) = 1 - [1 - P(D_U,t_1)]^2$$

In addition to the time t_1 and the allowable ductility μ_a, this probability depends on the average frequency ν_P and the variance $E[\delta_P^2]$ of schematized plastic jumps (eq.4), which are the two basic parameters of the markovian model of the plastic drift D(t).

PARAMETERS OF THE MAKOVIAN MODEL (ν_P and $E[\delta_P^2]$)

At the precise transition time t_0, when the SDOF leaves the plastic domain and returns in the elastic one, one knows exactly its relative elastic displacement $X(t_0)=X_0=\pm X_e$ and its velocity $\dot{X}(t_0)=\dot{X}_0=0$. Consequently the continuation of the motion in the elastic domain is driven by the joint conditional probability

$P(X,\dot{X},t/X_0,\dot{X}_0\ t_0=0)$ which provides the statistics of displacement and velocity, given the initial conditions X_0 and $\dot{X}_0$. This probability is well established for linear SDOF excited by gaussian white noises [2]. It permits to numerically calculate:

- the probability P_1 to have an half-cycle plastic-elastic-plastic,
- the probability P_0 to have an elastic peak followed by a plastic excursion in the half-cycle later,
- the distribution of velocities at the crossing of elastic limits $P(V_e)$.

These data, combined with other assumptions, especially a Rayleigh distribution for $P(V_e)$ (which gives an exponential distribution for the amplitude of a single plastic jump), lead to original formula for the variance $E[\delta_p^2]$ and the frequency ν_p of schematized plastic jumps. For details see reference [4].

APPLICATIONS AND COMPARISONS TO TIME-HISTORY SIMULATIONS

The theoritical approach, previously described, has been tested by extensive time history simulations carried out by the code CR4S developped at FRAMATOME. The caracteristics of simulations reported here are:
$f_0 = 5.$ Hz; $\xi = 0.04$; $\sigma_X = 1.$; $X_e = 1.0,\ 2.5$; $T_{REAL} = 50.$s; $N_{REAL} = 250$
(respectively the ALS eigenfrequency, reduced damping, standard deviation of displacement, the elastic limits, the duration of one realization and the number of realizations)

The figures 5 and 6 present "theoritical" and simulation curves giving the SDOF collapse probability in terms of the allowable ductility μ_a, for two durations T=25.s and T=50.s. The two basic parameters, the variance $E[\delta_p^2]$ and the frequency ν_p of plastic jumps, are (fig.5) directly computed from simulation results, and (fig.6) calculated according to the numerical approach fully developped in [4]. In a general way one notes the good agreement between theory and simulation, more especially for the low collapse probabilities. Thus the figures 5 justify the markovian model of the plastic drift, whereas the figures 6 partly validate the theoritical calculation of parameters ν_p and $E[\delta_p^2]$. Nevertheless it appears that this calculation could be improved.

CONCLUSIONS

In order to study the probabilistic dynamic behavior of SDOF elasto-plastic systems excited by gaussian white noises, it is efficient to separate the plastic response from the elastic one. The plastic drift can be modelled as a markovian process whose basic parameters are the variance $E[\delta_p^2]$ and the frequency ν_p of schematized plastic jumps. It is recalled that these parameters can be numerically evaluated using for the Associated Linear System the joint conditional probability $P(X,\dot{X},t/X_0,\dot{X}_0,t_0=0)$. The collapse probabilities, given by such markovian models, are validated by results of time history simulations.

Such a study, which will be extended [4], recalls the well-known fact that ductile structures can withstand severe earthquakes, and in addition it points out the basic brownian nature of the plastic drift (see also [5]).

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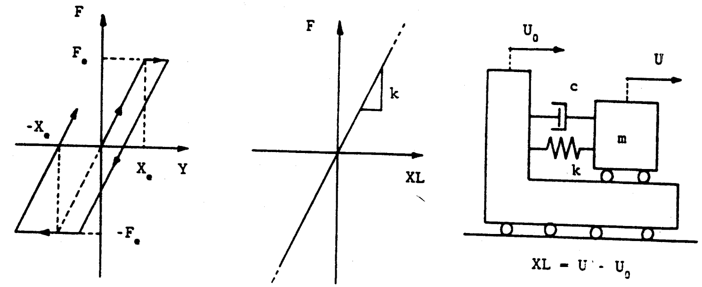


elasto-plastic curve

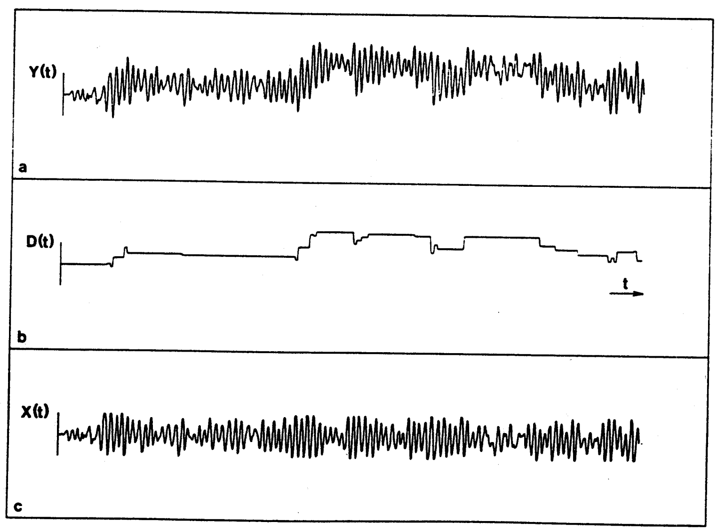
Associated Linear System

$XL = U - U_0$

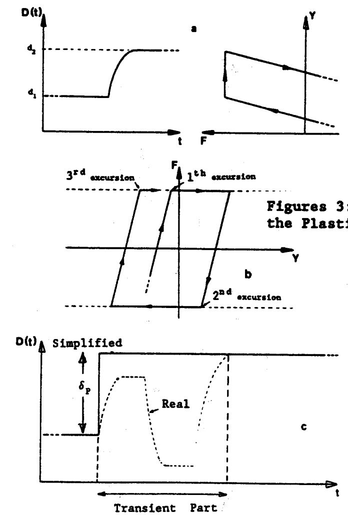
$f_0 = 1/2\pi \ (k/m)^{1/2}$

$c = 2 \ \xi \ (k.m)^{1/2}$

Figure 1: Elasto-Plastic System



Figures 2: Oscillator Response to a White Noise



Figures 3: Simplification of the Plastic Deformation D(t)

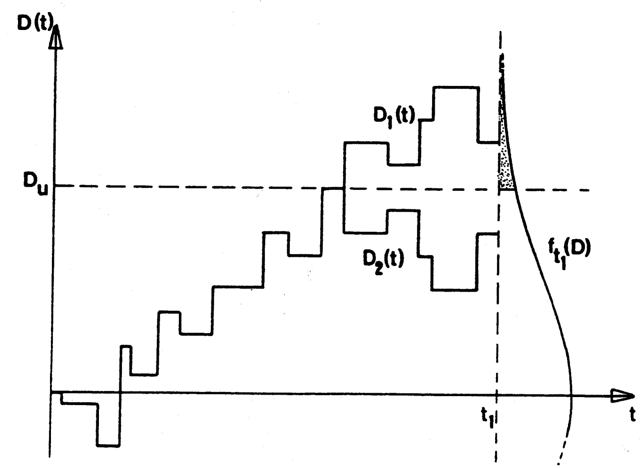
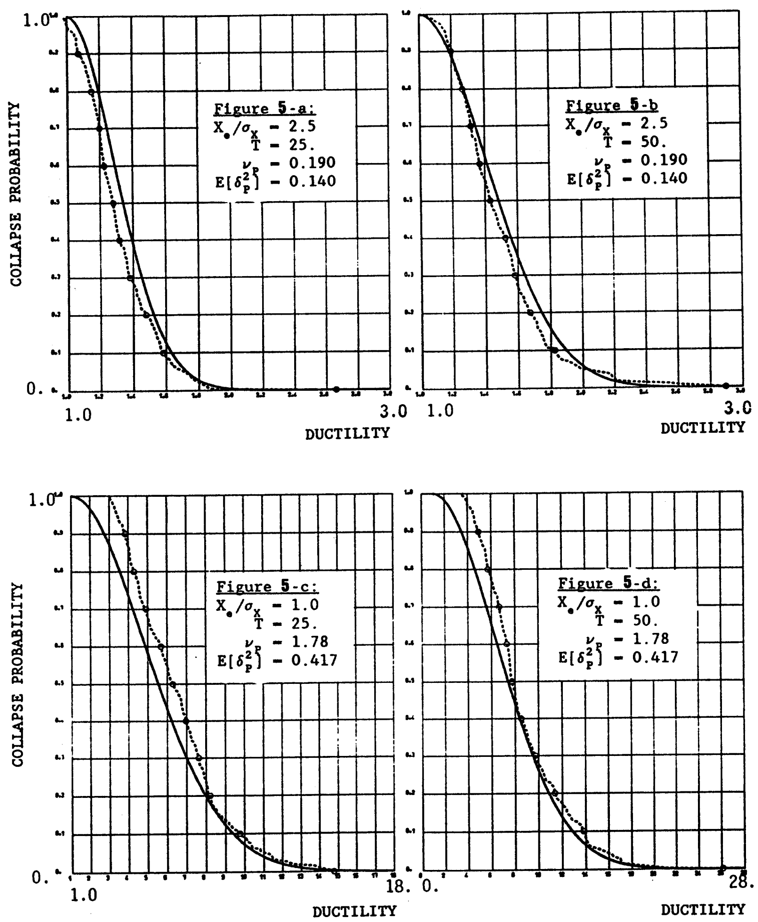
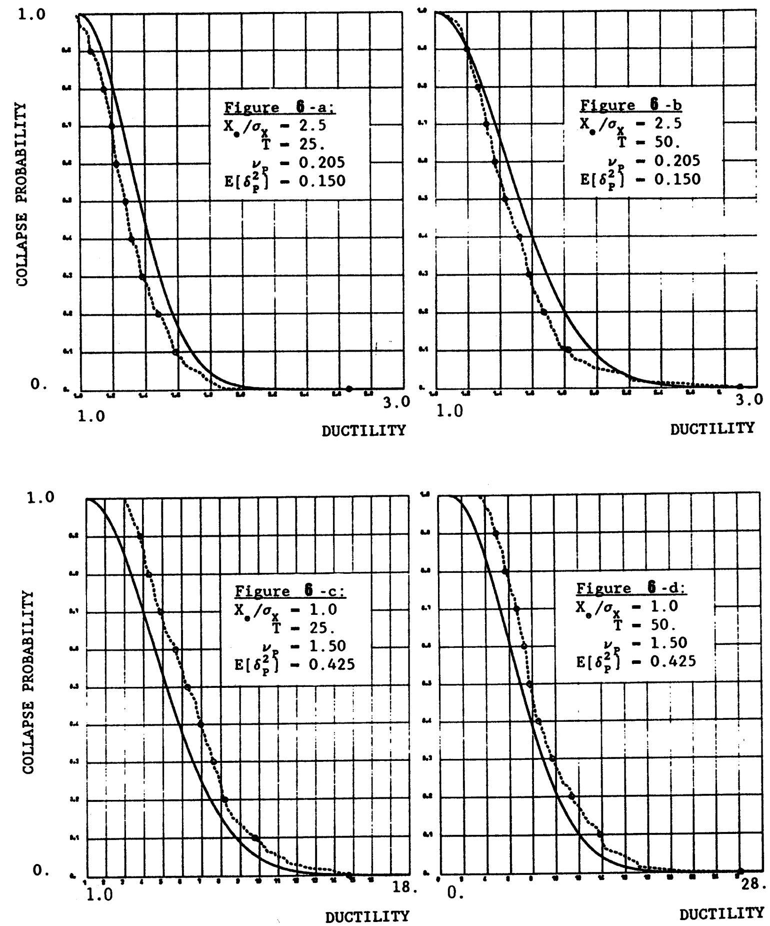


Figure 4: Schematized Plastic Process D(t)



Figures 5: Elasto-Plastic Oscillator Collapse Probability in terms of the allowable ductility (ν_p, $E[\delta_p^2]$ empirical)

Theory: ————
Simulation: -o------o-



Figures 6: Elasto-Plastic Oscillator Collapse Probability in terms of the allowable ductility (ν_p, $E[\delta_p^2]$ calculated)

Theory: ————
Simulation: -o------o-