Comparison of Approximative Markov And Monte Carlo Simulation Methods for Reliability Assessment of Crack Containing Components

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INTRODUCTION

Reliability assessments based on probabilistic fracture mechanics can give insight into the effects of changes in design parameters, operational conditions and maintenance schemes. Although they are often not capable of providing absolute reliability values, these methods at least allow the ranking of different solutions among alternatives. Due to the variety of possible solutions for design, operation and maintenance problems numerous probabilistic reliability assessments have to be carried out. This is a very laborous task especially for crack containing welds of nuclear pipes subjected to fatigue. The objective of this paper is to compare the Monte Carlo simulation method and a newly developed approximative approach using the Markov process ansatz for this task.

METHODS

Fracture Mechanics Model

The damage state of the considered component is given by the size of the most dangerous crack, which is assumed to be a semi-elliptic surface crack and can therefore be represented by the length of its semi-axes. The justification and limits of this procedure are discussed in (Schmidt and Schomburg 1985). The crack size is treated as a two-dimensional random variable whose distribution can be obtained by destructive and non-destructive testing results. The crack is growing due to cyclic loads. Stress corrosion cracking, creep and crack initiation are not taken into account. However, environmental effects can be incorporated via the fatigue crack growth relations. Due to the cyclic loads the crack may grow stably in a two-dimensional way and remain semi-elliptic or it may become locally or globally unstable. Subcritical crack growth "laws" and failure criteria were assumed to be random and can be taken from a variety of experimental results.

Transients

Besides the load cycle transients we also have to take into account the effects of inspections and repairs if such are carried out. Ultrasonic inspection and various leak detection as well as repair success data are obtainable from experimental results or expert experiences. The arrival times and the sequence of load, inspection and repair events are assumed to be known. For nuclear components whose failure behaviour is dominated by low cyclic fatigue, e.g. for some piping systems, the differentiation between arrival times distributed according to Poisson processes and those equally spreaded according to the expected values of the processes seems negligible with respect to failure probabilities (Harris et al. 1982).

Mathematical Approximations

The classical approach to include the scatter of material properties into cyclic crack growth is to randomize the crack growth relation, usually assuming a stochastic differential equation $da/dn = C \cdot f(\Delta K)$ as adequate, where a is the crack depth or length, n the cycle number, ΔK the stress intensity factor amplitude, f a known function (e.g.

Paris 's power law) and C the realization of a random variable (Virkler et al. 1979, Ostergaard and Hillberry 1983). In the two-dimensional case this formulation becomes

$$da/dn = C_a \cdot f(\Delta K_a) \quad \text{and} \quad db/dn = C_b \cdot f(\Delta K_b)$$

with C_a and C_b usually taken as i.i.d. random variables (rv). In both cases the random variables C resp. C_a and C_b keep the same values during all cycles. This has been shown not to reflect real crack growth data adequately (Kozin and Bogdanoff 1981). Keeping in mind that the differential formulation of cyclic fatigue is somewhat arteficial since by its definition it should be a difference equation, a form into which the above relation can be approximately retransformed by using Euler's polygonial algorithm, then the other extreme would be to introduce rv's C(n) or $C_a(n)$ and $C_b(n)$ which for all n are independent but possibly identically distributed. This leads to a markovian incremental crack growth process that overcomes the aforementioned inadequacy. For the true crack growth process, however, complete independency seems also not realistic. The same is true for the stochastic modelling of failure according to other mechanisms than subcritical crack growth (brittle, ductile or plastic limit load failures) which must not be excluded in a reliability assessment of nuclear components. Fortunately simulation studies showed no significant differences for the estimated failure probabilities except that their variation was larger in the case of total dependency as to expect and that the simulation time was significantly smaller (about 30%). On the other hand, complete independency is usually assumed for the effects of subsequent inspections and repairs. Under this assumption these transients can be incorporated in a combined markovian damage state process. This damage state can always be identified with the actual crack size (length and depth), thus being a two-dimensional Markov process (MP). Fatigue crack growth as a single step transition Markov chain (MC) was already modelled by (Bogdanoff (et al.) 1978/1980). Inspection effects could also be included in the model but it was only one-dimensional and still did not incorporate other failure mechanisms. It was not based on a crack growth relation but instead used actual crack growth data for certain stress amplitudes. One or more of this shortcomings also concern several other approaches to stochastic fatigue which all start from some specific assumptions of a randomized crack growth relation (e.g. Lin and Yang 1983, Ditlevsen 1986, Madsen et al. 1987, Ortiz and Kiremidjian 1988). A survey about various approaches is given by (Kozin and Bogdanoff 1987).

Monte Carlo Simulation

This method can be used to virtually simulate all stochastic processes and is therefore very generally applicable. Applications in reliability assessments are given e.g. by (Harris et al. 1982, Häberer et al. 1983, Schäfer et al. 1984). In this special case it is used to simulate the markovian damage process (see above). The simulation performed on a mainframe computer was carried out with the modified PRAISE code, originally developed by LLNL (cf. Harris et al. 1982) and adapted to the specific german situation (cf. Schmidt and Schomburg 1985). This version was again modified to be capable of simulating the markovian damage process (Schmidt 1989).
It has to be pointed out that for the estimation of very small failure probabilities sensitive variance reduction techniques have to be employed (cf. e.g. Brückner-Foit et al. 1989). In the PRAISE code this is done by stratification of the partial sample space Ω of all possible crack sizes expressed in the depth a and the depth to lenght ratio a/b (assuming $a \leq b$ always, cf. figure 2). The failure probability $P_F(t)$ as a function of time can be written as

$$P_F(t) = \int_{\Omega} q(t|(a,a/b)) \cdot f(a,a/b)\ d(a,a/b)$$

where f(a,a/b) is the normalized crack size density and q(t|(a,a/b)) is the probability that a crack of initial size (a,a/b) leads to failure until time t. Stratification means splitting up the domain Ω into mutually disjunctive parts Ω_i ($\Omega = \sum \Omega_i$). The conditional expectation of q(t|(a,a/b)) under the condition $(a,a/b) \in \Omega_i$ is obtained by crude Monte Carlo (for each Ω_i separately) by simulating each individual sample crack history until the projected component life time.

Markov Chain Approach

Preliminary results with this approach have already been discussed in (Schmidt 1987, Schmidt and Schomburg 1987). The principal idea is the following: Using the markovian

transition properties the failure probability $P_F(t)$ can also be expressed as the probability of reaching a specific damage state $L \subset S$ until time t, where S is the domain of all possible crack sizes now expressed for convenience in terms of depth a and length b. By little changes in the formulation of the original markovian damage process a related MP is described that gives the failure probability as

$$P_F(t_n) = \int_{S \setminus L} \nu_n(s,L)\, R_{n-1}(ds)$$

where $R_{n-1}(ds)$ is the damage state probability distribution at time t_{n-1} and ν_n is the transition probability belonging to the event occuring at time t_n. Using the fundamental relation

$$R_n(ds) = \int_S \nu_n(u,ds)\, R_{n-1}(ds)$$

the sequence of damage state probability distributions can be obtained recursively from the initial probability distribution R_0 by integration with $\nu_1,\ldots,\nu_n$. These transition probabilities depend on the actual transient and can easily be expressed in terms of the distributions of the rv involved in this transient at least for a generating subset of the Borel algebra on S. However, both, R_n and ν_n cannot be obtained analytically but must instead be gained by numerical approximation procedures. A natural way for such a procedure is to approximate the related MP by an MC. This is done in four steps:

1. discretization of the damage state space S into mutually disjunctive parts S_i ($S = \sum S_i$), which each represent one state of the new MC. For some reasons it is convenient that each corner point belonging to S makes its own state S_i and that the border lines between them are also partioned by the discretization (cf. fig. 3 and Schmidt 1989)
2. discretization of $\nu_n(u,ds)$ into $p_{i,j}(t_n) := \nu_n(u,S_j)$, if $S_i=\{u\}$ contains a corner point or into

$$p_{i,j}(t_n) := (1/\lambda_i(S_i)) \int_{S_i} \nu_n(u,S_j)\, \lambda_i(ds)$$

where λ_i is the one or two-dimensional Lebesgue measure belonging to S_i, if S_i is a line or area set.

3. discretization of R_0 into P_0 by defining $P_0(i) := R_0(S_i)$
4. successive calculation of the values $P_{n+1}(j) = \sum p_{i,j}(t_{n+1}) \cdot P_n(i)$

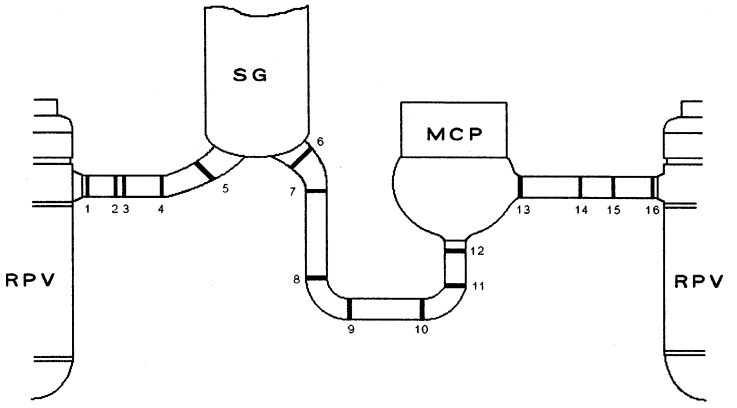
It can be shown that under certain conditions this approximation is consistent in a sense which especially means that the approximated failure probability converges towards the real failure probability when the discretization becomes ever finer (Schmidt 1989). The approximate calculation of failure probabilities is done here by a set of numerical integrations and matrix multiplications. It has to be emphasized that in particular the numerical effort to calculate the transition matrix $(p_{i,j})$ has to be spent only once for identical transients and its result, the transition matrix, can be used again in any sensitivity study for this component as long as this transient is still involved in the scenario under consideration.

APPLICATION

PWR Primary Cooling Pipe hot leg weld

In this paragraph the maximum stressed circumferential weld of a PWR primary cooling pipe weld is regarded (weld no. 1 in fig. 1). Essential data concerning this pipe are taken from (Bartholomé et al. 1983), a short listing is given below:

geometry: pipe with internal radius R_i = 380 mm and thickness h = 52 mm
material: 20 MnMoNi 55 (A533 B)
failure criterion: net-section stress exceeding flow stress
flow stress: normally distributed with M = 439 MPa and s = 24.6 MPa
crack growth law: cf. (Marshall 1982)
crack growth threshold: $\Delta K_0 = 3.16\ \text{MPa} \cdot \text{m}^{0,5}$
coefficients: m = 3, C lognormally distributed with $C_{50} = 4 \cdot 10^{-11}$ and $C_{90} = 1.6 \cdot 10^{-10}$
initial crack size distribution: depth and depth to length ratio distributions independent with exponential depth distribution (λ = 0.1295/ mm) and shifted exponential depth to length ratio distribution (τ = -1, λ = 1.65)
transients: 200 heat up/cool down cycles



RPV : reactor pressure vessel SG : steam generator MCP : main coolant pump

Figure 1: One of four loops of the primary cooling circuit of a pressurized water reactor with 16 circumferential welds

Stratification and Discretization

The stratification is done in two ways: a uniform partitioning of the depth axis a into m intervals and of the depth to length ratio axis a/b into n intervals (schematically shown in fig. 2) and a non-uniform partitioning, obtained by a heuristic optimization procedure (cf. Schmidt und Schomburg 1985, not shown here).

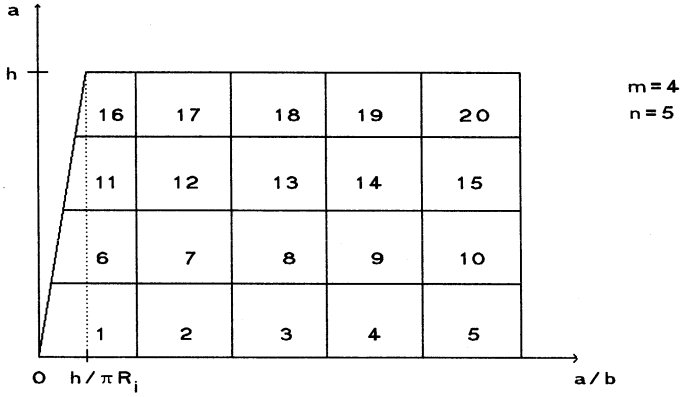


Figure 2: Schematical representation of the stratification in PRAISE

The discretization is performed by a uniform partitioning of the depth axis a into m intervals and of the length axis b into n intervals (cf. fig. 3), where the corner points F, G and B and the partitioning of their connecting lines K, L and U represent individual states.

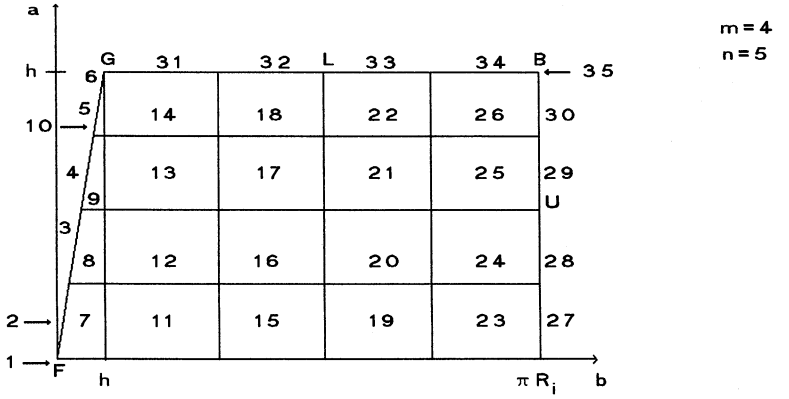


Figure 3: Schematical representation of the discretization in the Markov code

RESULTS

In the following figure the calculated failure probabilities for the considered weld and an assumed component lifetime of 40 years are shown for different stratification and discretization schemes.

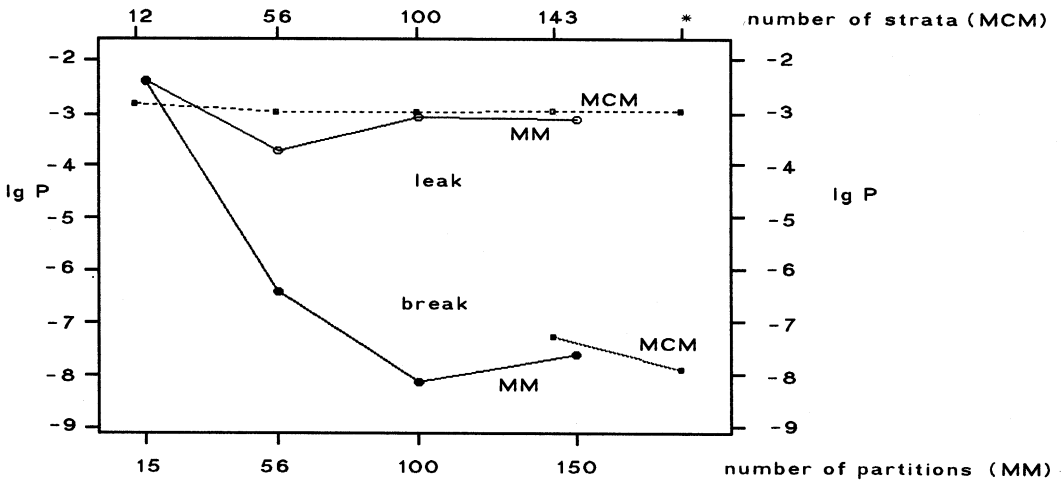


Figure 4 : Leak and break probabilities calculated with the Monte Carlo Method (MCM) and the Markov method (MM). The MCM-results marked with an asterix (*) were calculated using an optimized stratification scheme

Regarding the results it is obvious that the Markov chain approximation needs a certain degree of discretization to provide reliable results. But this is apparently also true for the Monte Carlo method when one looks at the small break probabilities. Here the Markov code gives at least conservative results even for small discretization numbers. On the other hand one may get better results from the Monte Carlo code when careful variance reduction schemes are employed, e.g. by iterative sampling, but this is very costly. With respect to CPU-times there are no significant differences for a one-time use. However, the Markov code spends most of the time for calculating the transition matrices . If these can be re-used in subsequent calculations only little additional time has to be spent (less than 10% of the mentioned time) while with the Monte Carlo code each following run costs about the same additional time as noted above. This advantage becomes even greater when repairs are considered that introduce new cracks in the component (Schmidt 1989). Another advantage not exploited here is that always during the calculation an approximation of the complete crack distribution is available without any extra memory needed and that the expected frequencies of failure can easily be obtained.

CONCLUSIONS

The Monte Carlo simulation is conceptually more simple and wider applicable, although in practice its application may be complicated, e.g. by the necessity to find an appropriate variance reduction scheme. But even with these at hand the computer time consumed in sensitivity studies is found to be usually very large. The Markov process approach is little more difficult to formulate and its discretization to a Markov chain model is quite straightforward, although similar but less difficult problems as in determining the variance reduction scheme may arise. The necessary computer time and accuracy strongly depend on the numerical integration procedures employed for the determination of the transition matrices. However, once the model is constituted, sensitivity studies dealing with different inspection and repair strategies cost significantly less computer time.

Thus approximative Markov methods for reliability assessments are a valuable complement to usual Monte Carlo simulation techniques, but cannot completely replace these. In safety problems at least for cross checking both methods should be employed simultaneously if possible.

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