Assessment of High Temperature Reliability with Probabilistic Fracture Mechanics Methods

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Introduction

Probabilistic methods to assess the reliability of components subjected to different types of loading are well established. Monte Carlo simulations are performed to compute the failure probability as a measure of the reliability of the component under consideration. If highly reliable components are considered, the application of variance reduction methods such as Importance Sampling is required in order to reduce the number of simulations which are necessary to achieve a sufficient degree of accuracy given by the statistical error. These calculations can be very tedious. Approximative methods such as the First Order Reliability Method (FORM) can be shown to yield estimates of the failure probability which deviate only by some 10% from the 'exact' Monte Carlo results. These calculations have been performed for various types of component subjected to static or fatigue loading [9].

Since the First Order Reliability Method is much easier to handle than Monte Carlo simulation with variance reduction, this is an important result, especially for the extended sensitivity studies which are an essential part of a probabilistic reliability study.

In this paper, FORM is used to determine reliability estimates for a component under creep loading at elevated temperatures. To the authors' knowledge, probabilistic methods have not yet been used in the field of high temperature loading. It seems, therefore, very promising to use the experience obtained from application of FORM in the fatigue regime and to demonstrate the benefits of the probabilistic fracture mechanics approach in case of creep loading.

The CEGB procedure published by Ainsworth et.al. [2] for deterministic lifetime assessment of structures operating in the creep range is used as the deterministic failure criterion. A reliability analysis is performed for a model component.

After a brief outline of the principles of probabilistic fracture mechanics and of the CEGB approach [2] for creep failure, the model component is defined. The corresponding fracture mechanical relations and the statistical distributions of the input variables are given. Results obtained with the First Order Reliability Method and the Second Order Reliability Method are compared with each other, and the accuracy of the approximate failure probability determined with these methods is checked with the help of Monte Carlo results.

Basic relations in probabilistic fracture mechanics

In order to apply probabilistic methods, it is necessary to have an adequate deterministic model which allows to decide whether a certain combination of basic variables corresponds to a safe state of the component or to a failure state. This failure criterion will be formulated by means of a failure function, g, which depends on the basic variables $X_1, \dots, X_n$, e.g. crack depth, a, aspect ratio, a/c, loading, σ, etc.. Failure occurs, if $g(x_1, \dots, x_n) < 0$. In the space of the basic variables, X_i, $g(\vec{x}) < 0$ defines the so-called failure domain, **F**. The failure probability, P_f, is given by

$$P_f = \int_{\mathbf{F}} f_1(x_1) \cdots f_n(x_n)\; \mathrm{d}x_1 \cdots \mathrm{d}x_n \tag{1}$$

where $f_1(x_1) \dots f_n(x_n)$ are the probability densities of the basic variables, which, for the sake of simplicity, are assumed to be stochastically independent.

Various methods are used to calculate P_f. The most common procedure is to use the Monte Carlo simulation with variance reduction by importance sampling or stratified sampling (see [3] and references therein).

An approximate solution of eq.(1) can be found by transforming the basic random variables $X_1, \ldots, X_n$ to standard normally distributed random variables $U_1, \ldots, U_n$. The transformed failure surface $g_u(u_1, \ldots, u_n) = 0$ is linearized in a suitably selected point $\vec{u}^*$ which can be determined by an iteration algorithm. The failure probability is then given by the standard normal distribution function $\Phi(-\beta)$ evaluated at the negative value of $\beta = |\vec{u}^*|$. This procedure is called First Order Reliability Method (FORM).

A value of the failure probability P_f which is more accurate than the one obtained with FORM is given by the Second Order Reliability Method (SORM) where the failure surface in the neighbourhood of $\vec{u}^*$ is approximated by a quadratic function.

In [9], it was shown that the accuracy of FORM is sufficiently high for sensitivity studies in probabilistic fracture mechanics. The deviation of the FORM results from the 'exact' Monte Carlo results was shown to be on the order of 10%. Moreover, the difference between FORM and SORM results could be used to estimate, at least qualitatively, the approximation error of the FORM analysis.

The CEGB procedure for lifetime assessment in the creep range

The main feature of the CEGB procedure [2] consists in comparing the desired service life, t_s, with times at which 'significant' events occur. These events are:

- incubation, characterized by the incubation time
  t_i, during which a defect blunts but does not show any significant growth;
- crack growth, characterized by the time
  t_g subsequent to initiation, during which the crack advances until it reaches a final size, a_F;
- continuum creep damage characterized by the time
  t_{CD} at which structural failure occurs by continuum damage mechanisms.

To simplify the subsequent calculations, the incubation time, t_i, is assumed to be zero. Thus, only t_{CD} and t_g have to be considered.

Time for structural failure by continuum damage

The time for failure by continuum damage is given by

$$t_{CD} = t_f(\sigma_{ref}) \tag{2}$$

where $t_f(\sigma_{ref})$ is the rupture time determined from uniaxial rupture data at a stress level equal to the reference stress σ_{ref} and a temperature equal to that of the considered component.

For continuum creep damage, a crack of depth a and length c in a component with the wall thickness w reduces the load carrying cross section but does not introduce any stress singularities ahead of the crack tip. Throughout the lifetime of the component the crack remains stationary. The reference stress is determined by calculating the plastic limit load of the cracked cross section and is given by

$$\sigma_{ref} = \frac{\sigma}{M(a/w,\ a/c)} \tag{3}$$

where $M(a/w,\ a/c)$ is a function depending on the crack configuration and on the loading mode.

Time for crack propagation

The time for crack propagation is given by

$$t_g = \int_{a_0}^{a_F} \frac{1}{\dot{a}} \mathrm{d}a \tag{4}$$

where a_0 and a_F denote the initial and the final defect sizes, respectively, and $\dot{a}$ is the crack propagation rate. For sufficiently long times, creep crack growth can be assumed to be controlled by the C^*-integral:

$$\dot{a} = k\ C^{*\,m} \tag{5}$$

where k and m are material constants. In most steels the exponent m is determined to be close to unity; $m = 1$ is used throughout this paper.

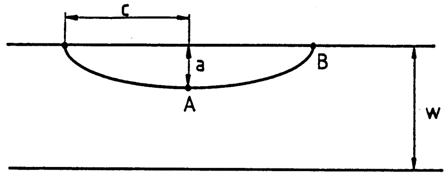

Figure 1. Two-dimensional surface crack.

Deterministic and probabilistic model

The CEGB procedure is applied to determine the failure probability of a model component subjected to creep loading.

In ref. [9], a probabilistic analysis was presented of a pipe elbow of the German fast breeder reactor SNR 300 under fatigue loading. In the present paper, a pipe with corresponding geometry and made of the same material (AISI 304 SS) is used as the model component. The service temperature is assumed to be $T = 973$ K. Thus, part of the input data used in [9] can be used in the probabilistic analysis.

A deterministic description of the component failure has to be given in terms of the failure function g, if FORM or SORM are utilized to calculate the failure probability P_f. Additionally, appropriate fracture mechanical relations have to be selected in order to apply the CEGB procedure to a pipe elbow. The material parameters and the statistical distributions of the basic variables will be summarized in the last part of this section.

Failure by continuum damage

For this failure mode, it is necessary to have creep rupture data and to know the material-independent function $M(.)$ of eq. (3). $M(.)$ is determined from the plastic limit load and is given in [6]

$$M\left(\frac{a}{w}, \frac{a}{c}\right) = \frac{2}{\pi}\left\{\left(1 - \frac{\sigma_u}{\sigma_y}\right)\frac{\sin\delta\cos\delta + \frac{\pi-\delta}{2} - \frac{\sin 2\delta}{4}}{1+\cos\delta} - \frac{a}{w}\sin\delta\right\} \tag{6}$$

where

$$\frac{\sigma_u}{\sigma_y} = 1 - \frac{\left(\pi - \delta\frac{a}{w}\right)(1+\cos\delta)}{(\pi-\delta)\cos\delta + \sin\delta} \tag{7}$$

and σ_u and σ_y denote the outer fibre compressive stress and the yield stress, respectively. The crack angle δ characterizes the length of the crack. Equations (6, 7) hold if $\sigma_u < \sigma_y$.

To obtain creep rupture data for specific temperature levels, a method of Cocks and Ashby [4] is used, which allows creep rupture data to be extrapolated to different temperature levels.

Data obtained by [4] and [11] are shown in Figure 2. A non-linear least-squares fit yields the following relation for a temperature of $T = 973$ K:

$$\lg(t_f) = -5.95 + \sqrt{265 - 100\lg(\sigma)} \tag{8}$$

The failure function g is therefore given by

$$g(a, a/c, \sigma) = t_f\left(\frac{\sigma}{M\left(\frac{a}{w}, \frac{a}{c}\right)}\right) - t_s \tag{9}$$

where w denotes the wall thickness of the pipe.

Failure by creep crack growth

Failure occurs, if the service time, t_s, exceeds the growth time, t_g, for a defect to reach its final depth, a_F. Similarly, the actual crack depth a after the desired service life t_s of a crack with initial depth a_0 and initial aspect

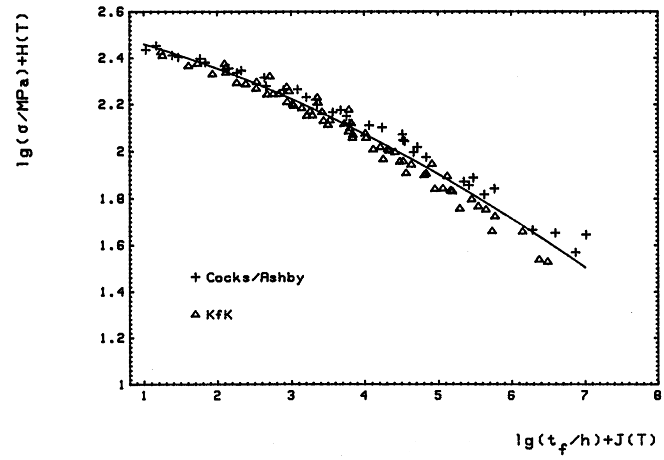

Figure 2. Master curve for creep rupture data: Data taken from [4] (+) and [11] (△).

ratio $(a/c)_0$ can be used in the failure criterion. The final crack depth for failure is assumed to be 80% of the wall thickness.

The failure function g is therefore given by

$$g(a,\ a/c,\ \sigma,\ A,\ k) = 0.8\,w - a(a_0,\ (\frac{a}{c})_0,\ \dots,\ t_s) \tag{10}$$

where the crack depth a after the desired service life t_s has to be determined by integration of a suitable crack growth law. The basic variables A and k will be defined below.

Creep crack growth is controlled by the parameter C^*, which can be estimated [1] by

$$C^* = \dot{\epsilon}_{ref}\,\sigma_{ref}\,R \tag{11}$$

with a characteristic length R which is given by $R = K^2/\sigma_{ref}^2$. The stress intensity factor $K = \sigma\sqrt{\pi a}\ Y(a/w,\ a/c)$ is calculated using a geometric function $Y(.)$ for a pipe under pure bending containing a circumferential crack [5]. The creep strain rate $\dot{\epsilon}_{ref}$ at the reference stress σ_{ref} is calculated using NORTON's law $\dot{\epsilon}_c = A\sigma^n$ with temperature dependent creep parameters A and n.

The creep crack growth law takes the following form:

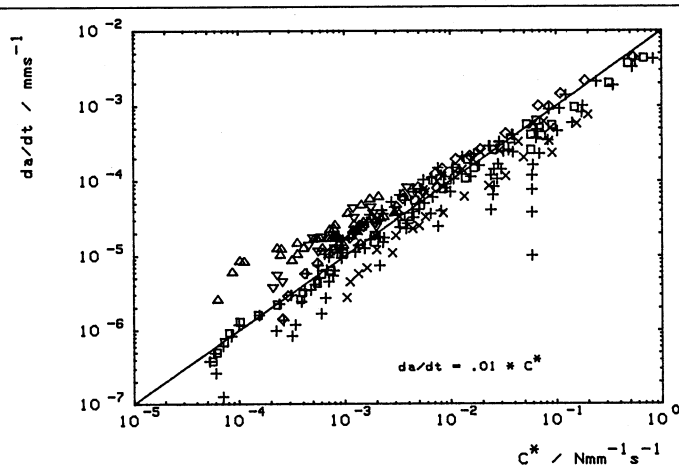

Figure 3. Compilation of creep crack growth data from the literature: (□ ◇ [7], △ ▽ [10], + × [12]).

$$\frac{da}{dt} = k\ C_A^* \qquad \text{and} \qquad \frac{dc}{dt} = k\ C_B^* \tag{12}$$

C_A^* and C_B^* are values of C^* at the deepest point A and at the surface B of the crack (see Figure 1).

The parameter A of the creep law and k of the creep crack growth law are the additional two basic variables of eq. (10).

In Figure 3, a compilation of creep crack growth data from the literature is shown together with the creep crack growth law of eq. (12) with the parameter $k = 0.01$.

Parameter values and distributions of basic variables

The pipe under consideration has an inner radius $R_i = 273.9$ mm and a wall thickness $w = 11$ mm. The service temperature is assumed to be $T = 973$ K. The distributions of the basic variables are:

- crack depth a - exponential distribution with parameter $\Lambda = 1\ \text{mm}^{-1}$,
- aspect ratio a/c - normal distribution with parameter $\mu = .52$ and $\sigma = .18$,
- loading σ - normal distribution with varying mean value and a coefficient of variation of 10%,
- creep parameter A in NORTON's law - lognormal distribution with $\mu = 3 \cdot 10^{-20}$ and $\sigma = 1$, where the unit of the stress is MPa and the strain rate is given in h^{-1},
- creep crack growth parameter k - lognormal distribution with parameters $\mu = 0.01$ and $\sigma = 1$.

The creep exponent in NORTON's law is $n = 10$.

Input variables			Probability of failure by continuum damage			
Service life in h	Parameter Λ of a-distr. in mm^{-1}	Loading σ in MPa	First-Order results	Monte-Carlo results	Second-Order results	Maximum of main curvatures
1000	1	50	$1.88 \cdot 10^{-5}$	$2.01 \cdot 10^{-5}$	$1.97 \cdot 10^{-5}$	0.020
1000	1	60	$3.85 \cdot 10^{-2}$	$3.98 \cdot 10^{-2}$	$3.92 \cdot 10^{-2}$	0.016
1000	0.5	50	$2.02 \cdot 10^{-5}$	$2.74 \cdot 10^{-5}$	$2.68 \cdot 10^{-5}$	0.102
1000	0.5	60	$3.94 \cdot 10^{-2}$	$4.36 \cdot 10^{-2}$	$4.26 \cdot 10^{-2}$	0.066
2000	1	40	$1.35 \cdot 10^{-8}$	$1.45 \cdot 10^{-8}$	$1.45 \cdot 10^{-8}$	0.022
2000	1	50	$7.19 \cdot 10^{-3}$	$7.57 \cdot 10^{-3}$	$7.36 \cdot 10^{-3}$	0.017
2000	0.5	45	$6.78 \cdot 10^{-5}$	-	$8.63 \cdot 10^{-3}$	0.095
2000	0.5	50	$7.43 \cdot 10^{-3}$	-	$8.31 \cdot 10^{-3}$	0.073
5000	1	35	$3.62 \cdot 10^{-7}$	$3.93 \cdot 10^{-7}$	$3.83 \cdot 10^{-7}$	0.021
5000	1	40	$1.02 \cdot 10^{-3}$	$1.05 \cdot 10^{-3}$	$1.05 \cdot 10^{-3}$	0.018
5000	0.5	40	$1.06 \cdot 10^{-3}$	$1.33 \cdot 10^{-3}$	$1.25 \cdot 10^{-3}$	0.082

Table 1. Failure probabilities calculated using the CEGB procedure: First- and Second-Order approximations compared with Monte-Carlo results ($T = 973$ K).

Results

The failure probability was calculated for each of the two failure modes described above. The First Order Reliability Method was used to obtain the dependence of the failure probability on the mean value of the applied loading, σ. For selected values of σ, Monte Carlo calculations were performed to establish confidence in the accuracy of the FORM results. Results are shown in Figure 4 for $t_s = 1000$ h and the two failure modes.

For increasing loading, continuum damage becomes the most important failure mode, whilst for small mean values of σ, failure by creep crack growth becomes dominant. The constant values of the failure probability for creep crack growth at a mean loading of less than about 50 MPa result from a particular feature of the failure criterion eq. (10). The failure function g becomes negative for all cracks with a_0 exceeding $0.8\,w$ so that the contributions to the failure probability coming from creep crack growth at very low stress levels are negligible. The total failure probability follows from:

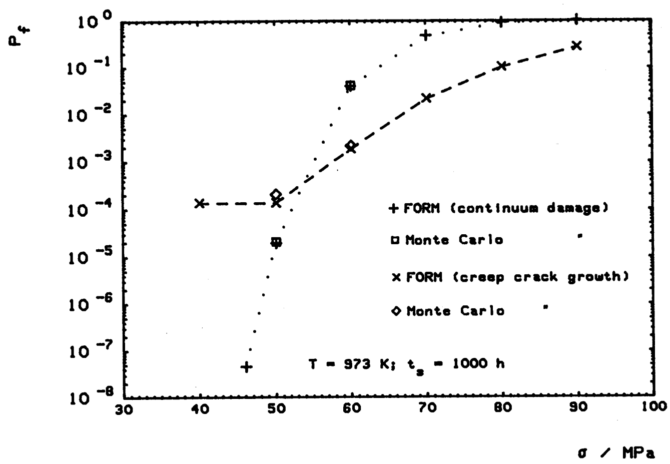

Figure 4. Results for failure after t_s = 1000 h.

$$P_{f,\ total} = 1 - (1 - P_{f,\ continuum\ damage})\,(1 - P_{f,\ crack\ growth}). \tag{13}$$

Figure 4 shows that the Monte Carlo results agree very well with the results obtained by the First Order Reliability Method. Table 1 contains results of Monte Carlo calculations for various combinations of input variables and parameters together with results of FORM and SORM. It can be seen that the deviations of FORM results from the Monte Carlo results are very small in those cases where the difference between FORM and SORM remains small. For slightly larger deviations between FORM and Monte Carlo results, SORM gives a very accurate estimate of the failure probability obtained by Monte Carlo simulation.

Thus, the deviation between FORM and SORM, which can also be expressed in terms of the main curvatures of the failure surface in the design point (see Table 1), gives a qualitative measure of the reliability of FORM results [9]. Hence, the First Order Reliability Method can be used to obtain sufficiently accurate estimates of the failure probability in the case of creep failure as well as in the case of fatigue failure.

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