A Probabilistic Analysis of an Internally Pressurized Cylinder with Axial Cracks

Henk Braam
Netherlands Energy Research Foundation, Petten, The Netherlands

SUMMARY

The computer code REFCO has been developed to quantify the influence of the statistical variation in fracture toughness, the yield stress and the crack geometry on the reliability of a flawed structure. The theory the code is based on and the application to an internally pressurized cylinder are presented in this paper.

1. INTRODUCTION

In a lot of steel structures flaws are existing. These flaws may have originated during fabrication, as a result of a welding process or during operation and can be detected by means of non-destructive examination (NDE). However, it appears that not all defects are detected, which means that even though all detected flaws have been repaired some flaws might still exist in the structure. Besides, the material properties like fracture toughness and yield stress show some scatter. A probabilistic analysis can be performed to gain insight into the influence of these uncertainties on the reliability of a flawed structure, thus providing a tool to quantify safety factors. For this purpose a computer program called REFCO (REliability of Flawed COmponents) has been developed. REFCO calculates the failure probability as a function of load for a flawed component, taking into account the statistical variation in the fracture toughness, the yield stress and the crack geometry. The load is a deterministic quantity, just as the geometry of the considered component. The main objective of REFCO is to assess the influence of the above mentioned quantities on the reliability of the considered flawed component. It is also possible to calculate the failure load as a function of crack length or crack depth assuming only one flaw to be present. At the moment calculations can be performed for a wide plate with a surface defect and an internally or externally axially cracked thick-walled cylinder. This paper gives a description of the theory the computer code is based on and the application to an internally pressurized cylinder.

2. METHOD

To assess the integrity of a flawed structure it is necessary to evaluate the behaviour of these cracks. All possible failure paths a cracked body can follow are shown schematically in figure 1 [1]. The calculation model in REFCO is based, however, on the assumption that the only two failure modes are cleavage fracture and plastic collapse of the remaining ligament, both without prior stable crack growth. Thus the failure paths that can be followed in the current model are the paths 1 and 6 in figure 1. Assuming that the two failure modes, viz. cleavage fracture and plastic collapse, are statistically independent the failure probability of a component with one crack with crack front length s is given by

$$Q_s = Q_f + Q_y - Q_f Q_y \tag{1}$$

where

Q_f: the failure probability as a result of cleavage fracture. Assuming that only cleavage fracture without prior stable crack growth occurs, the failure probability Q_f due to fracture can be obtained from the weakest link model (WLM) [2-4]. According to the WLM:

$$Q_f = 1 - \exp\left(- \frac{\overline{K^m}}{b_s^m}\right) \tag{2}$$

with: $b_s = b \left(\frac{s_t}{s}\right)^{1/m}$

$$\overline{K^m} = \frac{1}{s} \int_o^s \{K(s)\}^m \, ds$$

s : length of crack front
b, m: Weibull parameters obtained from specimens with thickness s_t
K(s): distribution of K along the crack front.

Q_y: the failure probability as a result of plastic collapse of the remaining ligament. The model for plastic collapse is based on the limit load analysis which calculates the maximum load (P_ℓ) that a given structure made of perfectly plastic material could sustain [5]. For the geometries in REFCO the limit load can be given by

$$P_\ell = G_y \sigma_f \tag{3}$$

where G_y is determined by the considered geometry and σ_f is the flow stress or the yield stress. So, the failure probability due to plastic collapse reads

$$Q_y(P) = P_r(P_\ell < P) = P_r(\sigma_f < P/G_y) \tag{4}$$

The last term of eq.(4) corresponds with the definition of the cumulative density function of the flow stress F_y, so

$$Q_y(P) = F_y(P/G_y) \tag{5}$$

To assess the integrity of a flawed structure the statistical variation in the crack geometry has to be taken into account. It is assumed that the cracks at hand are semi-elliptical surface cracks with depth a and depth to length ratio a/c (a/c < 1.0). The probability that one crack leads to failure is given by:

$$Q_1 = \int_o^t f_A(a) \int_o^1 f_C(a/c)\, Q_s(a, a/c)\, d(a/c)\, da \tag{6}$$

The distribution functions $f_A(a)$ and $f_C(a/c)$ are supposed to be independent. Eq. (6) is based on one crack. In case N_o cracks are present the failure probability is given by

$$Q_{N_o} = 1 - \exp(-N_o Q_1) \tag{7}$$

In the literature a number of solutions is given for f_A and f_C describing the as fabricated situation [6]. If the structure is subjected to NDE before entering into service the distribution function has to be adjusted. A lot of information about the nondetection probability of cracks is known from the PISC-studies [7]. The nondetection probability in REFCO is given by

$$P_{ND}(a) = \exp(-\lambda a) \tag{8}$$

In case all detected flaws will be repaired the distribution of cracks when entering into service is given by

$$\tilde{f}_A(a) = \frac{f_A(a)\ P_{ND}(a)}{{}_o\int^t f_A(a)\ P_{ND}(a)\ da} \tag{9}$$

while the number of cracks will reduce to

$$N = N_o\ {}_o\int^t f_A(a)\ P_{ND}(a)\ da \tag{10}$$

3. RESULTS

The computer code REFCO was applied to an internally pressurized thick walled cylinder with inner radius R = 200 mm and wall thickness t = 25 mm. Only internal axial cracks are assumed to exist and the number of cracks in the as fabricated situation is N_o = 10. The applied distribution functions for material properties and crack geometry are given in table 1. Figure 2 shows the failure probability of the considered cylinder as a function of internal pressure P. Besides the total failure probability Q_N the failure probability due to cleavage fracture Q_{Nf} and the failure probability due to plastic collapse Q_{Ny} are presented. It appears that for low values of P the failure is determined by cleavage fracture, hence in the following failure is assumed to occur to cleavage fracture only. The influence of the mean value and the scatter of fracture toughness can be derived from figure 3. As could be expected the failure probability decreases for increasing values of μ, the mean value of fracture toughness and for decreasing values of σ/μ, the coefficient of variation. A specific reliability level can be achieved by a proper combination of μ and σ/μ. From figure 4, showing this relation between μ and σ/μ for P = 22.5 MPa, it can be derived that the influence of the scatter becomes more important for higher values of μ.

The influence of the NDE is shown in figures 5 and 6. λ = 0.041 means that a crack with depth a = 12.5 mm has a nondetection probability of 60%, while the nondetection probability of the same crack is 20 % for λ = 0.129. Figure 6 shows that $|dQ_{Nf}/d\lambda|$ is decreasing with increasing values of μ while this decrease is most strongly for low values of λ. Low values of λ imply a high nondetection probability, or an NDE technique which is not too good. This implies that improving the NDE technique is most successful for materials with a low toughness. For higher values of μ the profit of improving the NDE technique is only small.

4. CONCLUSIONS

The computer code REFCO provides a tool to quantify the influence of statistical variation in fracture toughness, flow stress and crack geometry on the reliability of a flawed structure. For a construction with an assumed number of cracks originated during fabrication a specific reliability level can be achieved by a proper combination of mean value and scatter in the material properties and the nondetection probability. The material parameters depend on the choice of the material, while the applied NDE method determines the nondetection probability. The failure probability due to cleavage fracture decreases with improving NDE technique. However, the quantitative behaviour of this improvement depends on the current value of nondetection probability and the fracture toughness of the material.

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Table 1. Distribution functions

Quantity	Distribution	Mean Value	Coefficient of Variation
Crack Depth 1)	Exponential	1.667 mm	1.0
Crack Length	Normal	0.55 mm	0.327
Flow Stress	Normal	550. MPa	0.05
Fracture Toughness	Weibull	μ MPa√mm	σ/μ

1) As fabricated

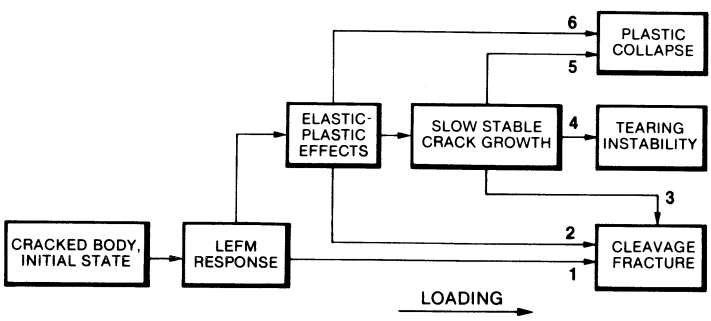

Figure 1: The different failure paths a cracked body can follow [1].

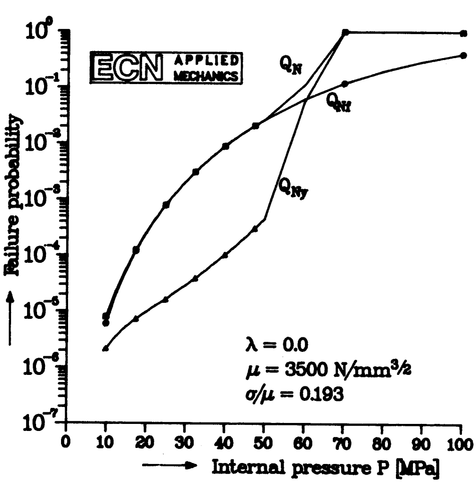

Figure 2: The failure probability of a thick walled pressurized cylinder; Q_N: total; Q_{Nf}: due to cleavage fracture; Q_{Ny}: due to plastic collapse.

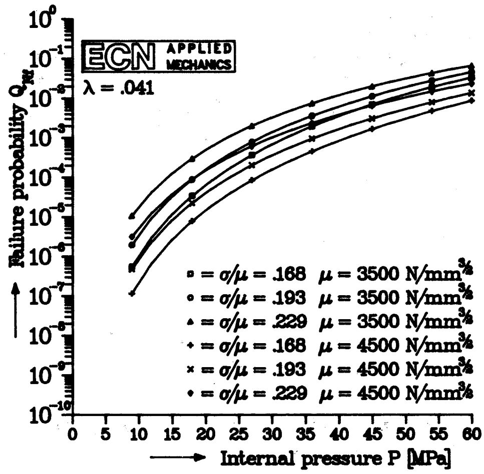

Figure 3: The failure probability for a number of values of μ and σ/μ, the mean value and the coefficient of variation of the fracture toughness.

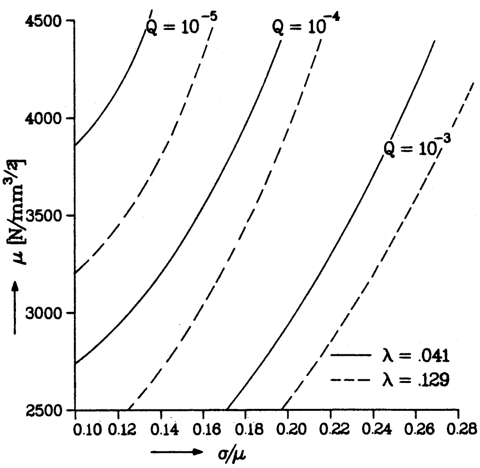

Figure 4: The relation between μ and σ/μ for a specific failure probability level.

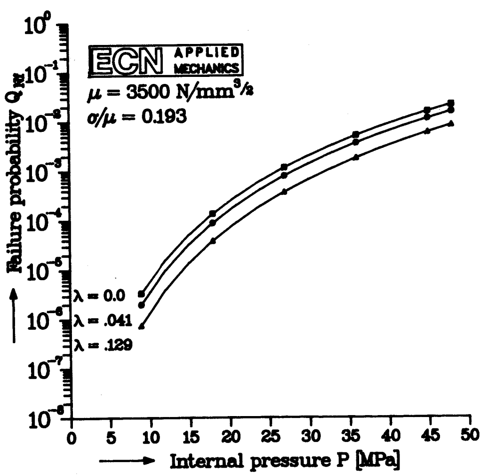

Figure 5: The failure probability as a function of load for a number of nondetection levels.

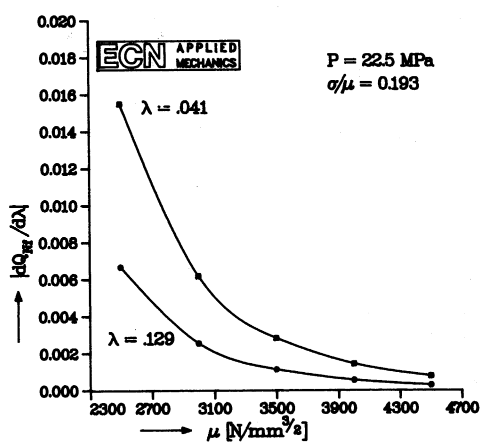

Figure 6: The influence of NDE on the failure probability as a function of mean value of fracture toughness.