Statistical Analysis of Concrete Creep Effects

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INTRODUCTION AND NATURE OF THE PROBLEM

The principal sources of uncertainty in concrete creep effects are the following: 1) uncertainty in the stochastic evolution in time of the mechanism of creep (internal uncertainty); 2) uncertainty in the prediction of the properties of the materials; 3) uncertainty in the stochastic evolution of environmental conditions; 4) uncertainty of the theoretical models; 5) errors of measurement . The random nature of concrete creep (and shrinkage) effects was subject of care not by far : on this subject see the ample review of the chapter 5 of the proceedings of the RILEM Symposium on creep and shrinkage of concrete, 1986. The late beginning of the studies on this subject is perhaps due to their theoretical and computational complexity: nevertheless, since creep and shrinkage affect features of concrete structures as the residual prestressing force in prestressed sections, the stress redistribution in steel-concrete composite beams, deflections and deformations, stress distributions in non-homogenous structures, reactions due to delayed restraints and creep buckling, these studies are very important.This paper is aimed to find the statistics of some of these effects taking into the account the third type of source of uncertainty.

Bazant and his co-workers (Madsen-Bazant, 1983; Bazant-Liu, 1985) looked for the statistics of some concrete creep effects: they neglected the third type of uncertainty, i.e. the stochastic character of environmental conditions, assumed a random, but constant in time value for both temperature and humidity.The temperature and the humidity of the air are the main external variables that influence concrete creep: they stated that, if these quantities have normal statistics as well as the other parameters, the effects have also normal statistics. But perhaps it is not proved that temperature and humidity have normal statistics: it will be investigated here. Moreover their variability in time will be taken into account : temperature and humidity will be schematized as a stationary and periodic stochastic process with period equal to one year; in each month they derive always from the same statistics different from other months . So the stochastic variation in time of the two basical quantities is easily simulated with Montecarlo Techniques.

DETERMINING THE STATISTICS OF TEMPERATURE AND HUMIDITY

A general discourse is not possible in this field: as a matter of fact temperature and humidity vary from a place to another. The records regarding the averages over each month during 30 years at Meteorological Observatory of Venegono Superiore (40 km North from Milan) are here considered: they are subjected to the ordinary stati-

Table 1 : temperature

1	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2	Ex3	Ex1	N*	Ln3	Ex3	B*	Ln	Ln	Ln3	Ln3	B	N
3	Ex3	Ln3	N*	N	B	B*	Ln	N	Ln3	T	N	N
4	1.63	3.37	7.15	11.39	15.40	19.44	21.87	20.74	17.27	11.87	6.46	2.77
5	1.48	1.89	1.69	1.30	1.48	1.22	1.23	1.16	1.36	1.17	0.94	1.23
6	-0.35	-0.95	0.14	0.09	-0.28	0.36	0.61	-0.03	-0.37	-0.66	0.02	-.0005
7	-0.21	1.53	-0.51	-0.44	-0.44	-0.60	0.16	0.05	0.46	1.84	-0.75	-0.27

Legenda - 1: month. 2: best statistics. 3: choice of statistics based on MPPCC test. 4: average (degrees centigrade). 5: stand deviation. 6: skewness (γ_a). 7 : excess. *: best result in 2 tests on 3. B: beta. Ex1: extreme 1st type. Ex2 : extreme 2nd type. Ex3: extreme 3rd type. Ln: lognormal. Ln3 : lognormal with 3 parameters. N: normal. T: Student's t.

Table 2 : humidity

1	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2	B*	B	B*	Ex3	B	Ln	Ex3	Ln3*	Ex3	B	Ex3	Ex3
3	B*	N	B*	N	N	T	N	Ln3*	B	B	B	N
4	72.13	68.72	63.96	62.07	65.41	63.56	61.26	67.20	73.00	76.56	76.19	74.41
5	6.08	8.91	9.92	8.11	7.38	5.08	5.35	4.55	5.25	5.93	5.64	6.94
6	-0.38	-0.23	-0.35	-0.02	-0.04	0.12	-0.43	-0.63	-0.47	-0.18	-0.43	0.03
7	-0.72	-1.09	-0.75	-0.20	-0.79	0.19	-0.03	0.78	-0.36	-0.94	-0.42	-0.53

Humidity in percent.

stical tests, i.e. chi square, Kolmogorov-Smirnov and maximum probability plot correlation coefficient test (Ang-Tang, 1975; Filliben, 1975; Ellingwood, 1984). As a rule it was stated that the statistics prevailing in two tests on three is chosen as statistics of the quantity under examination ; unfortunately in many cases each test suggests a different statistics : in these cases the choice is based on other considerations, as personal judgement , observation of the histograms and of the statistical moments,etc. The results are summarized in the table 1 and 2. It must be outlined that in some cases the statistical tests gave no univocal indications for the choice of the statistics.

THE THEORETICAL MODEL

The analysis is performed using the well-known Bazant-Panula's model for concrete creep and shrinkage (Bazant-Panula, 1978 - 1979):

$$\varepsilon_{sh}(t, t_0) = \varepsilon_{sh\infty} \cdot k_h \cdot S(t, t_0) \quad (1)$$

$$J(t, t') = 1/E_0 + C_0(t, t') + C_d(t, t', t_0) - C_p(t, t', t_0) \quad ; \quad (2)$$

for the meaning of the symbols see the above-cited references: however ε_{sh} is the shrinkage strain, $J(t, t')$ the compliance function where E_0 is a conventional elastic modulus, C_0 gives the basic creep and takes into account the effects due to variations of temperature, C_d represents the increment of creep due to drying and C_p represents the decrease of creep after drying. When the strain history is given and the stress is looked for, the fundamental equation of creep

$$\varepsilon(t) = \varepsilon_{sh}(t) + \int_{t_0}^{t} J(t, t') \cdot d\sigma(t') \tag{3}$$

is a Volterra's integral equation, the solution of which is numerically performed. The simulation is based on Monte Carlo Techniques: in each realization a set of values for the random quantities constant in time is extracted. These quantities are: the content of cement, the water-cement ratio,the gravel-cement ratio and the compressive strength of the concrete . All these variables are listed in table 3 together with the deterministic quantities . All their statistics are normal ones.

The uncertainties in the internal mechanism of creep and in the theoretical model are covered by introducing 3 "prediction errors" (Ang-Tang, 1984) ψ_1, ψ_2 and ψ_3 : ψ_1 affects in multiplicative form the shrinkage, ψ_2 affects $1/E_0 + C_0$ and ψ_3 the difference $C_d - C_p$. Differently from Bazant and his co-workers ψ_1, ψ_2 and ψ_3 take here lognormal statistics: as a matter of fact in Monte Carlo Simulation with normal statistics there would be a small, but not negligible probability of extracting negative and so physically absurd values for those parameters. Each realization include a history of 330 months, i.e. 10000 days about : for each month a value for the average temperature and a value for the average humidity are extracted from the respective statistics; the parameters of the compliance function (2) are revised in consequence. For each example 3 or 5 samples with 10^4 realizations are generated: the results are examined with statistical methods in order to find the statistics relative to the effect under examination.

EXAMPLES AND RESULTS

Two examples are considered in the following : the latter is taken by Bazant and his co-workers for sake of comparison (Madsen-Bazant, 1983; Bazant-Liu,1985).

Specimen subject to uniaxial compression - A concrete specimen (Table 3, I) with uniaxial compressive strength f_c equal to 34.5 N/mm² is loaded at 10.35 N/mm² (0.3 f_c) at the beginning of April: the load is kept constant during 10000 days. The results for the final strain ε for 5 samples each with 10000 realizations are summarized in the Table 4:the results of the former lines are got using the first choice for the statistics of temperature(Table 1) and humidity (Table 2); the latter (in parentheses) are based on the statistics according to MPPCC test. No significant differences between the two sets are observable. In Fig. 1 there is an example of histogram of ε: the relevant skewness and the two modes must be outlined. So a mixed statistics is suggested for ε :

$$f_\varepsilon(\varepsilon) = C \cdot \frac{(\varepsilon - a)^{q-1} \cdot (b - \varepsilon)^{r-1}}{(b - a)^{q+r-1}} \tag{4}$$

In the Eq.4 the parameters take the following values when $0.141186 \cdot 10^{-3} \le \varepsilon \le 0.45 \cdot 10^{-3}$: C = 272.27875, a = $0.141186 \cdot 10^{-3}$, b = $0.45 \cdot 10^{-3}$, q = 4.282583, r =5.290190, while, when $0.45 \cdot 10^{-3} \le \varepsilon \le 0.157412 \cdot 10^{-2}$, we have C = 3.23138, a = $0.45 \cdot 10^{-3}$, b= $0.157412 \cdot 10^{-2}$, q = 1.954030, r = 5.072549. In this case the statistics of ε, that is a mixture of 2 Beta densities, is very far from a normal one.

Three other samples were generated, each with 10000 realizations : in the first with fixed and deterministic temperature and humidity respectively equal to 20 °C and 60% the average strain was of $0.381633 \cdot 10^{-3}$ with $\sigma_\varepsilon = 0.256328 \cdot 10^{-3}$. In the

Table 3 : principal variables

Parameter		h_0	D (mm)	k_s	a_1	c kg/m^3	$\frac{w}{c}$	$\frac{s}{c}$	$\frac{g}{c}$	ψ_1	ψ_2	ψ_3	t_0
I)	E[ ]	1.0	83.(3)	1.55	1.0	450	0.46	1.66	2.07	1	1	1	8
	v	0	0	0	0	0.10	0.10	0.1	0.1	-	0.23	0.13	-
II)	E[ ]	1.0	350	1.00	1.0	385	0.42	2.1	2.7	1	1	1	7
	v	0	0	0	0	0.1	0.1	0	0.1	.14	0.23	0.13	-

Legenda - E[ ] is the operator of average, v the COV. h_0 is the initial relative humidity at which the specimen was in moisture equilibrium . D: effective thickness. k_s: shape factor. a_1: parameter depending on cement type.c: cement in mass. w/c: water/cement. s/c: sand/cement. g/c: gravel/cement . t_0 : age when drying begins. II) refers to the example of the prestressed section.

Table 4 : results for compressed specimen

SAMPLE	$\bar{\varepsilon} \cdot 10^3$	$\sigma_\varepsilon \cdot 10^3$	γ_a	e
1	.497735	.335718	1.0403	-0.3217
	(.495241)	(.334907)	(1.0413)	(-0.3182)
2	.500338	.308958	0.9873	-0.3850
	(.483284)	(.301214)	(0.9937)	(-0.3506)
3	.502101	.308952	0.9667	-0.4448
	(.482491)	(.302363)	(0.9904)	(-0.3608)
4	.497050	.336714	1.0378	-0.3292
	(.488701)	(.304394)	(0.9781)	(-0.3880)
5	.499676	.309961	0.9845	-0.3931
	(.482227)	(.302632)	(0.9896)	(-0.3632)

$\bar{\bar{\varepsilon}} = \Sigma_i \bar{\varepsilon}_i = 0.499380 \cdot 10^{-3}$ ($\bar{\bar{\varepsilon}} = 0.486389 \cdot 10^{-3}$), $\bar{\sigma}_\varepsilon = \Sigma_i \sigma_{\varepsilon i} = 0.320060 \cdot 10^{-3}$ ($0.309102 \cdot 10^{-3}$), $\bar{\gamma}_a = \Sigma_i \gamma_{ai} = 1.0033$ (0.998596), $\bar{e} = \Sigma_i e_i = -0.3747$ (-0.3575) (i = 1 , 5; γ_a = skewness; e = excess).

second sample temperature and humidity took normal statistics with averages of 11.613 °C and 68.706% respectively (averages of the monthly averages) and were kept constant in time: the first 2 moments of the final strain are $\bar{\varepsilon} = 0.498352 \cdot 10^{-3}$, $\sigma_\varepsilon = 0.306990 \cdot 10^{-3}$. In the third sample temperature was N(23°, 2.99°) and humidity N(0.60, 0.078), both constant in time:the results are $\bar{\varepsilon} = 0.400605 \cdot 10^{-3}$, $\sigma_\varepsilon = 0.262400 \cdot 10^{-3}$. The good approximation of the second sample compared with the "exact" results of table 4 is dignous to be noted as well as the first and the third approach, that is equal to Bazant's one, undervalue the final strain . Perhaps the results are rather sensitive to the choice for temperature and humidity.

Loss of prestressing in a prestressed section - The same section considered by Bazant and co-workers (Madsen-Bazant, 1983, p.124, fig.7) was chosen ; there is only a difference in the concrete strength , that takes here the value of 34.5 N/mm^2. In the above cited reference the datum is lacking, while in Bazant and Liu

Table 5 : results for the prestressed section

SAMPLE	$\overline{N}$	σ_N	γ_a	e
1	2.218748	0.521585	-2.4568	5.2608
2	2.217535	0.523875	-2.4549	5.3449
3	2.215453	0.528374	-2.4758	5.3601

$\overline{N}$, σ_N : mega-Newton. $\overline{\overline{N}} = 1/3\ \Sigma_i \overline{N}_i = 2.217245$, $\overline{\sigma}_N = 1/3\ \Sigma_i \sigma_{Ni} = 0.524611$, $\overline{\gamma}_a = 1/3\ \Sigma_i \gamma_{ai} = -2.462521$, $\overline{e} = 1/3\ \Sigma_i e_i = 5.321895$ (i = 1, 3)

(1985) the value 54 N/mm^2 is indicated: the results for this example are strongly different since in the older reference the final prestressing force N is 2.87 MN, in the latter it is 3.49 MN and this value is not substantiated. So a direct comparison is not possible: in this approach N takes lower values partly because of the lower concrete strength. The results of 3 samples with 10000 realizations and the global averaged moments are in Table 5: the results refer to the final value of the prestressing force at 10^4 days. Only 3 samples were generated: as a matter of fact a sample with 10000 realizations requires 55 hours of CPU on a Vax computer! The applied force is 3.6 MN; after the elastic losses we have a mean value of N = 3.015 MN and the final loss on this value is 26.5% (18.4% Madsen-Bazant, 42.9% Bazant-Liu). A curve N versus time is shown in Fig.2. In Fig. 3 there is a plot that is largely and negatively skewed : so the interpolating curve is a compound of 2 Beta densities with negative skewness ; their expression is like Eq. (4) and the parameters are not given for lack of space . the mean coefficient of variation of N at 10^4 days is 0.237 against 0.065 by Madsen-Bazant and 0.044 by Bazant-Liu.

CONCLUDING REMARKS

Some concrete creep effects are here studied from a statistical point of view: the randomness and the variability in time of the principal environmental quantities (temperature and humidity of air) are taken into account. The study leads to the following conclusions: (1) temperature and humidity of air in general have non-normal statistics and only in some cases normal ones. (2) The statistics of the two effects here considered are non-normal ones and considerably skewed. (3) The variability in time of the parameters influencing creep seems to increase the strains. (4) The use of Monte Carlo simulation in order to simulate the variability of temperature and humidity is very expensive, but it is the best tool able to give information on the higher moments of the statistics.

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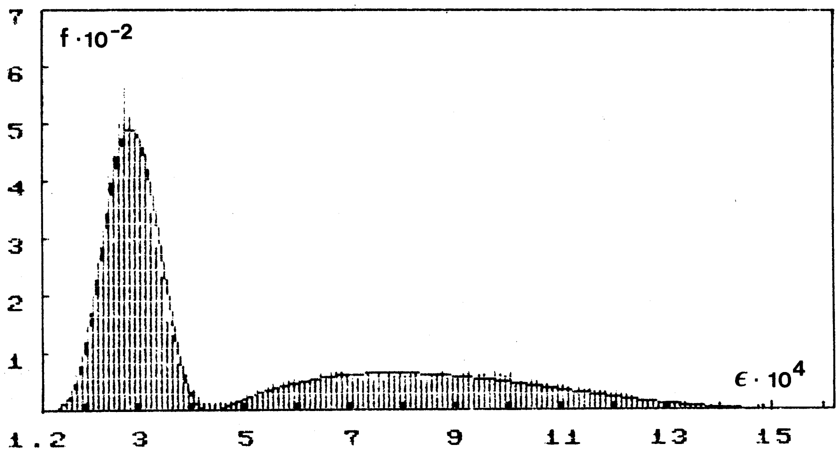

Fig. 1 - Plot of the 1st sample for the compressed specimen; the interpolating curve is given by Eq. 4.

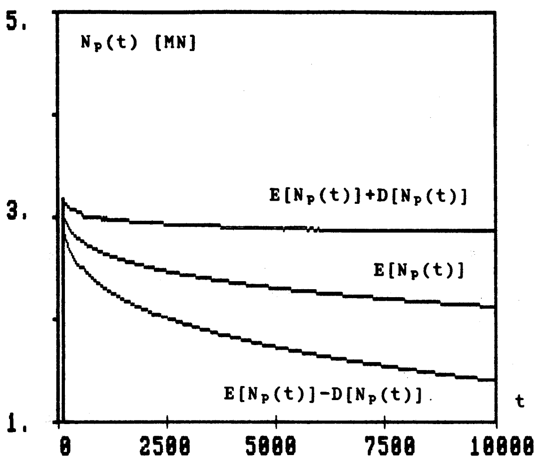

Fig. 2 - Curve prestressing force N_p versus time t (average values). E[ ]: average. D[ ] : standard deviation.

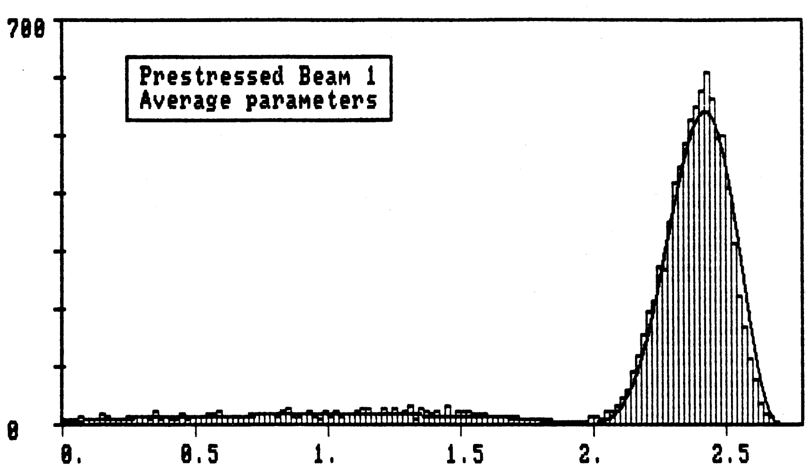

Fig. 3 - Plot of the histogram of the 1st sample (prestressed section). Interpolating curve : two Beta densities.