Random Responses of Thick Plates

Zeng Deshun
Tongji University, Shanghai, PRC
G. Landgraf
Technische Universität Dresden, Dresden, GDR

ABSTRACT

In this paper, the random responses of thick plates are analyzed, in which the thick plates are regarded as continual. The loads are described by random functions. Analytical expressions for dynamic characteristic of thick plates and general expressions for random responses of thick plates are given.

INTRODUCTION

The structures with thick plates have been used extensively in national defence, mechanical engineering, chemical engineering, nuclear engineering, civil engineering, etc. Various theories have been established to deal with the problems of elastic plates, which include the classical theory of thin plates, the improved theory of thick plates, three dimensional elastic theory. The classical theory cannot be expected to hold for plates, whose thicknesses are large with respect to the width. It also cannot be expected to hold when the wave number is large and the distribution of loads is nonuniform. However, it is difficult to obtain the analytical solutions for plates with various boundary conditions on the basis of three-dimensional elastic theory. Recently, more attentions have been paid to the improved theory, in which some of the suppositions in the classical theory are neglected and the effects of rotatory inertia and shear deformation are retained. It is applied more extensively than the classical one.

In this paper, the random responses of thick plates are analyzed, in which the thick plates are regarded as continual, by using the vibration equation of elastic thick plates with external and inner damping, deduced by the authors of this paper. The loads are described by random functions. Analytical expressions for dynamic characteristic of thick plates and general expressions for random responses of thick plates are given. The numerical results of the rectangular plates as examples are given herein. The results are compared with those obtained from the improved theory and the classical theory of plates.

VIBRATION EQUATION OF THICK PLATES WITH DAMPING

The essential viewpoint of Donnell's modification of the classical deflection method for plates is that the deflection w_s caused by the transverse shear strains would be added to the bending deflection w_f obtained from the classical theory for plates, making the total deflection of the middle plane of the plate, i.e.

$$w_t = w_f + w_s \tag{1.1}$$

Then the relationship between w_f and w_t is made

$$w_t=[1-\frac{\alpha h_0^2}{6(1-\nu)}\nabla^2]w_f \tag{1.2}$$

From Hamilton's principle we can obtain the improved vibration equation of elastic thick plates with damping

$$[\rho h_0 L\frac{\partial}{\partial t^2}+(c^*\rho h_0 L+c\nabla^2 D\nabla^2)\frac{\partial}{\partial t}+\nabla^2 D\nabla^2]w_f = [1-\frac{\alpha h_0^2}{6(1-\nu)}\nabla^2]q(x,y,t) \tag{1.3}$$

where, the operators $\nabla^2=\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}$, $L=[1-\frac{\alpha h_0^2}{6(1-\nu)}\nabla^2]^2-\frac{h_0^2}{12}\nabla^2$, c^* and c are the external and inner damping factor respectively, ν is Possion's ratio, h_0 is the thickness of the plate, ρ is the mass per unit volume, D is the flexural rigidity of the plate, the scale factor $\alpha = 3(8-3\nu)/20$, q is the transverse load.

Assuming that the general solution of (1.3) is

$$w_f(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}(x,y,t) \tag{1.4}$$

Where $W_{mn}(x,y,t)=\Phi_{mn}(x,y)Q_{mn}(t)$ (1.5)

Substituting (1.4) and (1.5) into (1.3) and using the orthogonality condition of the modal shape we can obtain

$$Q_{mn}(t)+2\xi_{mn}\omega_{mn}Q_{mn}(t)+\omega_{mn}^2 Q_{mn}(t) = p_{mn}(t) \tag{1.6}$$

where $\xi_{mn}=(c^*+c\omega_{mn}^2)/2\omega_{mn}$ (1.7)

For thick rectangular plates with simply supported edges we have

$$\Phi_{mn}(x,y)=\sqrt{2/a}\ \sin(m\pi x/a)\ \sqrt{2/b}\ \sin(n\pi y/b) \tag{1.8}$$

$$\omega_{mn}=\omega_{mn(c)}\{1+\frac{\pi^2 h_0^2}{a^2}(m^2+\frac{a^2}{b^2}n^2)[\frac{1}{12}+\frac{8-3\nu}{20(1-\nu)} + (\frac{8-3\nu}{40(1-\nu)})^2\frac{\pi^2 h_0^2}{a^2}(m^2+\frac{a^2}{b^2}n^2)]\}^{-1/2} \tag{1.9}$$

$$\omega_{mn(c)}=\frac{\pi^2}{a^2}\sqrt{\frac{D}{\rho h}}\ (m^2+\frac{a^2}{b^2}n^2) \tag{1.10}$$

For thick plates subjected to a concentrated load at (x_p,y_p)

$$q(x,y,t)=P(t)\delta(x-x_p,y-y_p) \tag{1.11}$$

we have

$$p_{mn}(t)=\frac{1}{\rho h_0}\frac{\omega_{mn}^2}{\omega_{mn(c)}^2}f_{mn}(x_p,y_p)P(t) \tag{1.12}$$

where

$$f_{mn}(x,y)=[1-\frac{\alpha h_0^2}{6(1-\nu)}\nabla^2]\Phi_{mn}(x,y) \tag{1.13}$$

ANALYTICAL EXPRESSIONS FOR DYNAMIC CHARACTERISTIC OF THICK PLATES

The frequency response function of $Q_{mn}(t)$ for the concentrated load input P(t) is

$$H_{Qmn}(\omega)=\frac{\frac{1}{\rho h_0}\frac{\omega_{mn}^2}{\omega_{mn(c)}^2}f_{mn}(x_p,y_p)}{(\omega_{mn}^2-\omega^2)+i2\xi_{mn}\omega_{mn}\omega} \tag{2.1}$$

From the Inversion of Fourier Transformation we can obtain the impulse response function of $Q_{mn}(t)$ for the concentrated load input P(t)

$$h_{Qmn}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}H_{Qmn}(\omega)e^{i\omega t}\,d\omega \tag{2.2}$$

The frequency response of $W_{mn}(x,y,t)$ is

$$H_{wmn}(x,y,\omega)=\frac{\frac{1}{\rho h_0}\frac{\omega^2_{mn}}{\omega^2_{mn(c)}}f_{mn}(x_p,y_p)\Phi_{mn}(x,y)}{(\omega^2_{mn}-\omega^2)+i2\xi_{mn}\omega_{mn}\omega} \tag{2.3}$$

The impulse response of $W_{mn}(x,y,t)$ is

$$h_{wmn}(x,y,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_{wmn}(x,y,\omega)e^{i\omega t}d\omega \tag{2.4}$$

The frequency response of $w_t(x,y,t)$ is

$$H_{wt}(x,y,\omega)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\frac{1}{\rho h_0}\frac{\omega^2_{mn}}{\omega^2_{mn(c)}}f_{mn}(x_p,y_p)f_{mn}(x,y)}{(\omega^2_{mn}-\omega^2)+i2\xi_{mn}\omega_{mn}\omega} \tag{2.5}$$

The impulse response of $w_t(x,y,t)$ is

$$h_{wt}(x,y,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_{wt}(x,y,\omega)e^{i\omega t}d\omega \tag{2.6}$$

GENERAL EXPRESSIONS FOR RANDOM RESPONSES OF THICK PLATES

Assuming that loads are described by stationary random functions. The mean value function of $w_t(x,y,t)$ is

$$E[w_t(x,y,t)]=E[P(t-\tau)]\int_{-\infty}^{\infty} h_{wt}(x,y,\tau)d\tau \tag{3.1}$$

or

$$E[w_t(x,y,t)]=E[p(t-\tau)]H_{wt}(x,y,0) \tag{3.2}$$

The spectral density of $w_t(x,y,t)$ at any point (x,y) is

$$S_{w_tw_t}(x,y,\omega)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}[1-\frac{\alpha h_0^2}{6(1-\nu)}\nabla^2]H^*_{w_{mn}}(x,y,\omega)[1-\frac{\alpha h_0^2}{6(1-\nu)}\nabla^2]H^*_{w_{kl}}(x,y,\omega)S_{pp}(\omega) \tag{3.3}$$

The auto correlation function of $w_t(x,y,t)$ at any point (x,y) is

$$R_{w_tw_t}(x,y,\tau)=\int_{-\infty}^{\infty} S_{w_t w_t}(x,y,\omega)e^{i\omega\tau}d\omega \tag{3.4}$$

Similarly, we can obtain the cross spectral density and the cross correlation function of $w_t(x,y,t)$ at any two points (x_1,y_1), (x_2,y_2)

$$S_{w_tw_t}(x_1,y_1,x_2,y_2,\omega)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}[1-\frac{\alpha h_0^2}{6(1-\nu)}\nabla^2]H^*_{w_{mn}}(x_1,y_1,\omega)[1-\frac{\alpha h_0^2}{6(1-\nu)}\nabla^2]H_{w_{kl}}(x_2,y_2,\omega)S_{pp}(\omega) \tag{3.5}$$

$$R_{w_tw_t}(x_1,y_1,x_2,y_2,\tau)=\int_{-\infty}^{\infty} S_{w_t w_t}(x_1,y_1,x_2,y_2,\omega)e^{i\omega\tau}d\omega \tag{3.6}$$

From (3.3)-(3.6) it is evident that

$$S_{w_tw_t}(x_1,y_1,x_2,y_2,\omega)=S_{w_t w_t}(x_2,y_2,x_1,y_1,\omega) \tag{3.7}$$

$$R_{w_tw_t}(x_1,y_1,x_2,y_2,\tau)=R_{w_t w_t}(x_2,y_2,x_1,y_1,\tau) \tag{3.8}$$

The variance functions of $w_t(x,y,t)$ are

$$\sigma^2_{w_tw_t}(x,y)= R_{w_tw_t}(x,y,0) \tag{3.9}$$

$$\sigma^2_{w_tw_t}(x_1,y_1,x_2,y_2)= R_{w_tw_t}(x_1,y_1,x_2,y_2,0) \tag{3.10}$$

NUMERICAL RESULTS AND CONCLUSIONS

The calculations are performed for the rectangular plates with simply supported edges subjected a random load concentrated in the centre of plates. Its spectral density $S_{pp}(\omega)=K$. The modules of elasticity $E=2.06\times 10^5 N/mm^2$, Possion's ratio $\nu = 0.3$, height-span ratios h_o/b = 0.05, 0.1, 0.2, 0.3, the length-width ratios a/b = 1.0, 1.25, 1.50, 1.75, the external damping c' = 0.98×10^{-3}, 1.96×10^{-3}, 2.94×10^{-3}, 4.9×10^{-3} $N\cdot sec/mm^3$ (where $c' = c^* \rho h_o$), the inner damping factor $c = 10^{-4}$, 2×10^{-4}, 3×10^{-4}, 5×10^{-4}sec, The results on $\sigma^2_{w_f w_f}(x,y)$ and $\sigma^2_{w_t w_t}(x,y)$ are given in the tables 1-6. The influences of h/b & a/b on the results are shown in Fig. 1.

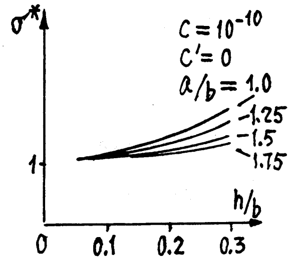

Fig. 1

From the numerical results it is evident that the estimate of the amplitude in the determinate vibration of thick plates can be done from the variance function of deflection, and the analysis should be done by the improved theory when the ratios of height-span and wave number are larger, and the classical theory cannot be expected to hold for the plates.

Table 1 $\frac{\sigma^2_{w_f w_f}(x,y)}{K}$ ($\frac{a}{b} = 1$, $\frac{h_o}{b} = 0.1$, $c = 0$, $c' = 9.8\times 10^{-9}$)

y/b \ x/a	0.0	0.25	0.5	0.75	1.0
0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	1.92×10^{-4}	3.84×10^{-4}	1.92×10^{-4}	0.0
0.5	0.0	3.84×10^{-4}	7.69×10^{-4}	3.84×10^{-4}	0.0
0.75	0.0	1.92×10^{-4}	3.84×10^{-4}	1.92×10^{-4}	0.0
1.0	0.0	0.0	0.0	0.0	0.0

Table 2 $\frac{\sigma^2_{w_f w_f}(x,y)}{K}$ ($\frac{a}{b} = 1$, $\frac{h_o}{b} = 0.2$, $c = 0$, $c' = 9.8\times 10^{-9}$)

y/b \ x/a	0.0	0.25	0.5	0.75	1.0
0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	2.34×10^{-5}	4.67×10^{-5}	2.34×10^{-5}	0.0
0.5	0.0	4.67×10^{-5}	9.34×10^{-5}	4.67×10^{-5}	0.0
0.75	0.0	2.34×10^{-5}	4.67×10^{-5}	2.34×10^{-5}	0.0
1.0	0.0	0.0	0.0	0.0	0.0

Table 3 $\dfrac{\sigma^2_{w_f w_f}(x,y)}{K}$ $\left(\dfrac{a}{b}=1,\ \dfrac{h_0}{b}=0.3,\ c=0,\ c'=9.8\times10^{-9}\right)$

y/b \ x/a	0.0	0.25	0.5	0.75	1.0
0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	6.79×10^{-6}	1.36×10^{-5}	6.79×10^{-6}	0.0
0.5	0.0	1.36×10^{-5}	2.71×10^{-5}	1.36×10^{-5}	0.0
0.75	0.0	6.79×10^{-6}	1.36×10^{-5}	6.79×10^{-6}	0.0
1.0	0.0	0.0	0.0	0.0	0.0

Table 4 $\dfrac{\sigma^2_{w_t w_t}(x,y)}{K}$ $\left(\dfrac{a}{b}=1,\ \dfrac{h_0}{b}=0.1,\ c=0,\ c'=9.8\times10^{-9}\right)$

y/b \ x/a	0.0	0.25	0.5	0.75	1.0
0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	2.27×10^{-4}	4.54×10^{-4}	2.27×10^{-4}	0.0
0.5	0.0	4.54×10^{-4}	9.08×10^{-4}	4.54×10^{-4}	0.0
0.75	0.0	2.27×10^{-4}	4.54×10^{-4}	2.27×10^{-4}	0.0
1.0	0.0	0.0	0.0	0.0	0.0

Table 5 $\dfrac{\sigma^2_{w_t w_t}(x,y)}{K}$ $\left(\dfrac{a}{b}=1,\ \dfrac{h_0}{b}=0.2,\ c=0,\ c'=9.8\times10^{-9}\right)$

y/b \ x/a	0.0	0.25	0.5	0.75	1.0
0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	4.60×10^{-5}	9.20×10^{-5}	4.60×10^{-5}	0.0
0.5	0.0	9.20×10^{-5}	1.84×10^{-4}	9.20×10^{-5}	0.0
0.75	0.0	4.60×10^{-5}	9.20×10^{-5}	4.60×10^{-5}	0.0
1.0	0.0	0.0	0.0	0.0	0.0

Table 6 $\frac{\sigma^2 w_t w_t(x,y)}{K}$ ($\frac{a}{b}=1, \frac{h_0}{b}=0.3$, $c = 0$, $c' = 9.8\times10^{-9}$)

y/b \ x/a	0.0	0.25	0.5	0.75	1.0
0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	2.82×10^{-5}	5.64×10^{-5}	2.82×10^{-5}	0.0
0.5	0.0	5.64×10^{-5}	1.13×10^{-4}	5.64×10^{-5}	0.0
0.75	0.0	2.82×10^{-5}	5.64×10^{-5}	2.82×10^{-5}	0.0
1.0	0.0	0.0	0.0	0.0	0.0

REFERENCES

Mindlin, R.D., Influence of Rotatory Inertia and Shear on Flexural Motions of Isotropic, Elastic Plates, J.of Appl. Mech. Vol.18, No.1, 31-38, 1951.

Timoshenko and Woinowsky-Krieger, Theory of Plates and Shells, McGRAW-HILL Book Company, INC, 1959.

Donnell, L.H., Beams, Plates, and Shells, 1976.

Zeng Deshun, Weng Zhiyuan, Application of Generalized Function to Dynamic Analysis of Thick Plates, Transactions of the 9th International Conference on Structural Mechanics in Reactor Technology, Vol.B,463-468, Lausanne, Switzerland, August 17-21, 1987.

Crandall(Ed.), S.H., Random Vibration, MIT Press, Cambridge, Mass., Vol.1, 1958; Vol.2, 1963.

Lin, Y.K., Probabilistic Theory of Structural Dynamics, McGRAW-HILL INC, 1967.

Zeng Deshun, die dynamische Analyse dicker Platten, Diss. TU Dresden, 1987.