Non-Axisymmetric Response of OH Coils to Lorentz Forces

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INTRODUCTION

Solenoids are often analyzed as homogeneous, axisymmetric structures. In some instances, this is an inadequate representation of the solenoid. The Compact Ignition Tokamak (CIT) ohmic heating solenoid, for instance, is constructed from stacks of thick-turn pancakes. Insulation is placed between metal turns in a pancake and between plates in the stack to electrically isolate individual turns. A stack is formed by placing pancakes on top of each other so that current flows in the same circumferential direction in each pancake, with the transition between pancakes occurring at the inner and outer joints. Such an arrangement is shown in Figure 1. It can be seen from this figure that the solenoid is an axisymmetric structure in an axially integrated sense. The non-axisymmetric response is driven by both the geometry and material composition of individual pancakes and the axial period of the stack.

Motivation for investigating the non-axisymmetric response is provided by the CIT project, which calls for OH coils with a peak central field of $\simeq$ 20 T. The axisymmetric response at this field results in peak stresses in the metal which are quite high. As departure from axisymmetry results in peak stresses above the axisymmetric level, it is desirable to determine accurately the non-axisymmetric response.

Figure 1 illustrates that the number of pancakes forming the basic structural unit depends on the turn loss per pancake; six pancakes form the basic structural unit when each pancake has a sixth-turn loss, for instance. Determination of the non-axisymmetric response necessitates the analysis of this basic unit. Because of the complexity associated with analyzing six or more pancakes, however, each of the analyses of the basic unit is conducted on two pancakes with boundary conditions applied to simulate the presence of other pancakes in the stack.

CALCULATION OF LORENTZ FORCES

The non-axisymmetric behavior of an individual pancake lies in the radial movement of current from one turn to the next. A typical pancake geometry is also shown in Figure 1. Each pancake consists of circular turns joined over a transition angle with a transition or joggle region. Methods for finding the optimum transition region topology are discussed in an accompanying paper (Witt, 1989).

In the circular sections of the pancakes, the Lorentz forces are purely radial; the body force has no moment about the central axis of the solenoid. A resultant radial force occurs because of an absence of one turn in the transition region and because the transition turns experience a lower

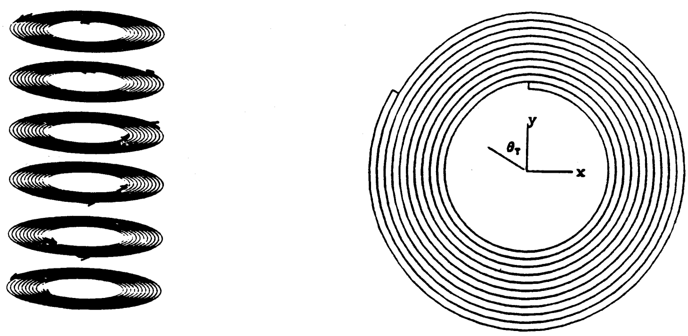

Figure 1 - Stack of Asymmetric Pancakes (Left) and Asymmetric Pancake Topology (Right)

magnetic field than the circular turns. The magnitude of this radial force varies approximately linearly with transition angle. A resultant circumferential force occurs because current is moved radially outward in the transition region. The magnitude of the circumferential force is insensitive to transition angle. To first order, the resultant radial force bisects the transition angle, and the resultant circumferential force is perpendicular to the resultant radial force. The resultant moment, the product of the resultant circumferential force and an average moment arm, is insensitive to transition angle as well.

NON-AXISYMMETRIC RESPONSE OF A DOUBLE PANCAKE

Attempts have been made to model the non-axisymmetric behavior of an isolated pancake, but the results are not meaningful. Unless a contrived set of boundary conditions is imposed on the pancake, the inner and outer joints are required to react the eccentric moment, and this produces highly asymmetric results. Superposition of the isolated pancake deflection profiles would indicate that adjacent pancakes have significantly different displacements. This is known not to be the case, and indicates the important role played by the plate-to-plate insulation in the solenoid. The least complicated analysis which produces reasonable results is based on an analysis of two pancakes with a piece of insulation between them. In this section, results are presented for double pancakes isolated from the rest of the solenoid; results accounting for the presence of the stack are discussed later.

In the double-pancake model, two pancakes are explicitly represented in a finite-element model. The first pancake is represented as in Figure 1, while the second pancake is represented as the reflection of the first pancake about its y-axis. The turn-to-turn insulation is modelled with spring elements. It has been found that modelling the plate-to-plate insulation with springs produces results in the **metal and turn-to-turn insulation** of the pancake similar to those obtained with more sophisticated plate-to-plate representations. The principle shortcoming of the springs is that linear deformation through the thickness is assumed. A second model, in which the plate-to-plate insulation is modelled with three-dimensional brick elements, indicates that deformation through the thickness of the insulation disk is nearly parabolic. The artificial plate-to-plate shear stress distribution obtained from the springs has little impact on the behavior in the metal, however, because the load per unit length from the plate-to-plate shears is, with the exception of the outermost turn, much less than the load per unit length from the electromagnetic forces.

Shear stresses in the plate-to-plate insulation are obtained from the second model. Each pancake is represented as a continuous structure; turn-to-turn insulation is not explicitly included. The plate-to-plate insulation is modelled with orthotropic brick elements, with two such elements through the thickness.

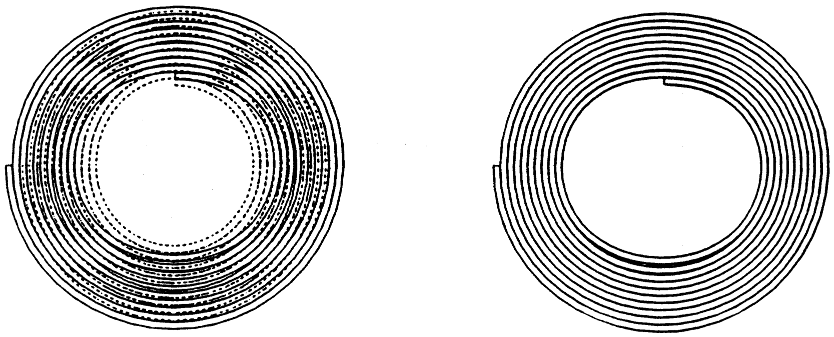

Figure 2 - Double Pancake Deflection Profiles; at Left, Superimposed on Undeflected Shape

The spring stiffness of the insulation is calculated assuming a G10 normal elastic modulus and shear modulus of 14 and 5.5 GPa respectively (Ledbetter, 1980). Turn-to-turn springs are removed where tension develops. The approach of iterative spring removal proves to be more efficient than that of using gaps, which exhibits a relatively slow convergence. The stiffness of the metal is calculated using a Youngs modulus of 168 GPa. The results which follow were obtained for pancakes with a 90° transition angle (quarter-turn loss). Variations of the results with transition angle are described at the end of the section.

Displacements/Stresses in Metal

In each model, the ends of the inner and outer joints are constrained not to rotate; this follows from the symmetry of the connections with adjacent pancakes. No other boundary conditions are prescribed. The deflection profile in Figure 2 shows the movement of one pancake relative to its undeflected shape; for a central field of 20 T, deflections are 1.4 mm at the inside of the first turn across from the outer joint. The deflections of the second pancake are equal to those of the first to within one part in a thousand. This occurs because the plate-to-plate insulation is stiff, and the plate-to-plate shears are very small. Each pancake tries to move away from the other along the x-axis, with the result being a "dilation" of the pancakes in the x-direction; the magnitude of the dilation is on the order of the axisymmetric displacement. This dilation is responsible for local gap distributions in the pancakes. Initial calculations with turn-to-turn tension allowed indicates peak radial tensile stresses would be on the order of 15 MPa.

The axisymmetric response can be estimated very crudely from the theory of thick shells. The equivalent pressure exerted by the magnetic field is $p \simeq \frac{b+a}{2a} \times \frac{B^2}{2\mu_o}$. For 20 T and $b \simeq 2a$, this gives an equivalent pressure of about 240 MPa. From the theory of thick shells, the hoop stresses on the inside and the outside of the shell for $b \simeq 2a$ are $\sigma_i = \frac{5}{3}p$ and $\sigma_o = \frac{2}{3}p$; for the equivalent 240 MPa pressure, the hoop stresses are 400 MPa and 160 MPa respectively. Membrane stresses near these levels are visible in contour plots of the pancakes (not presented here) with peak stresses near 500 MPa.

Stress Distributions in Turn-to-turn Insulation

Figures 3 and 4 illustrate the distributions of turn-to-turn shear and normal stresses. Results are plotted as a function of θ, where θ is defined as the clockwise rotation from the y-axis. The circumferentially-averaged radial stress rises from nearly zero to peak between turns 5/6, then falls back to zero towards the outer turn. There is significant variation in the circumferential direction, however, because of the dilation of the pancakes. Radial stresses tend to be weakly compressive in the top and bottom halves of the plates, while they tend to be strongly compressive across the outer joints. The peak radial tensile stress (if tension

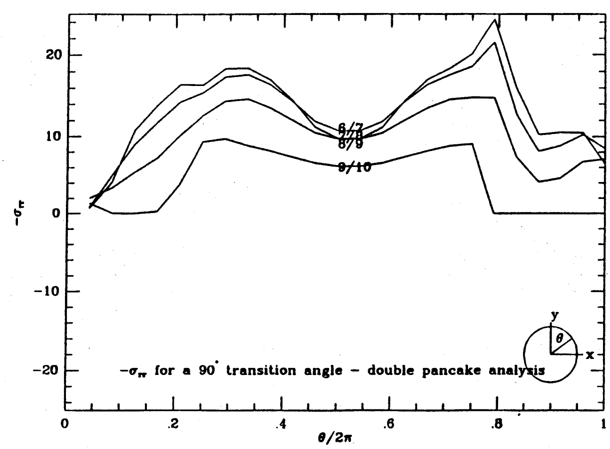

Figure 3 - Radial Stresses Between Turns 6/7-9/10 in Isolated Double Pancake, 90° Transition Angle

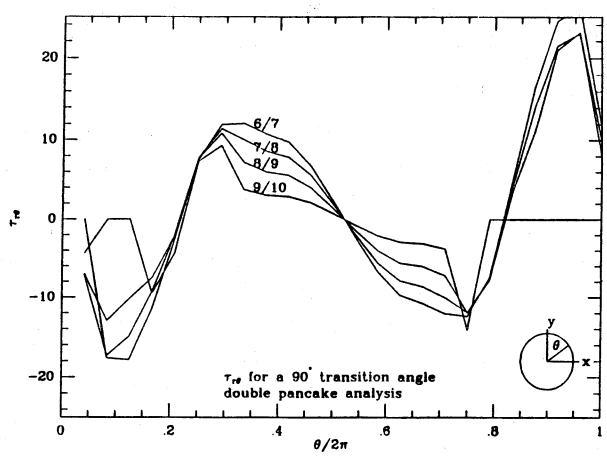

Figure 4 - Shear Stresses Between Turns 6/7-9/10 in Isolated Double Pancake, 90° Transition Angle

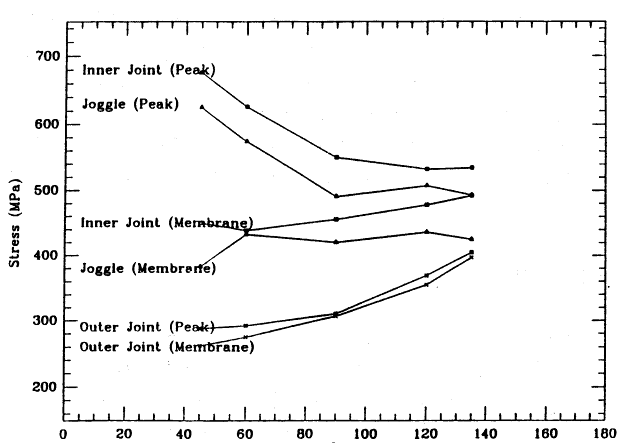

Figure 5 - Peak and Membrane Stresses in Inner Joint, Joggle and Outer Joint vs. θ_T in Isolated Double Pancakes

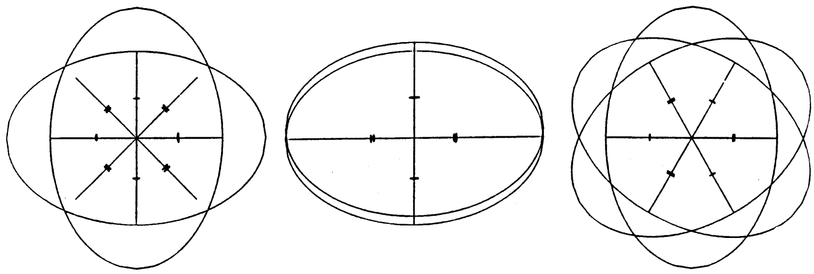

Figure 6 - Superposition of Isolated Double Pancake Deflection Profiles in Stack for 45°, 90° and 60° Transition Angles Respectively

were allowed) is approximately 15 MPa, and occurs where the inner joint tries to separate from the second turn. The peak radial compressive stress is just under 25 MPa and is located between turns 5/6 and 6/7 near the outer joint. Turn-to-turn shear stress distributions are shown in Figure 4, with peak shears of about 25 MPa.

Stress Distributions in Plate-to-Plate Insulation

The $\tau_{z\theta}$ and τ_{zr} distributions in the plate-to-plate insulation are obtained from the second model in which the insulation is modelled explicitly with three-dimensional brick elements. This model generates the same results in the metal as the model with plate-to-plate springs, but generates substantially different results in the plate-to-plate insulation. Results from the three-dimensional representation indicate that the deformation is approximately parabolic. The presence of the plate-to-plate insulation allows the eccentric moment to be reacted over the face of the pancake rather than over the inner and outer joints; because the pancake surface area is large, this is accomplished through very low shear stresses. Average values are a fraction of an MPa, with peak values no larger than 10 MPa.

Variation of Double Pancake Results with Transition Angle

The above results are valid for pancakes with a quarter-turn loss (90° transition angle). When the transition angle is varied, the dilation remains unchanged. The dilation appears to be driven by the eccentric moment, and, since this is a weak function of transition angle, the dilation is as well.

Peak stresses in the metal, on the other hand, are strong functions of transition angle. Figure 5 plots peak stresses in the inner joint, outer joint and innermost joggle regions as functions of transition angle for the double pancake model. As the transition angle decreases from 90°, pronounced increases in peak stress in both the inner joint and the joggle occur.

The turn-to-turn behavior also changes with transition angle. The gap distributions are characterized as subtending a certain angular region and penetrating a certain number of turns into the pancake. As the transition angle decreases, the angular range of the gap distribution around the inner joint decreases. The gaps also penetrate more deeply into the turns around the inner joint. Referring to Figure 3, the distributions of $-1 \times \sigma_{rr}$ rise more sharply from zero, then become a weaker function of angular position around the turn, then fall more sharply to zero as the gap distribution is again approached. The magnitudes of the shears away from the inner joint also decrease from the values shown in Figure 4 as the transition angle decreases, while the peak values just on either side of the gap distributions increase sharply.

NON-AXISYMMETRIC RESPONSE OF A STACK

The above analyses modelled the interaction of two pancakes. A complete analysis requires a number of double pancakes to be coupled together to form the basic structural unit illustrated

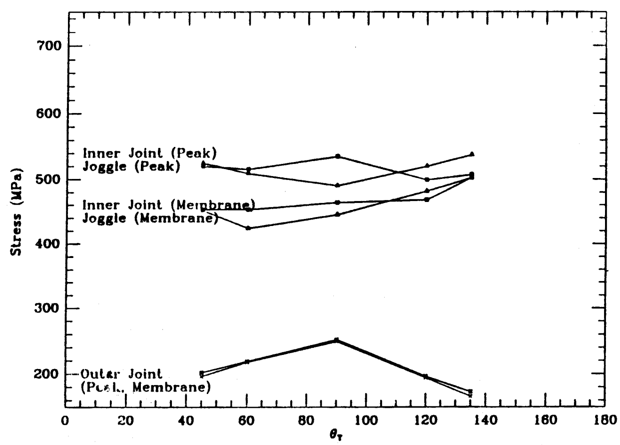

Figure 7 - Peak and Membrane Stresses in Inner Joint, Joggle and Outer Joint vs. θ_T in Double Pancakes with Equivalent Stack Boundary Conditions

in Figure 1. Because a very large number of degrees-of-freedom is involved in such a model, an alternative approach is taken. Figure 6 shows the dilated deflection profiles of double pancakes superimposed on each other to form stacks of 45°, 60° and 90° transition angle pancakes. In the stack of 45° pancakes, each double pancake is rotated 90° from its adjacent neighbors; the major axes of the dilated pancakes are perpendicular to each other. The dilation of the individual double pancakes, when communicated through the very stiff plate-to-plate insulation, tends to interfere destructively, producing a more axisymmetric deformation. In a stack of 90° pancakes, the major axes of adjacent, dilated double pancakes are parallel. Equivalent stack results are obtained from the double pancake model by invoking cyclic symmetry conditions; nodes lying on one radius of one pancake are required to have the same radial deflections as nodes lying on a radius of the second pancake rotated by twice the transition angle. This is shown with the use of tick marks in Figure 6.

Peak stresses in the pancakes with stack boundary conditions now have the values shown in Figure 7. Values are again plotted for the inner joint, outer joint and joggle regions. The solutions with equivalent stack boundary conditions generally yield a less pronounced dependence on transition angle than those of the double pancake model.

CONCLUSIONS

The non-axisymmetric behavior of a solenoid constructed from thick-turn pancakes has been presented. It has been found that an isolated double pancake assumes an oval or dilated deflection profile under the influence of its Lorentz forces, and that peak stresses are a strong function of transition angle. When additional boundary conditions simulating the presence of the stack are prescribed, the dilation characteristic of an isolated double pancake may be either weakly or strongly attenuated, depending on whether the major axes of adjacent, dilated double pancakes are parallel or perpendicular to each other. In the latter case, the dilation of adjacent double pancakes tends to interfere destructively, producing axisymmetric deformation. Peak bending stresses in the joggle and inner joint are weaker functions of transition angle in these cases, with peak values about 20% larger than the axisymmetric membrane stresses.

REFERENCES

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