Structural Analysis of Superconducting Coils for Fusion Reactors Taking into Account the Slip Between Conductors

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ABSTRACT

A finite element formulation of friction effects for multilayered superconductors based on partially bonded beam systems is shown. The stiffness matrix of superconductor elements is derived, which takes into account the effects of friction between conductors. Examples relating to superconductors proposed for the NET toroidal and poloidal field coils show that the effects of slip are not negligible and should be taken into account in a structural analysis of the coil system. Also parametric studies with varying widths and heights of the insulation layer are shown.

INTRODUCTION

Conductors for superconducting coils are usually made up of composite materials and the conducting packages are made of many layers of conductors and insulating material strips.

Since superconducting materials such as Nb_3Sn are strain sensitive, a precise determination of the strain field is of paramount importance especially when operating or fault conditions bring the superconductor close to the critical strain. Such an accurate strain analysis has to take into account the composite nature of the conductors.

The equivalent homogeneity

In stress and strain analyses of composite systems a main objective is to characterize the effective stiffness properties of the heterogeneous media. By effective stiffness properties we mean an average measure of the mechanical characteristics of the material, taking into account the properties of all phases of the heterogeneous media and their interactions. In the most general approach, usually an equivalent homogeneous material is considered, and the effective properties of the heterogeneous media are defined through the tensor C_{ijkl} by means of the relation

$$\langle\sigma_{ij}\rangle = C_{ijkl}\langle\varepsilon_{kl}\rangle \qquad (1)$$

where $\langle\sigma_{ij}\rangle$ and $\langle\varepsilon_{kl}\rangle$ are the average stress and the average strain corresponding to an imposed macroscopically homogeneous stress or deformation field on the representative volume element. Similar volume averaging definitions can be used in the nonlinear case (Cowin, 1977).

Transversely isotropic media

If, as in systems of aligned fibers, a symmetry in the plane normal to that of one main direction (in the example, the fiber direction) can be assumed, the analysis may be simplified. Such media are named transversely isotropic.

The stress strain relationship of such a heterogeneous medium can be written in the following form

$$\{\sigma_i\} = [C_{ij}]\ \{\varepsilon_j\} \tag{2}$$

where

$$\sigma_{11} = C_{11}\ \varepsilon_{11} + C_{12}\ \varepsilon_{22} + C_{12}\ \varepsilon_{33} \tag{3}$$

$$\sigma_{22} = C_{12}\ \varepsilon_{11} + C_{22}\ \varepsilon_{22} + C_{23}\ \varepsilon_{33} \tag{4}$$

$$\sigma_{33} = C_{12}\ \varepsilon_{11} + C_{23}\ \varepsilon_{22} + C_{22}\ \varepsilon_{33} \tag{5}$$

$$\sigma_{12} = 2\ C_{66}\ \varepsilon_{12} \tag{6}$$

$$\sigma_{23} = (C_{22} - C_{23})\ \varepsilon_{23} \tag{7}$$

$$\sigma_{31} = 2\ C_{66}\ \varepsilon_{31} \tag{8}$$

in which five independent constants C_{ij} appear, which characterize the independent properties of the media (Christensen, 1979).

They can be expressed by means of the engineering properties, that are E_{11} (uniaxial modulus), K_{23} (plane strain bulk modulus), $v_{12}=v_{31}$ (Poisson's ratios) and $\mu_{12}=\mu_{31}$ and μ_{23} (shear moduli):

$$C_{11} = E_{11} + 4\ v_{12}^2\ K_{23} \tag{9}$$

$$C_{12} = 2\ K_{23}\ v_{12} \tag{10}$$

$$C_{22} = \mu_{23} + K_{23} \tag{11}$$

$$C_{23} = -\ \mu_{23} + K_{23} \tag{12}$$

$$C_{66} = \mu_{12} \tag{13}$$

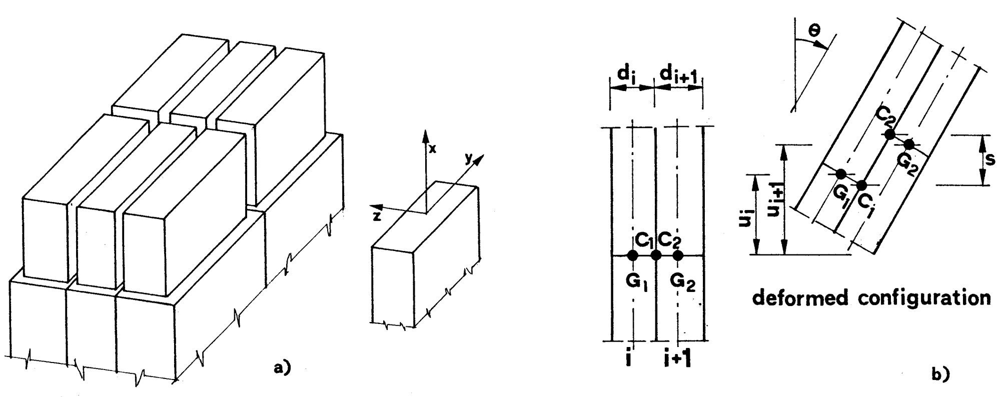

Figure 1. Slip in pancake: a) Pancake element and single conductor reference coordinates system; b) Slip between layers in bending and shear (longitudinal planes xy or xz) or in torsion (transversal plane yz).

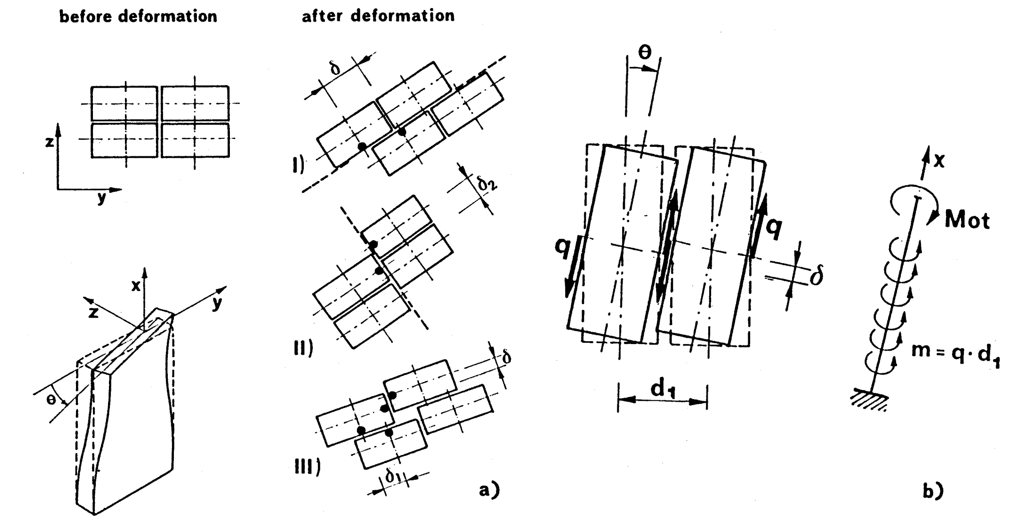

Figure 2. Torsional deformation system: a) Conductor and pancake deformation systems; b) Reaction torsional couples system on a single conductor.

Equations (2) may be inverted

$$\{\varepsilon_i\} = [S_{ij}]\ \{\sigma_j\} \qquad (14)$$

From the symmetry $C_{ij} = C_{ji}$ it follows that $S_{ij} = S_{ji}$. These coefficients may be expressed as

$$S_{11} = 1 / E_{11} \qquad (15)$$

$$S_{12} = - v_{21} / E_{22} \qquad (16)$$

$$S_{22} = S_{33} = 1 / E_{22} \qquad (17)$$

$$S_{23} = - v_{32} / E_{22} \qquad (18)$$

$$S_{44} = S_{66} = 1 / \mu_{12} \qquad (19)$$

$$S_{55} = 1 / (2\ \mu_{23}) \qquad (20)$$

These coefficients will be obtained for superconductor pancakes with epoxy insulation by means of a partially bonded beam element model.

THE PARTIALLY BONDED BEAM ELEMENT MODEL

The partially bonded beam element model, based on the finite element formulation of partially bonded layered beam elements, was developed for shear walls (Chakrabarti, Nayak et al, 1987). It has been successively extended (Gori and Schrefler, 1989) to account for bending, shear and torsion and is applied here to magnet pancakes, made up of many layers of conductors and insulations in two directions. Particular emphasis is given to the behaviour of the pancake under torsion.

In a partially bonded layered beam system, the inter-layer strips at the interfaces introduce additional degrees of freedom to the nodes. The shear that occurs at the interface (in the two tangential directions of the sliding surface) can be adequately represented by a linear relationship between shear stress and slip displacement. This inter-layer slip can be described generally by the displacements in the adjoining layers and the slopes of the partially bonded system at the section, corresponding respectively to the longitudinal or transversal direction of the beam (Fig. 1), by means of the following equation

$$s_i = (u_{i+1} - u_i) + 1/2\ (d_{i+1} + d_i)\ \Theta \qquad (21)$$

in which u_i is generally the component of displacement of the i-th layer in the direction parallel to the sliding direction of s_i. In particular u_i corresponds to the axial displacement of the centroid of the cross-section of the beam for bending and shear deformations, and to the transversal displacement of the centroid for torsional deformation; while Θ represents, respectively, the slope of the transverse displacement component ($\Theta = -\ dw/dx$), and the torsional rotation angle. The basic assumptions made for the finite element formulation of partially bonded beams are: 1) The interface between two adjoining layers constitutes a continuous medium; 2) Strain distributions across the section are linear; 3) In each cross-section, all the layers deflect by the same amount; 4) Buckling of the individual layers does not occur; 5) Slip and shear stress relationship is linear.

Assuming as unknown variables transverse displacement and axial displacement of a layer and the inter-layer slips of all the interfaces, it is possible to build the stiffness matrix of the multi-layered partially bonded layered beam element. This may be evaluated as

$$K^e = \int_L B^T\ D\ B\ ds = \sum_{k=1}^{G} B_k^T\ D_k\ B_k\ \frac{L}{2}\ C_i \qquad (22)$$

APPLICATION TO SUPERCONDUCTING PANCAKES

Superconducting winding pancakes, see for istance (NET status Report, 1987), can be represented as partially bonded layered beam elements. In fact the layers of epoxy insulation connecting adjoining conductors form a series of parallel slip surfaces in two orthogonal directions. In the paper (Gori and Schrefler, 1989), the stiffness coefficients for bending and shear deformations in the two principal directions of the cross section have been derived for a conductor element of a pancake in epoxy insulation. Here torsion is considered in detail.

If we consider a single conductor beam element fixed at one end and subjected to a twisting couple M_{ot} at the other end, assuming the deformation system I) of Fig.2.a), a torsional deformation Θ and a slip δ between two adjoining layers occur at the generic section. The slip, for the antisymmetry, is the same for the two parallel layers (Fig. 2.b) and can be expressed, for equation (21), by

$$\delta = d_1\ \Theta \qquad (23)$$

where d_1 is the pitch between two conductors. The corresponding reaction along the two contact surfaces (Fig. 2.b) is given by

$$q = k_{si}\ \delta \qquad (24)$$

where k_{si} can be expressed, for a continuum joint,in a linear approach (Chakrabarti, Nayak, et al. 1987), by

$$k_{si} = \frac{q_i}{s_i} = \frac{E_e\ w}{2\ Sf\ (1+v_e)\ f} \qquad (25)$$

where f and w are thickness and width of the epoxy joint.

Along the beam, a variable unknown distributed couple m is then applied

$$m = q\ d_1 \qquad (26)$$

and the torsional moment is

$$M_t = M_{ot} - \int_x^l m \, dx \qquad (27)$$

The stress-strain relationship,

$$\frac{d\Theta}{dx} = \frac{M_t}{G \, J_p} \qquad (28)$$

after substituting the previous equations, and imposing the boundary conditions, gives the following solution

$$\Theta = \frac{M_{ot}}{2 \, G \, J_p \, \alpha} \left[e^{\alpha x - \alpha l} - e^{-(\alpha x - \alpha l)} + e^{\alpha l} - e^{-\alpha l} \right] \qquad (29)$$

where

$$\alpha = \left[\frac{k_{si} \, d_1^{\,2}}{G \quad J_p} \right]^{1/2} \qquad (30)$$

From equation (29), the torsional stiffness coefficient may be obtained as

$$\frac{M_{ot}}{\Theta} = \frac{2 \, G \, J_p \, \alpha}{e^{\alpha l} - e^{-\alpha l}} \qquad (31)$$

EXAMPLE

The stiffness coefficient of equation (31) has been evaluated, as an example, for two typical conductor layouts proposed for the NET coils. Their geometrical characteristics are described in (Schrefler and Gori, 1988). One conductor (KfK) has a rectangular section (37.0x16.5 mm), and the other (SIN) an almost square section (26.1x25.2mm). Materials properties of (Marinucci and Was-

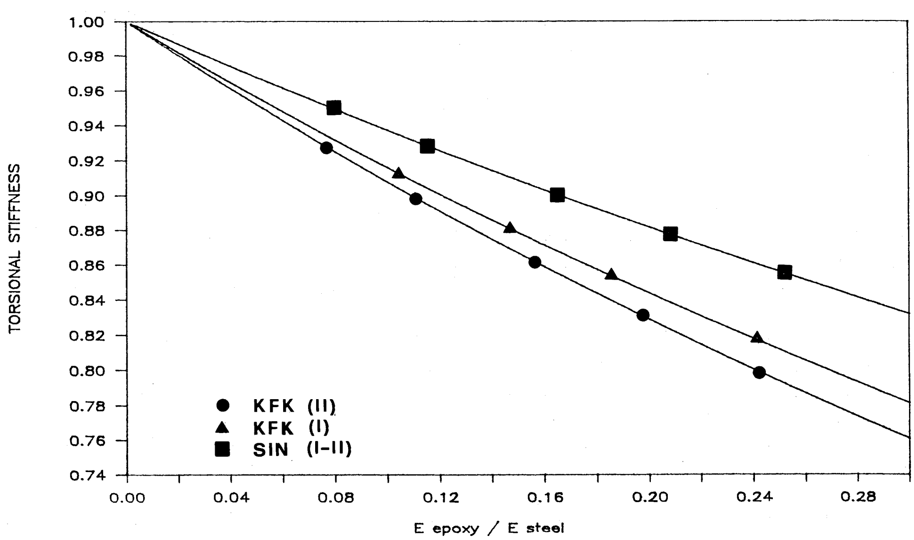

Figure 3. Variation of the torsional stiffness versus the ratio between epoxy and steel Young's moduli.

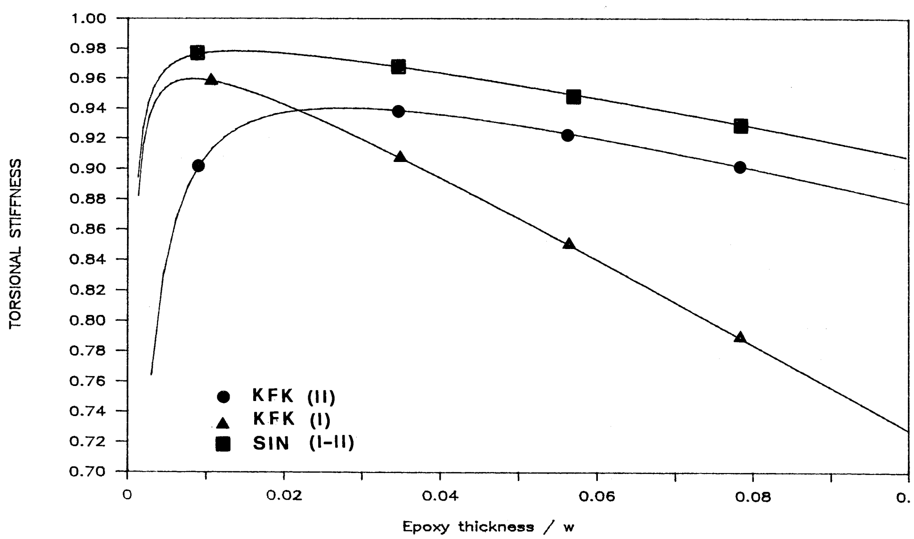

Figure 4. Variation of torsional stiffness versus the ratio between epoxy and conductor thicknesses.

sermann, 1986) have been assumed for the reference cases.

In the reported parametric investigation, the influence of the normal Young's modulus of the epoxy insulation (Fig. 3) and of the thickness of its layer (Fig.4) has been taken into account. For the rectangular conductor two different curves are shown, corresponding to deformation systems I) and II) of Fig. 2.a.

CONCLUSIONS

Examples relating to superconductors proposed for the NET toroidal and poloidal field coils show that the effects of slip and friction between conductors through the epoxy layers should be taken into account in a accurate finite element structural analysis of the coils themselves. The reason is a strong reduction of the torsional stiffness and a possible heating due to the slip itself. For this purpose the partially bonded beam element approach seems very useful.

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