Finite Element Analysis of Magnetically Induced Vibrations of Conductive Plates in TOKAMAK Reactors

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ABSTRACT

An efficient and consistent finite element formulation and its numerical implementation for fully coupled analysis of transient field-structure interactions in conducting plates are developed and corroborated numerically. The Fusion Electromagnetic Induction Experiments(FELIX) are modeled and the results are shown to be in very good agreement with the measured field data.

INTRODUCTION

When a structural component made of conductor is placed in a transient magnetic field, the dynamic behavior of the structure depends on both the mechanical and electromagnetic characteristics of the system. The motion of the structure, which is the product of the Lorentz force due to the field-structure interaction, induces additional eddy currents and further modifies the dynamic characteristics of the system. These coupling effect between the electromagnetic field and response of the structure is of importance in high energy devices. In fusion technology, for example, plasma disruption induces a large eddy eddy current and consequently a considerable amount of mechanical deformation can be generated in the first wall, divertor and limiter in the TOKAMAK. In order to design these structural components safely and economically and ensure the integrity of the whole structure, the field-structure interaction must be fully understood.

The numerical modeling of the mutual field-structure interaction has received little attention until recently. Most of the electromechanical problems treated so far are concerned with "uncoupled" cases only, where the known magnetic field determines the body force and hence the motion and deformation of the structures. Becker and Pillsbury(1976), Miya et al.(1980) and Yuan(1981) are some of the only few cases where the mutual coupling is taken into account in the finite element framework. Bialek and Weissenburger(1986) and Hua et al.(1986) reported their beam models based on a current mesh network method for the electromagnetic aspect and the finite element technique for the mechanical aspect. These studies, however, employ electromagnetic vector potentials, either magnetic or current potential, or current in discrete circuit element as a state variable to represent electromagnetic fields in their formulations resulting in higher order finite elements, loss of numerical accuracy, excessive number of unknowns and complicated boundary conditions. Moreover, the numerical formulations for electromagnetic and mechanical fields are not consistent. In most cases, a numerical model for eddy current calculation is data-linked to an available structural analysis code.

An efficient continuum-based finite element formulation and its numerical procedures for fully coupled analysis of magnetically induced transient vibrations of conductive plates are presented and corroborated numerically in this paper. The proposed finite element model is different from existing numerical models in many respects. First, it is consistent such that the field equations from the theory of magneto-elasticity are used directly in the finite element formulation, and that all finite elements thus derived require only C^o-continuity. Secondly, it employs magnetic field vector as the state variable to overcome many disadvantages shared by the usual potential formulations. Thirdly, this model can be applied to general geometries and field configurations in contrast to other models whose applications are rather restricted to certain fixed geometries and field configurations. A linear, isotropic, homogeneous conductor with a finite conductivity is assumed in the study. It is also assumed that there is no free charge density in the region of interest and the conductor is idle, i.e., carries no load current.

GOVERNING EQUATIONS

Governing equations of magneto-elasticity for nonferrous conductors are as follows :

$$\nabla \times \mathbf{E} + \dot{\mathbf{B}} = 0, \qquad \nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{H} - \dot{\mathbf{D}} = \mathbf{J}, \qquad \nabla \cdot \mathbf{D} = 0 \tag{1}$$
$$\nabla \cdot \mathbf{T} + \rho(\mathbf{f} - \ddot{\mathbf{u}}) + \mathbf{J} \times \mathbf{B} = 0$$

with

$$\mathbf{T} = \mathbf{C} : \mathbf{e}, \qquad \mathbf{e} = (\nabla \mathbf{u} + \mathbf{u}\nabla)/2$$
$$\mathbf{B} = \mu \mathbf{H}, \qquad \mathbf{D} = \varepsilon \mathbf{E}, \qquad \mathbf{J} = \sigma(\mathbf{E} + \dot{\mathbf{u}} \times \mathbf{B}) \tag{2}$$

where **H** is the total magnetic field, **B** the magnetic induction, **E** the electric field, **D** the electric displacement, **J** the current density, **T** the stress tensor, **u** the mechanical displacement, ρ the mass density, σ and ε the electric conductivity and permittivity, respectively, μ the magnetic permeability, **e** the elastic strain tensor, **C** the elasticity tensor. Super dots $\dot{}$, $\ddot{}$ and a colon : denote the first and second partial differentiations with respect to time and the inner product of the second order tensor, respectively. $\mathbf{u}\nabla$ denotes a transpose of the displacement gradient $\nabla\mathbf{u}$.

The total magnetic field intensity **H** is decomposed into two parts, namely, the incident field $\mathbf{H}^o$ and reactive field **h**. Maxwell's equations (1a-d) can then be reduced to a second order equation (wave-type) in terms of the reactive magnetic field vector as

$$\nabla \times \nabla \times \mathbf{h} + \sigma\mu\dot{\mathbf{h}} + \varepsilon\mu\ddot{\mathbf{h}} - \sigma\mu\nabla \times (\dot{\mathbf{u}} \times \mathbf{h}) = 0. \tag{3}$$

Equations (3) are further subject to the divergence-free condition, i.e.,

$$\nabla \cdot \mathbf{h} = 0 \tag{4}$$

to ensure the uniqueness of solution. Thus (1e), (3) and (4) must be solved simultaneously to determine the primary unknowns **h** and **u** under appropriate boundary and initial conditions.

It is noted that the usual quasi-stationary assumption for good conductors is abandoned to render the electromagnetic field equations of wave-type rather than of diffusion-type for computational reason. With both electromagnetic and mechanical field equations being hyperbolic, we can utilize a same hyperbolic driver for both subsystems in transient time integration procedure to enhance the computational efficiency.

SEMI-DISCRETE FINITE ELEMENT EQUATIONS

The weak formulation associated with the initial boundary value problem is obtained by proceeding along standard lines(e.g., Hughes, 1987). In the development, however, the coupled system is divided into two subsystems, namely, the electromagnetic and mechanical subsystem. The C^o-approach is taken in the development of the finite elements for both subsystems.

Electromagnetic Subsystem

The field formulation(Lee and Prevost, 1988) is employed since it has many advantages over the potential or circuit formulations. A penalty method is used in the formulation to embed the divergence-free condition as a constraint.

Mechanical Subsystem

A displacement-type two-dimensional plate element(Hughes, 1987) based on the Reissner-Mindlin plate theory, which accommodates shear deformation, is adopted.

The associated coupled matrix problem may be obtained by discretizing the domain occupied by the conducting solid into the non-overlapping finite elements for each subsystem. Each element is defined by nodal points at which shape functions are prescribed. The Galerkin counterpart of the weak formulation is expressed in terms of the shape functions and will give rise to the coupled nonlinear matrix equations in the following form :

$$\begin{bmatrix} \mathbf{M}^E & 0 \\ 0 & \mathbf{M}^M \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{d}}^E \\ \ddot{\mathbf{d}}^M \end{bmatrix} + \begin{bmatrix} \mathbf{C}^E & \mathbf{C}^{ME} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{d}}^E \\ \dot{\mathbf{d}}^M \end{bmatrix} + \begin{bmatrix} \mathbf{K}^E & 0 \\ \mathbf{K}^{EM} & \mathbf{K}^M \end{bmatrix} \begin{bmatrix} \mathbf{d}^E \\ \mathbf{d}^M \end{bmatrix} = \begin{bmatrix} \mathbf{F}^E \\ \mathbf{F}^M \end{bmatrix} \quad (5)$$

where the coefficient matrices $\mathbf{M}^E$,$\mathbf{C}^E$ and $\mathbf{K}^E$ are equivalent to, respectively, mass, damping and stiffness matrices in dynamic system and the superscripts E and M stand for the electromagnetic and mechanical subsystem respectively. The nonlinear and time dependent coefficient matrices $\mathbf{C}^{ME}$ and $\mathbf{K}^{EM}$ represent the magnetic damping and stiffness due to the mutual field-structure interaction effects, viz., the Lorentz force and velocity effect in Ohm's law respectively. The external force $\mathbf{F}^E$ and $\mathbf{F}^M$ represent the prescribed force for each subsystem. Since we are concerned only with the magnetically induced vibrations, the prescribed mechanical force $\mathbf{F}^M$ may be set to zero.

The coupled nonlinear matrix equation (5) can be, in principle, solved simultaneously as one computational entity. This simultaneous solution procedure, however, requires that large non-symmetric stiffness and damping matrices be formed at each time step. Besides the difficulty and inefficiency of the formation and solution of a larger, non-symmetric matrix equation, the same finite element mesh must be used for both subsystems Since the electromagnetic subsystem requires finer meshes in general, this limitation will result in an excessive number of structural elements.

TIME INTEGRATION PROCEDURE

The magneto-elastic interactions are intrinsically nonlinear effects : as seen in (1)-(3) or in (5), the coupling terms are nonlinear even though material in each subsystem is assumed linear. Special attention has been expended for efficient integration of the nonlinear coupled equations (5).

Partitioned Analysis

By transferring the nonlinear coupling terms which would perturb the smooth functioning of a conventional field analyzer to the right-hand-side as the pseudo-external forces, a partitioned transient analysis procedure (Park and Felippa, 1983) can be utilized. Thus, instead of (5), the following system of equations shall be solved :

$$\begin{bmatrix} \mathbf{M}^E & 0 \\ 0 & \mathbf{M}^M \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{d}}^E \\ \ddot{\mathbf{d}}^M \end{bmatrix} + \begin{bmatrix} \mathbf{C}^E & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{d}}^E \\ \dot{\mathbf{d}}^M \end{bmatrix} + \begin{bmatrix} \mathbf{K}^E & 0 \\ 0 & \mathbf{K}^M \end{bmatrix} \begin{bmatrix} \mathbf{d}^E \\ \mathbf{d}^M \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{F}}^E \\ \overline{\mathbf{F}}^M \end{bmatrix} \quad (6)$$

where all the non-symmetric time-dependent coefficient matrices are now rendered symmetric and constant. The external force vectors $\overline{\mathbf{F}}^E$ and $\overline{\mathbf{F}}^M$ now consist of two parts : the "true" prescribed external force and the pseudo-external force due to the coupling effects from the magneto-elastic interaction.

Implicit two-parameter, one-step predictor-corrector method of Newmark family (Newmark, 1959) is employed for the time integration of each hyperbolic subsystem. Coupled effects are integrated explicitly for each iteration using interpolation between the two different but conformal meshes. The flow chart of the algorithm is shown in Table 1. The solution state of the coupled system is advanced by the parallel execution of coupled matrix equations.

Iterative Technique

Iterations are performed at each time step to ensure numerical stability by balancing out the nonlinear coupling effects. Modified Newton-Raphson(MNR) method is employed for the iteration since the coefficient matrices are constant and do not need to be updated. They are formed and factorized only at the beginning once and for all. In subsequent iterations, the nonlinear coupling effects and hence the out-of-balance forces are calculated using the information (e.g., $\mathbf{d}^E$ and v^M) obtained from the previous iteration and corrections to the solution are computed. Iterations are repeated until the last approximation has converged satisfactorily to the correct solution. The flow chart of the iterative scheme is shown in Table 1.

Coupled Meshes

Different but conformal meshes are utilized for the electromagnetic and mechanical subsystems. The conformity of the two meshes can be achieved, for example, by dividing each bilinear quadrilateral plate element into several electromagnetic elements.

1. Initialize and Form Effective Mass :

$$\mathbf{M}^* = \mathbf{M} + \alpha\Delta t\ \mathbf{C} + \beta\Delta t^2\ \mathbf{K}$$

2. Form Predictors :

$$\tilde{\mathbf{v}}_{n+1} = \mathbf{v}_n + (1-\alpha)\Delta t\,\mathbf{a}_n$$

$$\tilde{\mathbf{d}}_{n+1} = \mathbf{d}_n + \Delta t\ \mathbf{v}_n + \frac{1}{2}(1-2\beta)\Delta t^2\ \mathbf{a}_n$$

3. Start Iteration : $i = 0$

$$\mathbf{d}_{n+1}^{(i)} = \tilde{\mathbf{d}}_{n+1}$$

$$\mathbf{v}_{n+1}^{(i)} = \tilde{\mathbf{v}}_{n+1}$$

$$\mathbf{a}_{n+1}^{(i)} = 0$$

4. Form Out-of-Balance Force :

$$\Delta\mathbf{F}^{E^{(i)}} = \overline{\mathbf{F}}^{E}_{n+1}(\mathbf{d}_{n+1}^{E^{(i)}}, \mathbf{v}_{n+1}^{M^{(i)}}) - \mathbf{M}^E\mathbf{a}_{n+1}^{E^{(i)}} - \mathbf{C}^E\mathbf{v}_{n+1}^{E^{(i)}} - \mathbf{K}^E\mathbf{d}_{n+1}^{E^{(i)}}$$

$$\Delta\mathbf{F}^{M^{(i)}} = \overline{\mathbf{F}}^{M}_{n+1}(\mathbf{d}_{n+1}^{E^{(i)}}) - \mathbf{M}^M\mathbf{a}_{n+1}^{M^{(i)}} - \mathbf{C}^M\mathbf{v}_{n+1}^{M^{(i)}} - \mathbf{K}^M\mathbf{d}_{n+1}^{M^{(i)}}$$

5. Convergence test :

If $\dfrac{|\Delta\mathbf{F}^{(i)}|}{|\Delta\mathbf{F}^{o}|} \le TOL_F$ and $\dfrac{|\Delta\mathbf{d}^{(i)}|}{|\Delta\mathbf{d}^{o}|}\, Q \le (1-Q)\, TOL_D$, GO TO 9 ;

or CONTINUE

where $Q = \max\left[\dfrac{|\Delta\mathbf{d}^{(i)}|}{|\Delta\mathbf{d}^{(i-1)}|}, \dfrac{|\Delta\mathbf{d}^{(i-1)}|}{|\Delta\mathbf{d}^{(i-2)}|}\right]$

6. Solution :

$$\mathbf{M}^*\,\Delta\mathbf{a}^{(i)} = \Delta\mathbf{F}^{(i)}$$

7. Update :

$$\mathbf{a}_{n+1}^{(i+1)} = \mathbf{a}_{n+1}^{(i)} + \Delta\mathbf{a}^{(i)}$$

$$\mathbf{v}_{n+1}^{(i+1)} = \tilde{\mathbf{v}}_{n+1} + \alpha\,\Delta t\ \mathbf{a}_{n+1}^{(i+1)}$$

$$\mathbf{d}_{n+1}^{(i+1)} = \tilde{\mathbf{d}}_{n+1} + \beta\,\Delta t^2\ \mathbf{a}_{n+1}^{(i+1)}$$

8. $i \leftarrow i + 1$; GO TO 4
9. $n \leftarrow n + 1$; GO TO 2

Table 1. Flow chart for coupled hyperbolic system

THE FUSION ELECTROMAGNETIC INDUCTION EXPERIMENTS(FELIX)

Fortran programs have been developed based on (1)-(4) and the numerical procedures described above, and incorporated into a general-purpose finite element code DYNA-FLOW (Prevost, 1983). To examine the applicability of the proposed numerical model, a FELIX problem (Hua et al., 1986) is modeled and the numerical results are compared to the measured field data.

Problem Geometry and Field Configuration

An aluminum plate, 48.7 cm long, 10 cm wide and 3.175 mm thick, is clamped 7.6 cm from one end. The plate is subjected to a constant background field(B_x) of 0.5 tesla in longitudinal direction and driven by a exponentially decaying field in transverse direction. The transverse field is given by $B_y(t) = B_0 \exp(-t/t_d)$ where the initial field intensity B_0 is 0.055 tesla and the decaying time constant t_d is 11.6 milliseconds (Fig. 1).

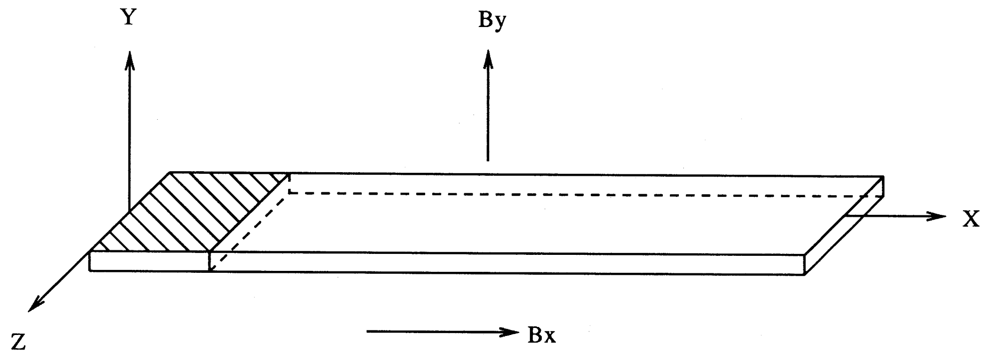

Figure 1 : Geometry and field configuration of the FELIX problem

Finite Element Meshes

The mechanical mesh consists of 12 uniform bilinear quadrilateral plate elements and each node is assigned 3 degrees-of-freedom, a transverse displacement and two rotations. Each plate element is divided into four uniform quasi-3D electromagnetic elements ; thus the electromagnetic mesh consists of 48 elements and each node is assigned 1 degree-of-freedom, the reactive magnetic field in transverse direction.

Implementation

The Newmark parameters in the time integration were chosen as $\alpha = 0.65$ and $\beta = (\alpha+0.5)^2/4 = 0.33$. This choice of α introduces a slight numerical damping ($\alpha = 0.5$ corresponds to no numerical damping) and the selected value for β maximizes high frequency numerical dissipation. The calculation was performed for five hundred constant time steps with $\Delta t = 0.4$ milliseconds. No viscous damping was introduced. The maximum 20 iterations were allowed and the convergence tolerance was set to 10^{-4}.

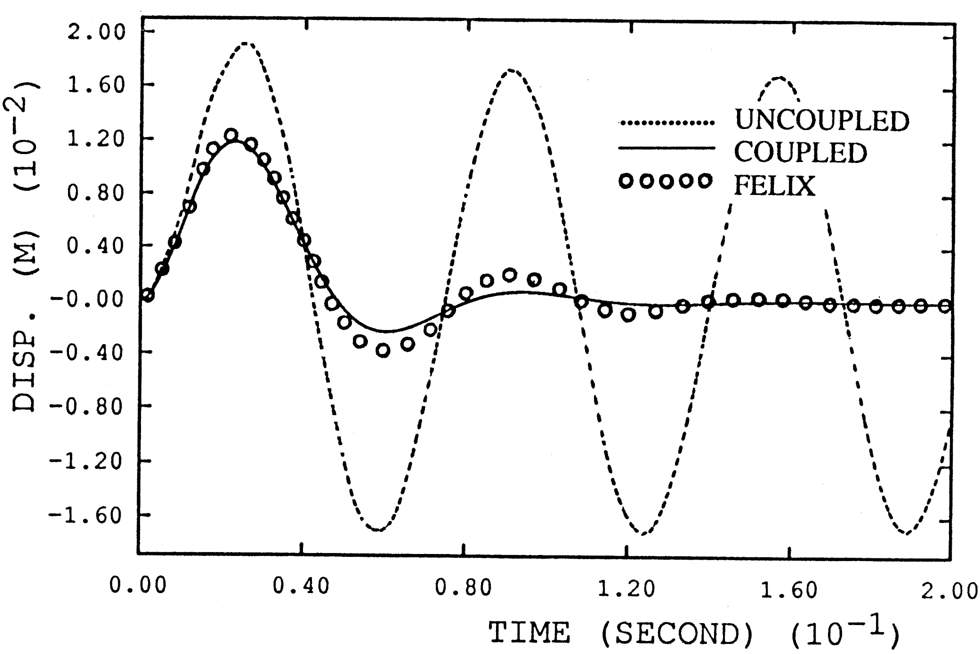

Figure 2 : Time histories of the tip displacement of the FELIX plate
(computed vs. measured)

Results

Computed histories of the responses from the fully coupled analysis are compared with those of the uncoupled analysis. Magneto-elastic interaction has damped the electromagnetic and mechanical responses significantly from the level obtained from the uncoupled analysis. Fig. 2 shows comparison of the mechanical response histories at the tip of the cantilevered plate. Note that the maximum displacement at the tip of the beam from the coupled analysis is about 40 % smaller than that from the uncoupled case. The results from the coupled analysis are shown to be in very good agreement with the measured field data.

CONCLUSIONS

A numerical model for fully coupled analysis of magnetically induced transient vibrations of conducting plates is presented and the coupling effects are examined while validating the proposed model. Numerical results show that the transient responses, both electromagnetic and structural, obtained from the fully coupled analysis are quite different from those of the uncoupled analysis. While the maximum plate deflection and eddy current are reduced considerably due to the magnetic damping from the level obtained from the uncoupled analysis, some local responses are in fact increased due to the field-structure interactions. The change of directions and the branching of the eddy current flows, which can never be predicted in the uncoupled analysis, are also observed in the fully coupled analysis. Numerical results suggests that, in structural components in high energy devices such as the TOKAMAK reactors which are subjected to strong transient magnetic fields, the field-structure interactions are so significant that the uncoupled analysis should never be used. Overall, the proposed finite element formulation together with its numerical procedures provides an efficient and versatile tool for the coupled analysis of field-structure interactions in conductive structures.

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