Transient Thermoelastic In-Plane and Bending Behaviors of Cross-Ply and Angle-Ply Laminated Slabs

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1 INTRODUCTION

In recent years, Metal Matrix Composites have been developed actively as new materials, which may be adaptable for the super high temperature environment for the structural components of fusion reactor or space-plane. Then, as one of the analytical modeling of the composite materials, laminated plates show very characteristic mechanical behavior compared with uni-homogeneous plates, then the thermal stress problems of these laminated plates are considered to be important from analytical viewpoint. However, the reports concerned with these problems are rarely found out. So far as we know, the thermal stress problems of simply supported laminated plates have been reported by Tauchert and his co-worker. These papers (Tauchert et al, 1980) (Wu et al, 1980), however, have treated no more than the thermal stress problems under the steady temperature distribution.

In the present paper, we have analyzed the thermal stress problem of the laminated slabs in a transient state of heating. And, we have assumed the infinitely long laminated slabs consisting of orthogonal or oblique pile of layers having orthotropic material properties, which are corresponded to the so-called cross-ply and angle-ply laminates. And then, we have solved the thermoelastic problem under the condition of uniformly distributed heat supply from upper and/or lower surfaces of the slabs. Introducing a method of Laplace transform to the fundamental equation of temperature field, we have obtained the temperature solution with aid of the residue theorem, and the characteristic behavior of thermal stresses in a transient state are evaluated by using the elementary plate theory. As an illustration, we have carried out numerical calculations for the 5-layered cross-ply and angle-ply laminated slabs.

2 ANALYSIS

We now consider two infinitely long, laminated slabs made of n layers as shown in Fig.1, which thickness is B. We assume that each layer is composed of dissimilar plate with orthotropic material properties in x-y plane. And we consider an cross-ply laminate in which principal axis for each layer is parallel to the x or y axis, and an angle-ply laminate in which the principal axis 1 for i-th layer is alternated with ply-angle θ_{0i} to the x axis. Let b_i be the thickness of the i-th layer, and the origin of a local coordinate z_i is taken at the bottom side of i-th layer.

2.1 TEMPERATURE DISTRIBUTION

We assume that the cross-ply and angle-ply laminated slabs are initially at zero temperature and are suddenly heated uniformly from the lower and upper surface by the surrounding media with their temperature given by T_a and T_b. We denote the relative heat-transfer coefficients on the lower and upper surfaces by h_a and h_b. Then the heat conduction problem becomes to one-dimensional one governed by a coordinate z or z_i. Making use of the local coordinate z_i, the heat conduction equation for i-th layer in dimensionless form is shown as

$$\bar{T}_{i,\tau}=\bar{\kappa}_{zi}\bar{T}_{i,\bar{z}_i\bar{z}_i} \quad ; \; i=1\sim n \tag{1}$$

And, the initial and thermal boundary conditions in dimensionless form are taken in the following forms

$$\tau=0 \quad : \quad \bar{T}_i=0 \quad ; \; i=1\sim n \tag{2}$$

$$\bar{z}_1=0 \quad : \quad \bar{T}_{1,\bar{z}_1}-H_a\bar{T}_1=-H_a\bar{T}_a \tag{3}$$

$$\bar{z}_i=\bar{b}_i,\bar{z}_{i+1}=0 \quad : \quad \bar{T}_i=\bar{T}_{i+1} \tag{4}$$

$$\bar{\lambda}_{zi}\bar{T}_{i,\bar{z}_i}=\bar{\lambda}_{zi+1}\bar{T}_{i+1,\bar{z}_{i+1}} \quad ; \; i=1\sim(n-1) \tag{5}$$

$$\bar{z}_n=\bar{b}_n \quad : \quad \bar{T}_{n,\bar{z}_n}+H_b\bar{T}_n=H_b\bar{T}_b \tag{6}$$

in which a comma denotes partial differentiation with respect to the subscript it follows. In the above expressions (1)-(6), we have introduced the following dimensionless values.

$$\begin{aligned}&\bar{T}_i=T_i/T_0,\; (\bar{T}_a,\bar{T}_b)=(T_a,T_b)/T_0,\; \tau=\kappa_0 t/B^2,\; \bar{\kappa}_{zi}=\kappa_{zi}/\kappa_0\\&(\bar{z},\bar{z}_i)=(z,z_i)/B,\; \bar{b}_i=b_i/B,\; \bar{\lambda}_{zi}=\lambda_{zi}/\lambda_0,\; (H_a,H_b)=(h_a,h_b)B\end{aligned} \tag{7}$$

where, T_i is temperature change, t is time, κ_{zi} is thermal diffusivity in z direction, λ_{zi} is thermal conductivity in z direction. Moreover, T_0, κ_0 and λ_0 are typical values of temperature, thermal diffusivity and thermal conductivity, respectively. Introducing a method of Laplace transform, the solution of equation (1) can be obtained with aid of residue theorem so as to satisfy the conditional equations (2)-(6). The transient part of temperature solution is given as follows:

$$\bar{T}_{iu}=\sum_{j=1}^{\infty}\frac{2exp(-\omega_j^2\tau)}{\omega_j\Delta'(\omega_j)}(\bar{A}_i cos\beta_i\omega_j\bar{z}_i+\bar{B}_i sin\beta_i\omega_j\bar{z}_i) \tag{8}$$

where Δ is a determinant of the $2n\text{x}2n$ matrix $[a_{kl}]$, and the coefficients $\bar{A}_i$ and $\bar{B}_i$ are defined as determinant of the matrix similar to the coefficient matrix $[a_{kl}]$, in which the $(2i-1)$-th column and $2i$-th column are each exchanged by the constant vector $\{c_k\}$. The non-zero elements a_{kl} and c_k among the coefficient matrix $[a_{kl}]$ and the constant vector $\{c_k\}$, are given as follows:

$$\begin{aligned}&a_{11}=-H_a,\; a_{12}=\beta_1\omega,\; a_{2i,2i-1}=cos\beta_i\omega\bar{b}_i,\; a_{2i,2i}=sin\beta_i\omega\bar{b}_i\\&a_{2i,2i+1}=-1,\; a_{2i+1,2i-1}=-\bar{\lambda}_{zi}\beta_i sin\beta_i\omega\bar{b}_i,\; a_{2i+1,2i}=\bar{\lambda}_{zi}\beta_i cos\beta_i\omega\bar{b}_i\\&a_{2i+1,2i+2}=-\bar{\lambda}_{zi+1}\beta_{i+1} \quad ; \; i=1\sim(n-1)\\&a_{2n,2n-1}=H_b cos\beta_n\omega\bar{b}_n-\beta_n\omega sin\beta_n\omega\bar{b}_n\\&a_{2n,2n}=H_b sin\beta_n\omega\bar{b}_n+\beta_n\omega cos\beta_n\omega\bar{b}_n,\; c_1=-H_a\bar{T}_a,\; c_{2n}=H_b\bar{T}_b\end{aligned} \tag{9}$$

where β_i and $\Delta'(\omega_j)$ in Eqs.(8) and (9) are defined as

$$\beta_i=(1/\bar{\kappa}_{zi})^{1/2},\; \Delta'(\omega_j)=d\Delta/d\omega|_{\omega\to\omega_j} \tag{10}$$

and ω_j represents the j-th positive root of the following transcendental equation.

$$\Delta(\omega)=0 \tag{11}$$

On the other hand, the steady part of temperature solution is omitted here for the sake of brevity.

2.2 THERMAL-STRESS ANALYSIS OF CROSS-PLY LAMINATED SLAB

We shall now evaluate the transient thermal stresses of an cross-ply laminated slab by using the elementary plate theory. The stress-strain relations for i-th layer can be written in dimensionless form as

$$\begin{Bmatrix} \bar{\sigma}_{xxi} \\ \bar{\sigma}_{yyi} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11i} & \bar{Q}_{12i} \\ \bar{Q}_{12i} & \bar{Q}_{22i} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_{xxi} - \bar{\alpha}_{xi}\bar{T}_i \\ \bar{\varepsilon}_{yyi} - \bar{\alpha}_{yi}\bar{T}_i \end{Bmatrix} \tag{12}$$

where

$$\bar{Q}_{11i}=\bar{S}_{22i}/D_i,\ \bar{Q}_{12i}=-\bar{S}_{12i}/D_i,\ \bar{Q}_{22i}=\bar{S}_{11i}/D_i,\ D_i=\bar{S}_{11i}\bar{S}_{22i}-\bar{S}_{12i}^{\,2} \tag{13}$$

On the other hand, denoting the strain components at the lower side (z=0) of the laminated slab and the curvatures as $(\varepsilon_{xx}{}^0, \varepsilon_{yy}{}^0)$ and (κ_x, κ_y) respectively, the strains in an arbitrary position of i-th layer in dimensionless form are shown as follows:

$$\bar{\varepsilon}_{xxi} = \bar{\varepsilon}_{xx}{}^0 + \bar{z}\,\bar{\kappa}_x,\ \bar{\varepsilon}_{yyi} = \bar{\varepsilon}_{yy}{}^0 + \bar{z}\,\bar{\kappa}_y \tag{14}$$

In the above expressions (12)-(14), the following dimensionless quantities are introduced.

$$\begin{aligned} &(\bar{\varepsilon}_{kli}, \bar{\varepsilon}_{kl}{}^0) = (\varepsilon_{kli}, \varepsilon_{kl}{}^0)/(\alpha_0 T_0),\ \bar{\sigma}_{kli} = \sigma_{kli}/(\alpha_0 E_0 T_0),\ \bar{S}_{kli} = S_{kli}E_0 \\ &\bar{Q}_{kli} = Q_{kli}/E_0,\ (\bar{\kappa}_x, \bar{\kappa}_y) = (\kappa_x, \kappa_y)B,\ (\bar{\alpha}_{xi}, \bar{\alpha}_{yi}) = (\alpha_{xi}, \alpha_{yi})/\alpha_0 \end{aligned} \tag{15}$$

where ε_{kli} is the strain component, σ_{kli} is the stress component, S_{kli} is the elastic compliance constant, Q_{kli} is the elastic stiffness constant, α_0 and E_0 are the typical values of the coefficients of linear thermal expansion and the Young's modulus of elasticity, respectively. Substituting Eq.(14) into Eq.(12), the thermal stress components of each layer can be written in the following form

$$\begin{Bmatrix} \bar{\sigma}_{xxi} \\ \bar{\sigma}_{yyi} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11i} & \bar{Q}_{12i} \\ \bar{Q}_{12i} & \bar{Q}_{22i} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_{xx}{}^0 + \bar{z}\,\bar{\kappa}_x \\ \bar{\varepsilon}_{yy}{}^0 + \bar{z}\,\bar{\kappa}_y \end{Bmatrix} - \begin{Bmatrix} \bar{\beta}_{xi} \\ \bar{\beta}_{yi} \end{Bmatrix} \bar{T}_i \tag{16}$$

where

$$\bar{\beta}_{xi} = \bar{Q}_{11i}\bar{\alpha}_{xi} + \bar{Q}_{12i}\bar{\alpha}_{yi},\ \bar{\beta}_{yi} = \bar{Q}_{12i}\bar{\alpha}_{xi} + \bar{Q}_{22i}\bar{\alpha}_{yi} \tag{17}$$

The above unknown constants $\bar{\varepsilon}_{xx}{}^0, \bar{\varepsilon}_{yy}{}^0, \bar{\kappa}_x$ and $\bar{\kappa}_y$ can be determined from the conditions of equilibrium of resultant force and resultant moment in x and y directions.

2.3 THERMAL-STRESS ANALYSIS OF ANGLE-PLY LAMINATED SLAB

We can evaluate the transient thermal stresses of an angle-ply laminated slab with the similar manner in the previous section. For i-th layer, the stress-strain relations in the principal direction of orthotropy can be written in dimensionless form as

$$\begin{Bmatrix} \bar{\sigma}_{11i} \\ \bar{\sigma}_{22i} \\ \bar{\sigma}_{12i} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11i} & \bar{Q}_{12i} & 0 \\ \bar{Q}_{12i} & \bar{Q}_{22i} & 0 \\ 0 & 0 & \bar{Q}_{66i} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_{11i} - \bar{\alpha}_{1i}\bar{T}_i \\ \bar{\varepsilon}_{22i} - \bar{\alpha}_{2i}\bar{T}_i \\ \bar{\varepsilon}_{12i} \end{Bmatrix} \tag{18}$$

where

$$\begin{aligned} &\bar{Q}_{11i}=\bar{S}_{22i}/D_i,\ \bar{Q}_{12i}=-\bar{S}_{12i}/D_i,\ \bar{Q}_{22i}=\bar{S}_{11i}/D_i \\ &\bar{Q}_{66i}=1/\bar{S}_{66i},\ D_i=\bar{S}_{11i}\bar{S}_{22i}-\bar{S}_{12i}^{\,2} \end{aligned} \tag{19}$$

Making use of the coordinate transformation, the stress components in x and y directions are

$$\begin{Bmatrix} \bar{\sigma}_{xxi} \\ \bar{\sigma}_{yyi} \\ \bar{\sigma}_{xyi} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11i}^{*} & \bar{Q}_{12i}^{*} & \bar{Q}_{16i}^{*} \\ \bar{Q}_{12i}^{*} & \bar{Q}_{22i}^{*} & \bar{Q}_{26i}^{*} \\ \bar{Q}_{61i}^{*} & \bar{Q}_{62i}^{*} & \bar{Q}_{66i}^{*} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_{xxi}-\bar{\alpha}_{xi}\bar{T}_i \\ \bar{\varepsilon}_{yyi}-\bar{\alpha}_{yi}\bar{T}_i \\ \bar{\varepsilon}_{xyi}-\bar{\alpha}_{xyi}\bar{T}_i/2 \end{Bmatrix} \tag{20}$$

where

$$\begin{aligned}
&\bar{Q}_{11i}^{*}=\bar{Q}_{11i}\bar{m}_i^4+2(\bar{Q}_{12i}+\bar{Q}_{66i})\bar{m}_i^2\bar{n}_i^2+\bar{Q}_{22i}\bar{n}_i^4 \\
&\bar{Q}_{12i}^{*}=(\bar{Q}_{11i}+\bar{Q}_{22i}-2\bar{Q}_{66i})\bar{m}_i^2\bar{n}_i^2+\bar{Q}_{12i}(\bar{m}_i^4+\bar{n}_i^4) \\
&\bar{Q}_{16i}^{*}=2(\bar{Q}_{11i}-\bar{Q}_{12i}-\bar{Q}_{66i})\bar{m}_i^3\bar{n}_i+2(\bar{Q}_{12i}-\bar{Q}_{22i}+\bar{Q}_{66i})\bar{m}_i\bar{n}_i^3 \\
&\bar{Q}_{22i}^{*}=\bar{Q}_{11i}\bar{n}_i^4+2(\bar{Q}_{12i}+\bar{Q}_{66i})\bar{m}_i^2\bar{n}_i^2+\bar{Q}_{22i}\bar{m}_i^4 \\
&\bar{Q}_{26i}^{*}=2(\bar{Q}_{11i}-\bar{Q}_{12i}-\bar{Q}_{66i})\bar{m}_i\bar{n}_i^3+2(\bar{Q}_{12i}-\bar{Q}_{22i}+\bar{Q}_{66i})\bar{m}_i^3\bar{n}_i \\
&\bar{Q}_{66i}^{*}=2(\bar{Q}_{11i}+\bar{Q}_{22i}-2\bar{Q}_{12i}-\bar{Q}_{66i})\bar{m}_i^2\bar{n}_i^2+\bar{Q}_{66i}(\bar{m}_i^4+\bar{n}_i^4) \\
&\bar{Q}_{61i}^{*}=\bar{Q}_{16i}^{*}/2,\ \bar{Q}_{62i}^{*}=\bar{Q}_{26i}^{*}/2 \\
&\bar{\alpha}_{xi}=\bar{\alpha}_{1i}\bar{m}_i^2+\bar{\alpha}_{2i}\bar{n}_i^2,\ \bar{\alpha}_{yi}=\bar{\alpha}_{1i}\bar{n}_i^2+\bar{\alpha}_{2i}\bar{m}_i^2 \\
&\bar{\alpha}_{xyi}/2=(\bar{\alpha}_{1i}-\bar{\alpha}_{2i})\bar{m}_i\bar{n}_i,\ \bar{m}_i=\cos\theta_{0i},\ \bar{n}_i=\sin\theta_{0i}
\end{aligned} \tag{21}$$

On the other hand, the strain components in an arbitrary position of i-th layer are given by the strains at the lower side (z=0) of the laminated slab (ε_{xx}^0, ε_{yy}^0, ε_{xy}^0) and the change of curvatures and twist (κ_x, κ_y, κ_{xy}). Substituting these equations into Eq.(20), the thermal stress components of each layer can be shown in the following form

$$\begin{Bmatrix} \bar{\sigma}_{xxi} \\ \bar{\sigma}_{yyi} \\ \bar{\sigma}_{xyi} \end{Bmatrix} = [\bar{Q}_i^{*}] \begin{Bmatrix} \bar{\varepsilon}_{xx}^0+\bar{z}\ \bar{\kappa}_x-\bar{\alpha}_{xi}\bar{T}_i \\ \bar{\varepsilon}_{yy}^0+\bar{z}\ \bar{\kappa}_y-\bar{\alpha}_{yi}\bar{T}_i \\ \bar{\varepsilon}_{xy}^0+\bar{z}\ \bar{\kappa}_{xy}/2-\bar{\alpha}_{xyi}\bar{T}_i/2 \end{Bmatrix} \tag{22}$$

The above unknown constants $\bar{\varepsilon}_{xx}^0$, $\bar{\varepsilon}_{yy}^0$, $\bar{\varepsilon}_{xy}^0$, $\bar{\kappa}_x$, $\bar{\kappa}_y$ and $\bar{\kappa}_{xy}/2$ are also determined from the conditions of equilibrium of resultant force and resultant moment in x and y directions.

3 NUMERICAL EXAMPLES AND DISCUSSION

As an illustration of numerical calculations, we assume that each layer of laminated slab consists of the same in-plane orthotropic plate, and consider the 5-layered angle-ply laminated slab composed from Al_2O_3 fiber reinforced aluminum composite, in which the principal directions of orthotropy are oriented at $\pm\theta_0$ to the x axis. The numerical results are presented for the following values.

$\bar{T}_a$=1.0, $\bar{T}_b$=1.0 (symmetric heat supply)

$\bar{T}_a$=0.0, $\bar{T}_b$=1.0 (unsymmetric heat supply)

$H_a=H_b=10.0$, $\bar{b}_i=0.2$ ($i=1\sim5$), $\theta_{01}=-\theta_{02}=\theta_{03}=-\theta_{04}=\theta_{05}=\theta_0$

θ_0=0°, 15°, 30°, 45°, 60°, 90°

And the material constants are taken as

$E_{1i}=2.2\times10^2$(GPa)$\equiv E_0$, $E_{2i}=1.38\times10^2$(GPa), $G_{12i}=4.8\times10$(GPa)

$\nu_{12i}=0.244$, $\alpha_{1i}=7.2\times10^{-6}$(1/°C)$\equiv\alpha_0$, $\alpha_{2i}=20.0\times10^{-6}$(1/°C) ; $i=1\sim5$

where E_{1i} and E_{2i} are Young's modulus of elasticity, G_{12i} is shear modulus, ν_{12i} is Poisson's ratio. The results of ply-angle θ_0=45° corespond to the case of cross-ply laminated slab.

The results under the condition of symmetric heat supply are given in Figs.2-4. Fig.2 shows the one-dimensional temperature variation along the layer. The temperature distribution along the layer shows the smooth curve, because the thermal diffusivity of the thickness direction in each layer is the same. Fig.3(a) and (b) show the thickness variations of normal stress $\bar{\sigma}_{xx}$ and shear stress $\bar{\sigma}_{xy}$. From Fig.3(a), as the laminated slab corresponds to the orthotropic plate with a thickness B in the case of ply-angle θ_0=0° or 90°, the stress distribution along the layer become to be continuous. From Fig.3(b), as the ply-angle increase, the distribution of shear stress $\bar{\sigma}_{xy}$ becomes large gradually. And in the case of the ply-angle θ_0=45°, the shear stress $\bar{\sigma}_{xy}$ becomes to maximum. Figs.4(a) and (b) show the variations of radial stress $\bar{\sigma}_{rr}$ and hoop stress $\bar{\sigma}_{\theta\theta}$ on $\theta=\theta_0$. From these figures, as the ply-angle increase, the discontinuity on the interfaces becomes remarkably. On the other hand, the results under the condition of unsymmetric heat supply are given in Figs.5-7. Fig.5 shows the one-dimensional temperature variation along the layer. Figs.6(a) and (b) show the variations of radial stress $\bar{\sigma}_{rr}$ and hoop stress $\bar{\sigma}_{\theta\theta}$ on $\theta=\theta_0$. Fig.7 shows the variation of curvatures κ_x, κ_y and twist κ_{xy} with time. We can be seen that the values of curvatures and twist depend on the ply-angle remarkably.

In the concluding remarks, we may extend our analytical development into the several thermoelastic problems of the anisotropic laminated slab such as simply-supported and non-uniformly heated one.

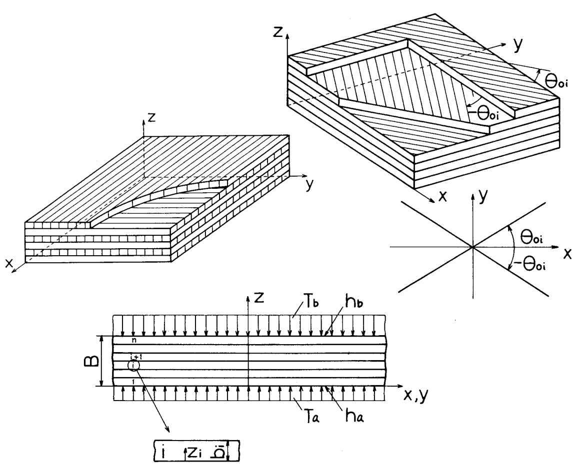


Fig.1. Coordinate system and thermal boundary conditions

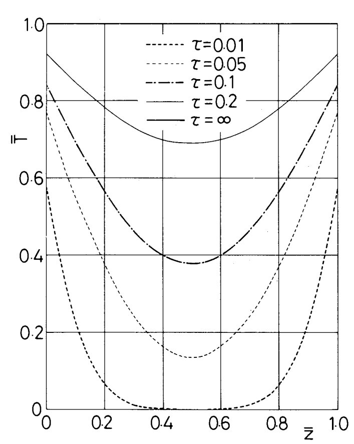
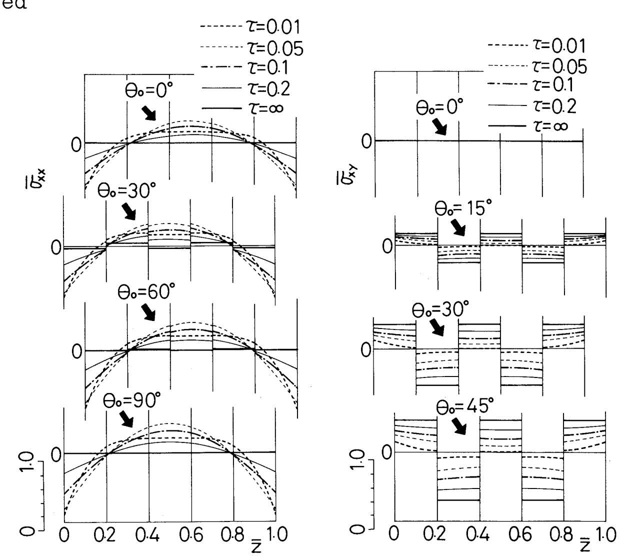
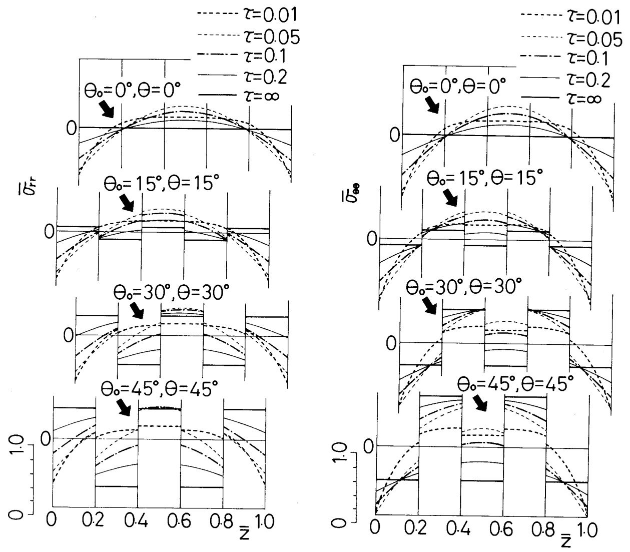


Fig.2. Temperature variation along the layer



(a) stress $\bar{\sigma}_{xx}$ (b) stress $\bar{\sigma}_{xy}$

Fig.3. Thermal stress variations along the layer



(a) stress $\bar{\sigma}_{rr}$ (b) stress $\bar{\sigma}_{\theta\theta}$

Fig.4. Thermal stress variations along the layer

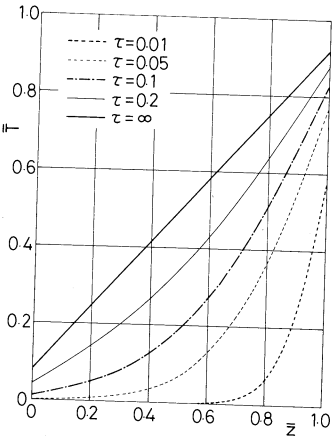
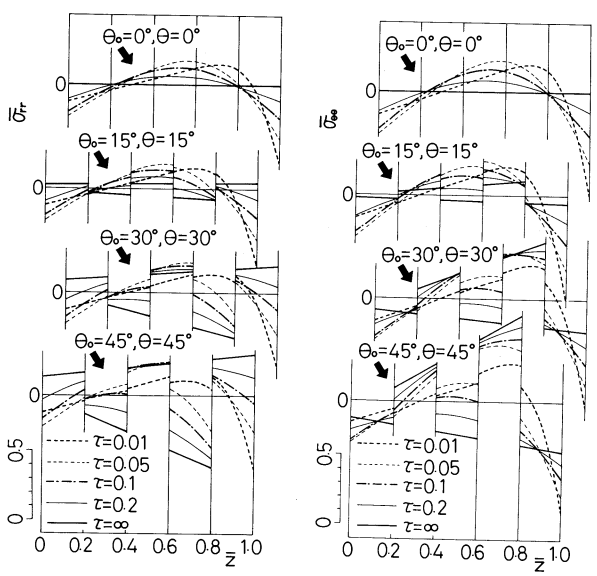


Fig.5. Temperature variation along the layer



(a) stress $\bar{\sigma}_{rr}$ (b) stress $\bar{\sigma}_{\theta\theta}$

Fig.6. Thermal stress variations along the layer

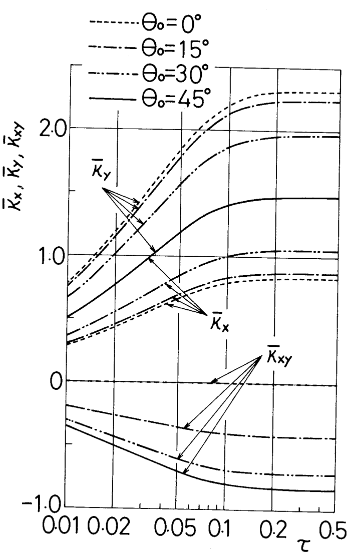


Fig.7. Variation of curvatures and twist with time

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