

# On the Treatment of Dependence in Making Decisions About Risk

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Much attention has been paid to the treatment of dependence in performing probabilistic risk assessments (PRA). For instance, causal dependencies (e.g., common cause failures, cascade failures, and intersystem dependencies) have been taken into account in PRAs beginning with the Reactor Safety Study (USNRC, 1975). In addition, beginning in the early 1980s, attention began to be paid to the issue of probabilistic dependence between the failure rates (Apostolakis and Kaplan, 1981) or seismic fragilities (Kaplan, 1985) of similar components, and the impact of such dependence on risk estimates. By now, it has been clearly demonstrated that failure to take either causal or probabilistic dependence into account in PRAs can lead to misleading results--typically, underestimates of the true risk.

However, there has been little attention to date on the effects of dependence in the area of decision making. The objectives of this paper are (1) to illustrate the potential importance of dependence in making decisions about risks, and (2) to present some ideas on how to communicate the effects of dependence to decision makers in a clear and easily comprehensible manner.

Two particular types of decisions will be of particular concern. The first involves evaluations of risk acceptability for a single technology that is in widespread use; e.g., at multiple facilities, or by multiple people. The second type of decision involves comparisons of two or more alternative technologies; e.g., two alternative designs, or a base case and various possible risk reduction strategies.

## **Evaluations of Risk Acceptability for a Single Technology**

A state-of-the-art PRA today typically involves some statement about the magnitude of the uncertainty that exists about the final result of the assessment. For example, in a risk assessment of a nuclear power plant, the results would typically include a range (i.e., a probability distribution) of possible core melt frequencies, rather than a single value. Similarly, in an assessment of the risks associated with a toxic or carcinogenic chemical, the results would typically include a range of possible dose/response values, rather than a point estimate.

However, such assessments are not likely to specify whether the uncertainty is associated primarily with population variability (Kaplan, 1983), or, instead, is due to a general lack of knowledge about the technology being evaluated, in which case the risk levels borne by different members of the population are likely to be highly correlated. This distinction can be particularly important for generic PRAs; e.g., risk analyses for a technology or an entire population of facilities, rather than for a specific facility. For example, if a generic PRA concludes that the risk of core melt is between  $10^{-5}$  and  $10^{-4}$  per year for a particular type of nuclear power plant, it may make a big difference to know whether some plants have a core melt frequency of  $10^{-4}$  and others a frequency of  $10^{-5}$ , or whether the same (albeit uncertain) core melt frequency applies to all plants of that type.

If all plants in the population have the same (or highly correlated) core melt frequencies, then a high risk at one plant cannot be canceled out by a low risk elsewhere, and the range of possible societal consequences for all plants taken together will be much broader. Thus, for example, the chance of having two or more core melt accidents over a period of several years will be greater in the correlated case than if the core melt frequencies at different plants are independent. In this case, if our societal utility function is nonlinear in the total number of accidents (e.g., if public opinion is more forgiving of the first accident than of subsequent events), then the technology in question could be much more hazardous on a societal basis than would be suggested by a simple multiple of the mean core melt frequency per plant-year.

Similarly, the backfit costs resulting from an accident will tend to be greater in the correlated case than in the case of independence since the increased risk estimates resulting from the accident will apply to a larger number of facilities. To see this, consider the different impacts of an accident caused by a generic industrywide design flaw (correlated across all plants in the population) versus an accident caused by a site-specific feature such as soil subsidence or offsite flooding. An accident caused by a design flaw is likely to make us reconsider the safety of all other facilities using the same design, and, hence, is likely to result in backfits or safety improvements being required at all similar facilities for a large total societal cost. By contrast, if it can be shown that the accident was caused by a feature that is truly site specific, and is unlikely to occur at other plants, safety improvements will be needed at only a single plant for a much lower total societal cost.

Much the same type of reasoning applies to the assessment of health effect risks; e.g., dose/response relationships for toxic chemicals. Here, the question is whether the assessed uncertainties represent variability between the most and least susceptible people in the exposed population, or, instead, are due to a general lack of knowledge about the hazards of the substance being evaluated. If the uncertainties are due to a general lack of knowledge, then the risk levels experienced by different members of the exposed population will tend to be highly correlated, and all individuals in the population can be expected to experience similar levels of risk. For instance, if the risk of the substance in question turns out to lie toward the high end of its assessed range, then the total risk (i.e., to all exposed people) could be extremely large. Since our societal utility function is almost certainly nonlinear in the total number of fatalities, the substance in question in this case could entail much greater societal risks than would be suggested by a simple multiple of the mean risk to a single individual. This is very different from a situation in which a high risk to one individual (e.g., someone with respiratory problems) is canceled out by a lower risk to other, less vulnerable, people, in which case the magnitude of the total societal risk is well-represented by the risk to an "average" individual.

Of course, most risk assessments will yield results that reflect a mixture of population variability and state-of-knowledge uncertainty. For instance, a PRA of a nuclear power plant will typically include some highly plant-specific features and also some elements of uncertainty (e.g., regarding degraded core phenomena) that would affect all plants in the population equally. This poses a problem for analysts trying to communicate their results to decision makers. On the one hand, a probability distribution may be misleading if important elements of correlation are not communicated. On the other hand, decision makers are likely to be perplexed by statements such as, "Sixty-five percent of the uncertainty is due to correlated factors," or "The correlation coefficient between the core melt frequencies of different plants in this population is estimated to be 0.65."

At present, perhaps the best approach is simply to explore with the decision maker any important nonlinearities in his or her utility function; e.g., risk aversion with respect to large numbers of fatalities, or an aversion to multiple accidents. Risk assessment results can then be presented in terms of the most relevant attributes. For example, decision makers who are risk averse with respect to multiple fatalities might wish to see distributions over the total number of fatalities likely to result from a particular hazardous substance, instead of distributions over the level of risk to a randomly selected individual. Thus, the aim of the risk analyst

should be to provide as much assistance as possible to the decision maker in processing the results of the analysis. This is in contrast to the usual approach in risk analysis, in which certain conventional figures of merit are used to describe the risks of hazardous technologies, without much regard for the specific needs of decision makers in the situation at hand.

### Comparisons of the Risks from Two or More Technologies

In some types of risk comparisons, the technologies being compared are so different that their risks are unlikely to be correlated. For example, although the risks from nuclear power plants and coal-burning power plants may both be quite uncertain, these uncertainties arise from very different sources; e.g., lack of knowledge about safety system reliabilities at nuclear power plants versus lack of knowledge about the health effects of sulfur dioxide emissions from coal plants. In this case, the difference in risk from using one technology rather than another can be determined simply by subtracting the two risk estimates, under the assumption that they are independent.

In other cases, however, the designs or technologies being compared may be quite similar, with only incremental differences between them; e.g., the use of more modern equipment to perform the same function, the addition of an extra safety system, or the elimination of one particular accident scenario through the correction of a design flaw. In this case, simple risk comparisons such as the one shown in Figure 1 are subject to possible misinterpretation. For instance, the comparison shown in Figure 1 suggests that Design B may not necessarily be better than Design A, since Point  $B_1$  represents a higher risk level than Point  $A_0$ . Simply subtracting the two risk levels in this case would give a result similar to that shown in Figure 2. According to that figure, there appears to be a significant chance that the desired risk reduction associated with changing from Design A to Design B could actually turn out to be a risk increase (as represented by the left-hand tail of the curve extending below zero).

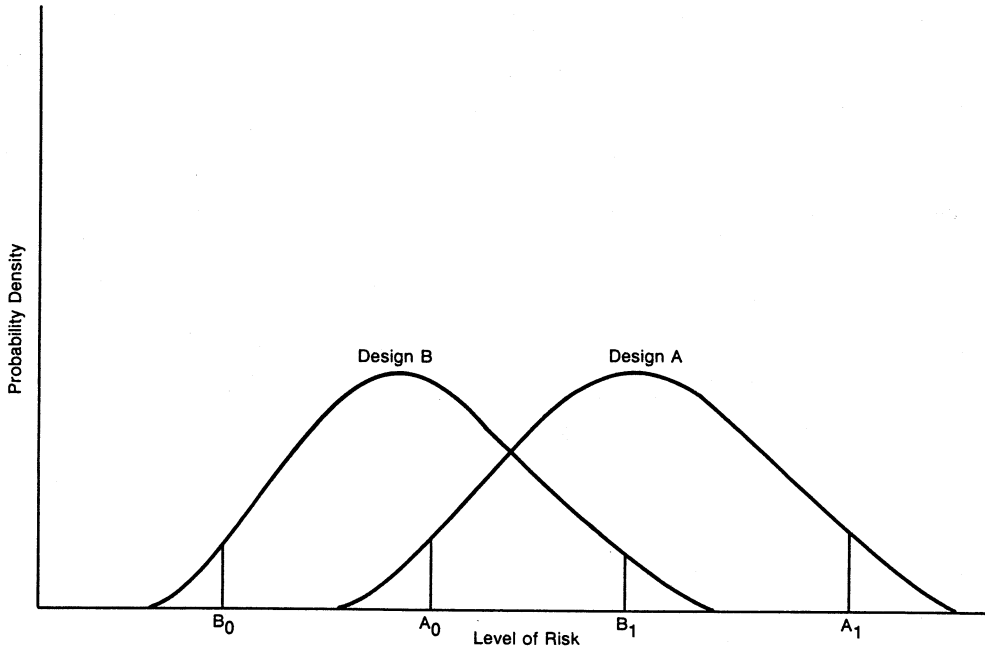
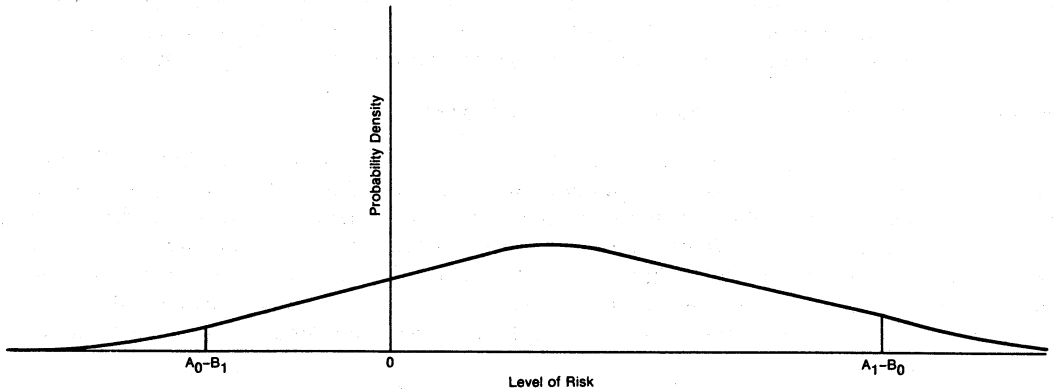


Figure 1. Comparison of the Risks of Two Alternative Designs



**Figure 2. Difference between the Risk Levels of Designs A and B (assuming they are independent)**

However, in some situations we may actually know for certain that Design B represents a reduction in risk. For example, Design A may represent the risk posed by a particular plant with a two-train auxiliary feedwater (AFW) system, while Design B represents the risk from the same plant with a three-train AFW system. In this case, much of the uncertainty reflected in Figure 1 is likely to stem from factors that are highly correlated between the two designs; e.g., uncertainty about initiating event frequencies, about the response of safety systems other than the AFW system, or about containment response to a core melt. The risks of Designs A and B in this case would clearly not be independent, and Figure 2 would therefore not be a valid description of the difference in risk between the two technologies.

The comparison shown in Figure 1 would still be valid, of course, but it may give misleading impressions to some decision makers. For example some decision makers may interpret Figure 1 to imply that the benefit associated with Design B is highly uncertain. However, the situation depicted in this figure is equally consistent with, say, a guaranteed factor of 3 improvement in a highly uncertain initial risk level. This would be the case, for instance, if high risk for Design A (Point  $A_1$ ) always occurred in conjunction with high risk for Design B (Point  $B_1$ ), and similarly for lower risk levels; e.g.,  $A_0$  and  $B_0$ .

To avoid such possible misinterpretations, risk analysts may wish to present not only "before-and-after" comparisons such as Figure 1, but also distributions for the actual magnitude of the risk reduction associated with a particular design change. This risk reduction can be represented either by an arithmetic difference or by a ratio of the two risk levels. Which approach is more appropriate in a particular context may depend on the type of safety improvement being evaluated, and also on the form of the overall PRA model being used.

For example, if the design change in question results in the elimination (or reduction in frequency) of one particular accident scenario, this might best be reflected by the arithmetic difference between the risk levels before and after instituting the improvement. An arithmetic difference\* will highlight the reduction in risk for the particular scenario affected by the

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\*Note that any uncertainty analysis must be performed for the entire quantity  $RISK_A - RISK_B$ , or, equivalently, for the difference  $(\lambda_j - \lambda_j')$ . Simply subtracting the distributions for  $RISK_A$  and  $RISK_B$  as if they were independent will not yield correct results, and, in particular, will not remove the confounding effects of uncertainty about the  $\lambda_i$  for  $i \neq j$ .

improvement, while eliminating the uncertainty associated with those accident scenarios that are unaffected by the improvement:

$$\text{RISK}_A - \text{RISK}_B = \sum_{i=1}^n \lambda_i - \sum_{i=1}^n \lambda_i' = \lambda_j - \lambda_j'$$

where

$\lambda_i$  = the frequency of scenario  $i$  in Design A.

$\lambda_i'$  = the frequency of scenario  $i$  in Design B.

$\lambda_i = \lambda_i'$  for all scenarios except scenario  $j$ .

By contrast, use of a ratio

$$\frac{\text{RISK}_A}{\text{RISK}_B} = \frac{\sum_{i=1}^n \lambda_i}{\sum_{i=1}^n \lambda_i'} = \frac{\sum_{i=1}^n \lambda_i}{\sum_{i=1}^{j-1} \lambda_i + \lambda_j' + \sum_{i=j+1}^n \lambda_i}$$

would tend to confound the reduction in risk for the scenario of interest with uncertainty about the frequencies of other scenarios.

However, ratios may be more appropriate for other types of safety improvements, such as the addition of an extra safety system; e.g., a second containment structure. In this case, the overall risk of a particular scenario can be expected to decrease by a factor equal to the failure probability of the new system. This will be reflected by the ratio\* of the risk levels before and after instituting the new safety system:

$$\frac{\text{RISK}_A}{\text{RISK}_B} = \frac{\prod_{i=1}^n P_i}{\prod_{i=1}^{n+1} P_i} = \frac{1}{P_{n+1}}$$

where

$P_i$  = the failure probability of system  $i$  ( $i = 1, \dots, n$ ).

$P_{n+1}$  = the failure probability of the new safety system added in Design B.

In this situation, use of the arithmetic difference between the risk levels.

$$\text{RISK}_A - \text{RISK}_B = \prod_{i=1}^n P_i - \prod_{i=1}^{n+1} P_i = \prod_{i=1}^n P_i (1 - P_{n+1})$$

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\*Here again, any uncertainty analysis must be performed for the overall ratio, or else for the quantity  $1/P_{n+1}$  in isolation. Simply taking the ratio of the distributions for  $\text{RISK}_A$  and  $\text{RISK}_B$  as if they were independent will not yield correct results.

would tend to confound the risk reduction resulting from the system of interest with uncertainty about the failure probabilities of other safety systems.

### **Conclusions**

Dependence can be important both in evaluations of risk acceptability for a single technology that is in widespread use (e.g., at multiple facilities, or by multiple people) and in comparisons of two or more alternative technologies; e.g., a base case and one or more possible risk reduction strategies. The role of dependence in making decisions about risk has received little attention to date, and is not always adequately understood, either by decision makers or by risk analysts.

Some suggestions have been presented here for how to communicate the impacts of dependence to decision makers. However, these ideas are only preliminary in nature. Further suggestions along these lines would be more than welcome, and could serve to stimulate discussion on this important topic.

### **References**

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