A Model for Describing Both the Anisotropic and Uniaxial Distributed Damage in Concrete

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ABSTRACT

We propose to describe here the behavior of damaged concrete by a phenomenological model. For this, we suppose that the main mechanism of degradation is the microcracking represented by an internal variable of damage. At first, a coupling with the elasticity leads to an anisotropy of the material. Second, the uniaxial behavior is described by a partition of the free energy in two terms. At least, the inelastic strains are taken into account by an appropriate coupling in the third terms of the free energy between damage and strains. We show simulations obtained by this model in order to illustrate the interest of these choices.

INTRODUCTION

In order to describe precisely the behavior of damaged concrete, it is necessary to know which are the main mechanisms responsible for the phenomenological observations. We can consider in a first approach, that the most important evolution in the characteristics of damaged concrete are :

i) the degradation of elastic properties of the material,

ii) the creation and the evolution of inelastic strains which are closely linked to the existence of microcracks inside concrete,

iii) the unilateral behavior of these microcracks which can be either closed or opened depending on the state of stress.

The aim of our work is to model these three points within the general framework of thermodynamics and to identify all the material parameters and the proposed evolution laws.

DAMAGE MODEL

1. Damage variables

We use to distinguish two kinds of damage process :

a) The propagation of microcracks around the aggregates and then inside the matrix is mainly responsible for the evolution of the stiffness of the material. As we know, when damaged, concrete can be anisotropic and so the damage variable $\mathbb{d}$ will be a second order tensor [1].

b) Under high hydrostatic pressure, micropores can collapse and this degradation will be represented by a scalar δ [2].

2. Inelastic strains

Many reasons are involved to explain the creation of permanent strains in concrete. Residual stresses due to shrinkage and fabrication of concrete may be a good explanation for the early stage of their development. But it seems correct to admit that the propagation of microcracks will induce inelastic strains. Indeed, some of these cracks keep still open without any external applied load. For that reason, we propose a coupling between damage and inelastic strains without any other formalism such as plasticity [3].

3. Uniaxial phenomena

We have shown [4] that whenever concrete is damaged, the previously created microcracks may close under the appropriate load. In a particular case -uniaxial compression after a damageable uniaxial tension- we have obtained the recovery of the initial stiffness of concrete although the elastic modulus was nearly zero in tension.

GENERAL FORMULATION

We postulate here the particular form of the Helmholtz free energy $\rho\psi$ for our problem as follow [5] :

$$\rho\psi = \mathrm{A} + \mathbb{B} : \varepsilon + \frac{1}{2}\mathbb{C}\, \varepsilon : \varepsilon$$

where A, $\mathbb{B}$ and $\mathbb{C}$ are material parameters :

$\mathbb{C}$ represents the elasticity tensor and $\mathbb{B}$ can be interpreted as a residual stress tensor.

$\mathbb{A} : \mathbb{B} = \mathrm{Tr}\,[\mathbb{A} \bullet \mathbb{B}]$.

$\mathbb{A} \bullet \mathbb{B}$ represents the tensorial product of $\mathbb{A}$ and $\mathbb{B}$.

STRAIN BASED FORMULATION

1. Without the "unilateral effect"

In that case, the free energy becomes :

$$\rho\psi = \frac{1}{2}\left[2\mu\,[(\mathbb{1} - \mathbb{d}) : \varepsilon : \varepsilon] + \lambda(1-\delta)\,\mathrm{tr}^2[\varepsilon]\right] + \mathbb{B} : \varepsilon$$

where $\mathbb{d}$ is the damage tensor and δ a scalar variable. In order to take into account the evolution of inelastic strains due to damage we propose to write $\mathbb{B} : \varepsilon$ as

$$\mathbb{B} : \varepsilon = \frac{1}{2}\left[(2\beta\,\mathbb{d} : (\mathbb{1} - \varepsilon) : \mathbb{d}] + \gamma\delta\,.(1 - \mathrm{tr}\,[\varepsilon])\,.\delta)\right]$$

where β and γ are also material parameters.

That form has been derived from previous works [6].

The behavior becomes :

$$\sigma = 2\mu\,(\mathbb{1} - \mathbb{d})\,.\varepsilon + \lambda(1-\delta)\mathrm{tr}[\varepsilon]\,\mathbb{1} - \beta\,\mathbb{d}\,.\,\mathbb{d} - \frac{\gamma}{2}\delta^2\,\mathbb{1}$$

2. With the "unilateral effect"

In order to simplify the formulation, we do not consider in this part inelastic strains ($\beta = \gamma = 0$). Furthermore we consider that when the sign of strains change (i.e cracks open or close) the unilateral behavior exists.

For that reason we have splitted the strain tensor :

$$\varepsilon = \varepsilon^+ + \varepsilon^-$$

ε^+ (resp. ε^-) is built with the positive (resp. negative) eigen value of ε.

We write the free energy as the sum of two terms :

$$\rho\psi = \rho\psi\,(\varepsilon^+) + \rho\psi\,(\varepsilon^-)$$

$$2\,\rho\psi = 2\mu\,\varepsilon^+ : (\mathbb{1} - \mathbb{d}^+) : \varepsilon^+ : (\mathbb{1} - \mathbb{d}^+) + \lambda\,(1 - \delta^+)^2\,(\langle\mathrm{tr}[\varepsilon]\rangle_+)^2)$$
$$+\ 2\mu\,\varepsilon^- : (\mathbb{1} - \mathbb{d}^-) : \varepsilon^- : (\mathbb{1} - \mathbb{d}^-) + \lambda\,(1 - \delta^-)^2\,(\langle\mathrm{tr}[\varepsilon]\rangle_-)^2)$$

where $\mathbb{d}^+$ and δ^+ are associated to ε^+

$\mathbb{d}^-$ and δ^- are associated to ε^-

and $\langle\mathrm{tr}[\varepsilon]\rangle_+ = 0$ if $\mathrm{tr}[\varepsilon] < 0$

$= \mathrm{tr}[\varepsilon]$ if $\mathrm{tr}[\varepsilon] > 0$

In the principal axes, the state law becomes :

$$\sigma = \frac{\partial\,(\rho\psi)}{\partial\varepsilon^+} + \frac{\partial\,(\rho\psi)}{\partial\varepsilon^-}$$

$$\sigma = 2\mu\,(\mathbb{1} - \mathbb{d}^+)\,.\,\varepsilon^+\,.\,(\mathbb{1} - \mathbb{d}^+) + 2\mu\,(\mathbb{1} - \mathbb{d}^-)\,.\,\varepsilon^-\,.\,(\mathbb{1} - \mathbb{d}^-) + \lambda(1-\delta)^2\,\mathrm{tr}[\varepsilon]\,\mathbb{1}$$

where $\mathbb{d}^+ = \begin{bmatrix} d_1^+ & & 0 \\ & d_2^+ & \\ 0 & & d_3^+ \end{bmatrix}$ $\mathbb{d}^- = \begin{bmatrix} d_1^- & & 0 \\ & d_2^- & \\ 0 & & d_3^- \end{bmatrix}$ in the principal axes.

If we consider the inelastic strains, the behavior becomes :

$$\sigma = 2\mu\,(\mathbb{1} - \mathbb{d}^+)\,.\,\varepsilon^+\,.\,(\mathbb{1} - \mathbb{d}^+) + 2\mu\,(\mathbb{1} - \mathbb{d}^-)\,.\,\varepsilon^-\,.\,(\mathbb{1} - \mathbb{d}^-) + \lambda(1-\delta)^2\,\mathrm{tr}[\varepsilon]\,\mathbb{1}$$
$$-\left\{\beta^+\,(\mathbb{d}^+\,.\,\mathbb{d}^+)^2 - \beta^-\,(\mathbb{d}^-\,.\,\mathbb{d}^-)^2 - \frac{\gamma}{2}(\delta^2)^2\,\mathbb{1}\right\}$$

EVOLUTION LAWS

We propose here to write the evolution laws of the variables $\mathbb{d}^+$, $\mathbb{d}^-$ and δ as follow

$$d_i^+ = d_i^+ (\mathbb{Y}^+)$$

$$d_i^- = d_i^- (\mathbb{Y}^-)$$

$$\delta = \delta (\tilde{y}) \text{ where } \tilde{y} = \frac{\text{elastic energy for virgin material}}{\text{elastic energy for damaged material}}$$

where d_i^+ (resp. d_i^-) are the components of the tensor $\mathbb{d}^+$ (resp. $\mathbb{d}^-$) in the principal axes.

Simulation

1. We shall show here some simulations of that model. First, we illustrate the elastic anisotropy described by the damage tensors $\mathbb{d}^+$ and $\mathbb{d}^-$ (figure 1). On that figure we have first damaged the material by a compression ($\sigma_1;\varepsilon_1$). Then, in order to know the elastic response of the material in the others directions we have plotted the curves ($\sigma_2 = \sigma_3$, $\varepsilon_2 = \varepsilon_3$) in tension and in compression. We see that the stiffness in tension is very low in that case and that corresponds in reality to the opening of many cracks (figure 2).
In opposite, the stiffness in compression ($\sigma_2 = \sigma_3$) is nearly the same as the initial one and that can be related to the closure of the cracks (figure 3).

2. We have then simulated the evolution of inelastic strains with our formulation for two cases : pure compression and pure tension (figure 4, figure 5).
It is interesting to remark that we can describe the evolution of the volume change of the material in that case. If we compare with experimental results (figure 6, figure 7) we see that the agreement is good. The anisotropy of the evolution of the inelastic strains is also well described as they are coupled with damage.

3. Figure 8 shows the unilateral effect for the particular case of pure tension followed by a compression. We remark that a change of the sign of the total strain is responsible for the uniaxial behavior as it is shown by the experimental results (figure 9).

CONCLUSION

We have built a general model well adapted to describe three important points of the behavior of concrete which are :

1. The anisotropy of the elastic constants of damaged concrete.

2. The evolution of inelastic strains due to damage.

3. The unilateral behavior of the damaged material.

The way to obtain the model is to built a free energy with two terms ; the first one is related to the residual stresses and the second one is related to the elasticity behavior. The last one is then splitted in two terms in order to describe the unilateral behavior of concrete.

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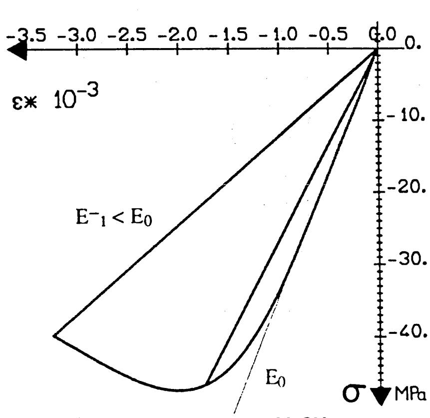

Figure 1 : UNIAXIAL COMPRESSION
Curve $(\sigma_1^-, \varepsilon_1^-)$

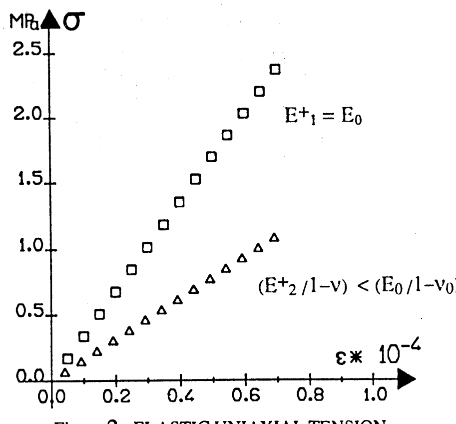

Figure 2 : ELASTIC UNIAXIAL TENSION
Curves $(\sigma_1^+, \varepsilon_1^+)$ and $(\sigma_2^+, \varepsilon_2^+)$

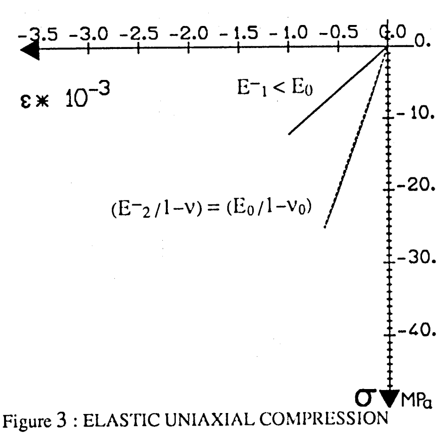

Figure 3 : ELASTIC UNIAXIAL COMPRESSION
Curves $(\sigma_1^-, \varepsilon_1^-)$ and $(\sigma_2^-, \varepsilon_2^-)$

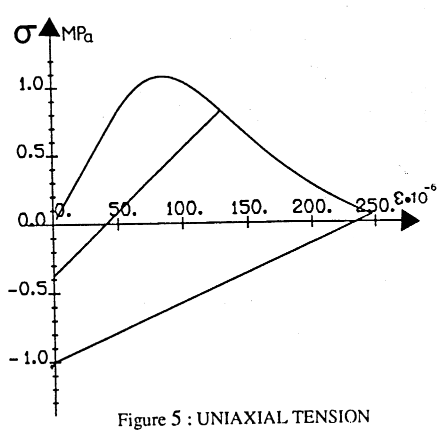

Figure 5 : UNIAXIAL TENSION
Curve $(\sigma_1^+, \varepsilon_1^+)$

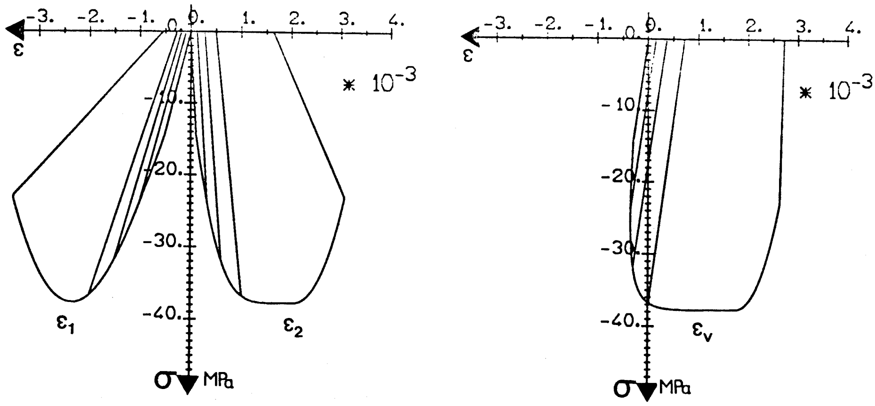

Figure 4 : UNIAXIAL COMPRESSION

Curves $(\sigma_1^-, \varepsilon_1^-)$, $(\sigma_1^-, \varepsilon_2^+)$ and $(\sigma_1^-, \varepsilon_v)$

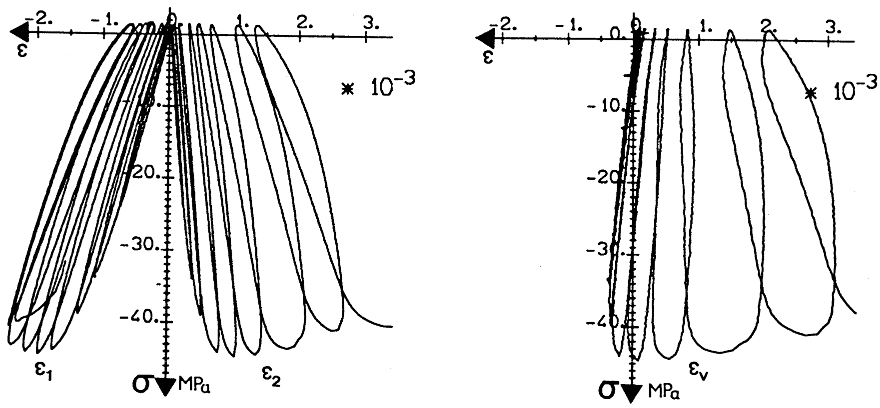

Figure 6 : UNIAXIAL COMPRESSION

Curves $(\sigma_1^-, \varepsilon_1^-)$, $(\sigma_1^-, \varepsilon_2^+)$ and $(\sigma_1^-, \varepsilon_v)$

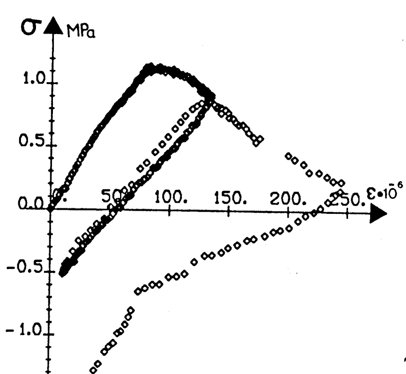

Figure 7 : UNIAXIAL TENSION

Curve $(\sigma_1^+, \varepsilon_1^+)$

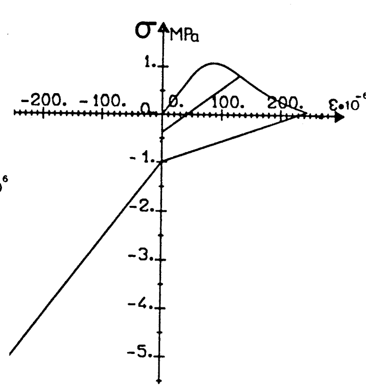

Figure 8 : UNILATERAL EFFECT

Curves $(\sigma_1^+, \varepsilon_1^+)$ and $(\sigma_1^-, \varepsilon_1^-)$

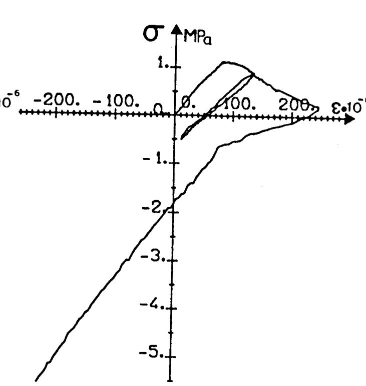

Figure 9 : UNILATERAL EFFECT

Curves $(\sigma_1^+, \varepsilon_1^+)$ and $(\sigma_1^-, \varepsilon_1^-)$