# Fracture and Aggregate Interlock Mechanisms in Reinforced Concrete

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## ABSTRACT

Nine tests regarding thin-webbed I-beams failing in shear because of stirrup yielding and concrete crushing are here considered. An incremental-iterative procedure for the nonlinear F.E. analysis of reinforced concrete structures is applied to three of the above beams. Solid concrete is modeled as a hypoelastic material, whilst cracked concrete (including fracture and aggregate interlock) is modeled with crack bands, according to the smeared crack approach.

## 1 INTRODUCTION

Concrete exhibits a complex response with various important nonlinearities regarding both the solid, undamaged concrete (as shown by the many biaxial and triaxial tests carried out so far) and the cracked concrete (either at the onset of cracking or after crack stabilization). As regards damaged concrete, both tensile and shear cracking have been lately investigated to a remarkable extent, also within the framework of fracture mechanics, and a few models for crack formation and propagation have lately become suitable to the introduction into existing F.E.codes. Furthermore several reliable models are now available [4] for modelling stabilized cracking, as required -for instance- by aggregate interlock. The constitutive laws regarding each type of non-linearity may be in principle used within an evolutive approach or a limit analysis approach, in the latter case with more limited results, but with far less numerical and computational effort [5].

In this paper, the ultimate analysis of regularly cracked r.c. thin-webbed beams failing in shear is performed by means of a modified ADINA program (evolutive analysis), in order to evaluate the ultimate load in shear of a few normal concrete beams, having I sections tested by Regan and Rezai-Jorabi [7]. The so-called Rough Crack Model for aggregate interlock is adopted [4].

In the evolutive analysis, a hypoelastic model for solid concrete is used, and tensile cracking is identified by means of failure surfaces. The smeared crack concept for fracture and aggregate interlock is adopted since it brings several advantages, mainly regarding the adjustment of the compliance and stiffness matrices with no topological changes in element connectivity.

As regards fracture, a compliance matrix-based approach is used. If the direction of the advancing crack is unknown, any mesh other than a square mesh is unsuitable because it would introduce a bias favoring some crack paths over others [2,3].

It seems very reasonable to assume $w=h/\cos\alpha_{cr}$ where $\alpha_{cr}$ is the angle between the crack axes and the mesh grid (crack orientation is fixed and coincides with the one at the onset of local cracking), and h is the mesh size. However, it appears that the above relation must be limited to $|\alpha_{cr}|<45°$. When the first principal stress becomes larger than the tensile strength $f_{ct}$ the modulus of elasticity in the strain softening range is computed according to Linear Fracture Theory [6]:

$$\delta_0 = 2G_F / f_{ct}\ , \qquad \lambda = 2G_F E_0 / f_{ct}^2\ , \qquad (1)$$

where $G_F$ is the fracture energy.

$$C_f = -w f_{ct}^2 / 2G_F = -w E_0 / \lambda, \qquad E_t = -E_0 / (\lambda / w - 1) \qquad (2)$$

As regards aggregate interlock, a stiffness matrix-based approach is used. According to the "Rough Crack Model" initially proposed by Bazant and Gambarova [4], the constitutive laws of aggregate interlock are:

$$\sigma_{nn} = a_{12}\, r \sqrt{\delta_n}\, \sigma_{nt} / h \qquad \text{(always compressive)} \qquad (3)$$

$$\sigma_{nt} = \tau_o \left(1 - \sqrt{2\delta_n / d_a}\right) r (f/g) \qquad (4)$$

$$f = a_3 + a_4 \, |r^3| \qquad g = 1 + a_4 r^4 \qquad h = (1 + r^2)^{1/4} \qquad (5)$$

where: $r = \delta_t / \delta_n$ , $a_{12} = 0.62$, $a_3 = 2.45/\tau_o$ , $a_4 = 2.44(1 - 4/\tau_o)$, $\tau_o = 0.25 f'_c$
$d_a$ =maximum aggregate size (20mm in the present paper).

The stiffness coefficients obtained by deriving Eqs.(3) and (4) are introduced into the stiffness matrix which is no longer symmetric [6].

## 2 INPUT DATA FOR THE ANALYSIS

The tests reported here regard a set of thin-webbed reinforced concrete I-beams designed to fail in shear [7]. In contrast to most earlier specimens for such tests, the shear reinforcement was not excessive and generally yielded before final collapse (see references in [7]). The geometry of the beams and their longitudinal reinforcement were constant through the series of nine tests apart from a few minor details. Geometry is given in Fig.1.

Three beams out of the nine tested by Regan are here considered, having the characteristics given in Table 1. In the analysis, the steel model is elastic-plastic with kinematic hardening and the mechanical properties of the steel are: $f_{yw} = 745.\text{MPa}$ $f_{yl} = 646.\text{MPa}$ $E_0 = 210{,}000.\text{MPa}$ $E_t = 5{,}000.\text{MPa}$.

The standard cylindrical strength $f'_c$ is introduced by reducing the cube strength given by Regan ($f'_c = 0.833 f_{cu}$).

The secant concrete modulus $E_c$ ($\sigma_c = 0.0 - 0.4 f'_c$) was evaluated according to the EUROCODE No.2 [1] and the initial tangent modulus $E_0$ was given the value $1.1E_c$

Still according to [1] the crushing strain of concrete is assumed $\varepsilon'_c = -.0022$.

## 3 RESULTS OF THE ANALYSIS

In all tests the final collapse was accompanied by web crushing. The results are summarized in Tables 1 and 2.

Table 1-Summary of Results.

| Beam | Concrete | | | | | Stirrups | | Ultimate load | | | Average values | | | [7] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| no. | $G_F$ N/mm | $f'_c$ MPa | $f_{cu}$ MPa | $f_{ct}$ MPa | $E_0$ MPa | s mm | $\rho_w f_{yw}$ MPa | $V_u$ [7] | $V_u$ kN | $V_u/b_w z$ MPa | $\alpha_{cr}$ Deg | $\theta_u^\sigma$ Deg | $\theta_u^\varepsilon$ Deg | $\theta_u^\sigma$ Deg |
| 1 | .20 | 42.92 | 51.5 | 3.26 | 37,807. | 100. | 5.26 | 273. | 293.8 | 9.66 | 43.2 | 28.5 | 40.6 | 30.5 |
| 4 | .28 | 60.08 | 72.1 | 4.33 | 43,278. | 175. | 3.01 | 280. | 301.3 | 9.91 | 43.0 | 24.7 | 41.4 | 18.5 |
| 8 | .13 | 27.92 | 33.5 | 2.79 | 33,660. | 70.0 | 7.51 | 210. | 253.2 | 8.33 | 43.1 | 30.1 | 41.1 | 45.0 |

Table 2-Angles in the web (Degrees).

| Beam | (Fig.1)Alignment n. | | | | | | | | | | | | [7] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1(z=126.44mm) | | | 2(z=198.56mm) | | | 3(z=251.44mm) | | | 4(z=323.56mm) | | | |
| no. | $\alpha_{cr}$ | $\theta_u^\sigma$ | $\theta_u^\varepsilon$ | $\alpha_{cr}$ | $\theta_u^\sigma$ | $\theta_u^\varepsilon$ | $\alpha_{cr}$ | $\theta_u^\sigma$ | $\theta_u^\varepsilon$ | $\alpha_{cr}$ | $\theta_u^\sigma$ | $\theta_u^\varepsilon$ | $\theta_u^\sigma$ |
| 1 | 50.1 | 27.3 | 42.2 | 43.6 | 33.4 | 41.6 | 42.6 | 24.2 | 40.1 | 36.6 | 28.9 | 38.6 | 30.5 |
| 4 | 50.0 | 21.8 | 44.2 | 43.2 | 31.6 | 42.5 | 42.3 | 20.6 | 40.3 | 36.5 | 24.9 | 38.5 | 18.5 |
| 8 | 50.1 | 27.0 | 42.8 | 43.4 | 36.1 | 42.0 | 42.4 | 25.6 | 40.4 | 36.6 | 31.7 | 39.1 | 45.0 |

With reference to Fig.2, the most interesting results regard the elements no.24-27, 34-37, which are marginally affected by the boundary conditions and exhibit a strong shear role.
In Fig.2,the small cyrcles pinpoint the steel elements for which the numerical results were compared with test results. The same comparison was performed in the concrete at the locations marked with a little cross.

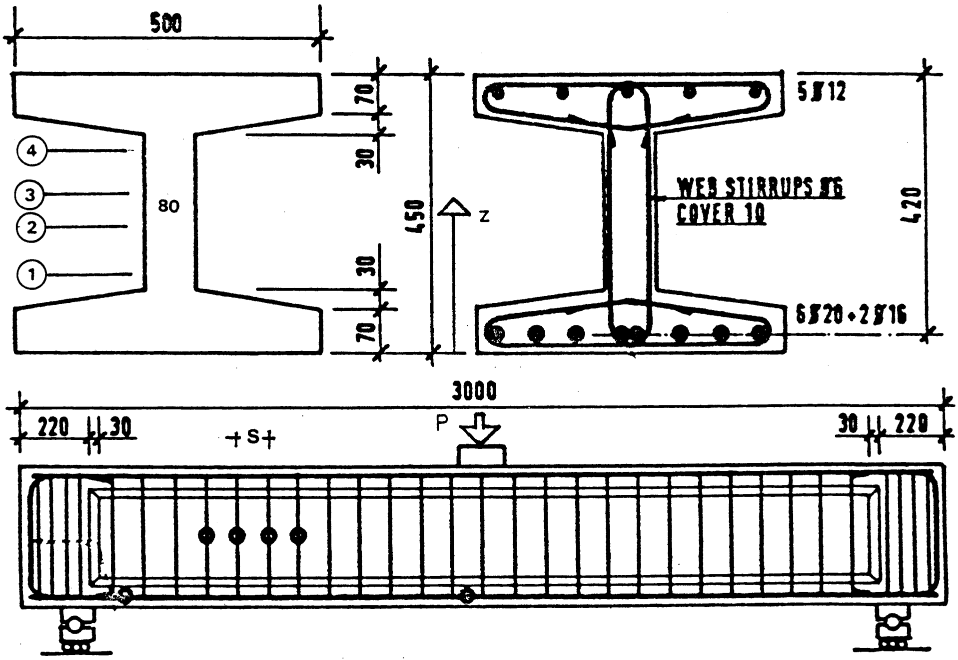


Fig.1-Details of the beams [7].

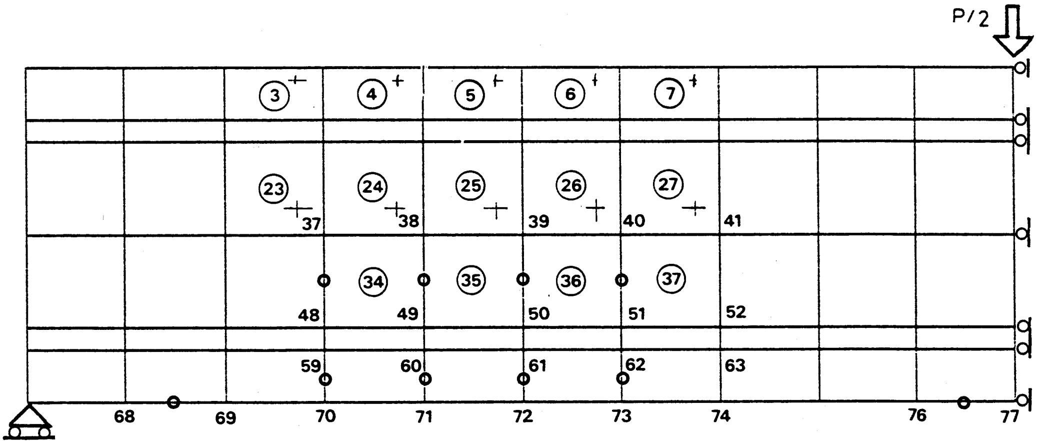


Fig.2-F.E.Mesh and Location of Measurement Points. Average values of concrete are computed in elements No.24-27, 34-37.

In Figs.3 and 4, typical stress and strain fields are shown for Beam No.4 and P= 540.kN.
The strains in the stirrups of the web are plotted in Fig.5. In all cases shown in Fig.5 the stirrups yield at failure, except the stirrups of beam 8.
Though stirrup yielding is described by the model, real stirrups exhibit a fairly softer response.

The average strains of the main reinforcement at the two instrumented sections are presented in Fig.6. The strains at midspan varied very little from beam to beam.

Fig.7 shows some typical diagrams of the strains at the top of a section very close to the center of the shear span.

The principal compressive stresses and strains in the web are given in Figs.8,9 where:

$\alpha_{cr}$ defines the crack orientation;

$\theta_u^\sigma$ is the second principal stress at collapse (compression);

$\theta_u^\varepsilon$ is the second principal strain at collapse (contraction).

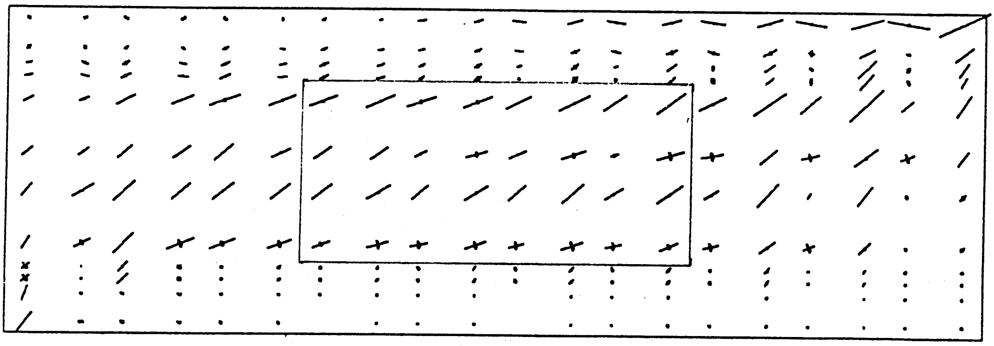


Fig.3-Stress Field for Beam No.4 P=548.KN ($\sigma_{min}$ =-38.5 MPa).The comparison between the analysis and the test [7] is limited to the zone inside the central rectangle.

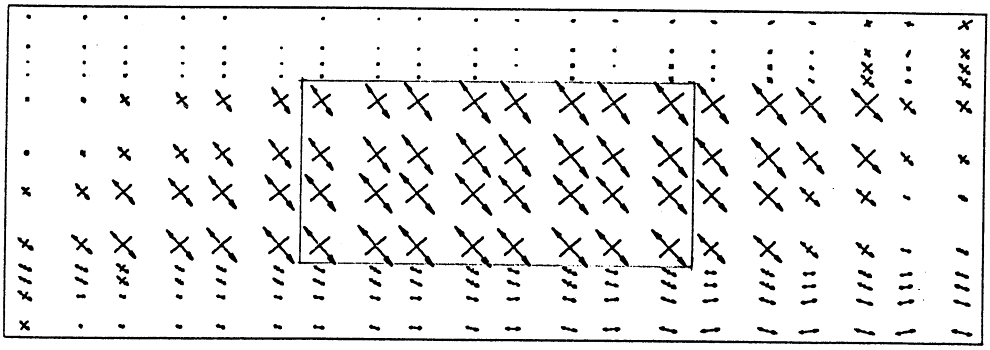
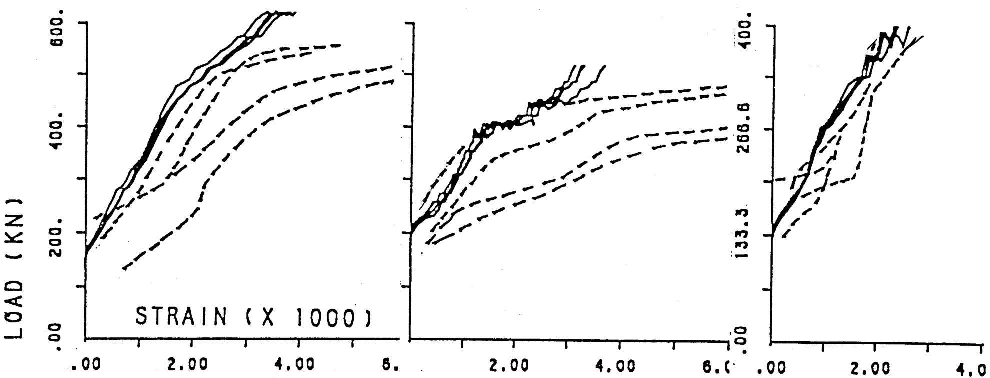


Fig.4-Typical Strain Field for Beam No.4 P=548.KN ($\varepsilon_{min}$ =0.0051).



Beam n.1 Beam n.4 Beam n.8

Fig.5-Strains in the Stirrups;(---)test;(——)analysis.El1.37-48..40-51 of the web.

In Fig.8, at increasing load level the principal compressive stress tends to a limit value (represented by a vertical asymptote), which shows that further load increases are possible because of resistant mechanisms other than the inclined compression field.

At the same time, the principal compressive strain (Fig.9) tends to flatten-off, i.e. to diverge, which means that the web is really at the onset of collapse, at the maximum loads applied during the analysis.

Furthermore, in the above-mentioned ultimate situations the maximum compressive stress in the concrete is not only very close to $f'_c$ (which is natural) but tends to flatten off (at increasing standard cubic strength of Fig.10), as suggested recently by some Authors and accepted also by Eurocode No.2, where $f^{1}_{cw}$ is formulated as follows:

$$f^{1}_{cw} = f'_c\,[0.7 - f'_c/200] \qquad (f^{1}_{cw}, f'_c = \mathrm{MPa}) \qquad (6)$$

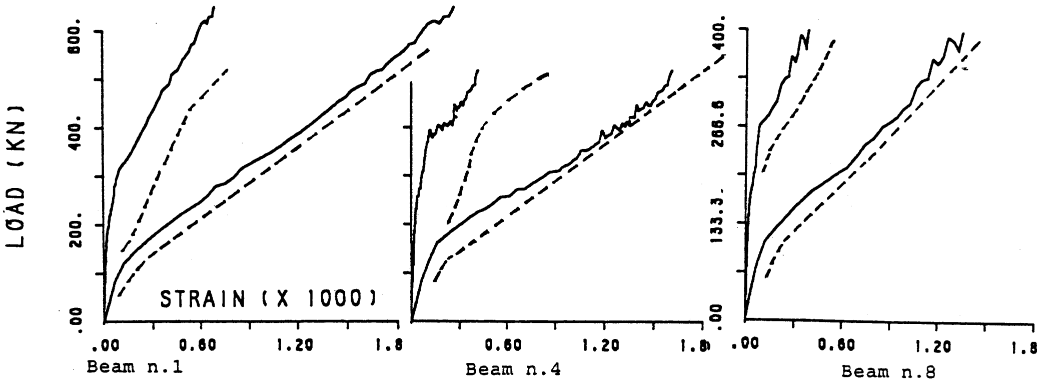


Fig.6-Strains of Main Reinforcement;(---)test;(——)analysis. El1.68-69..76-77.

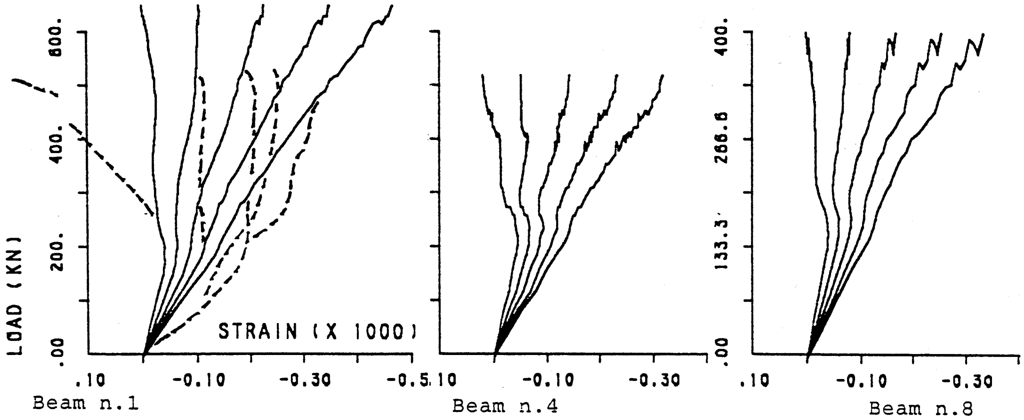


Fig.7-Strains of Top Surface;(---)test;(——)analysis. Elements 3,..,7.I.P.No.4.

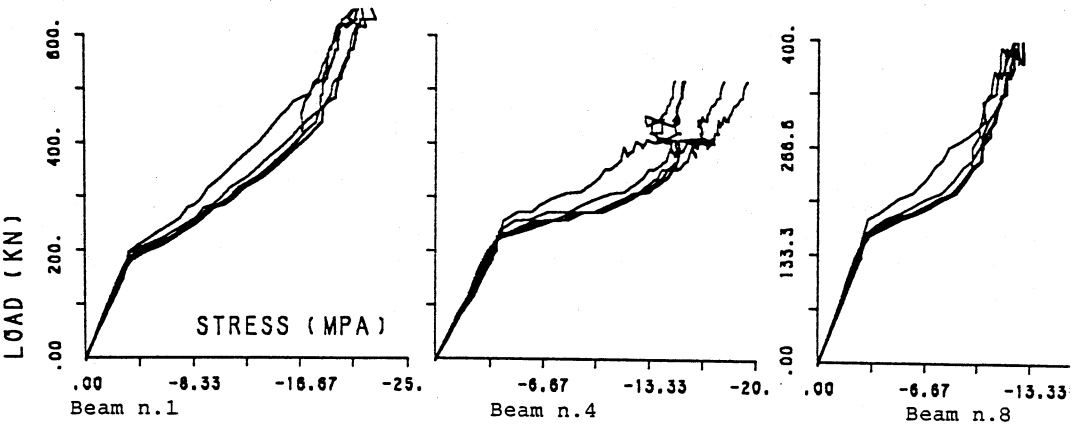


Fig.8-Principal compressive Stresses in the Web as obtained with F.E. Elements 23..27.I.P.No.3.Beam No.1$\theta_u^\sigma$ =28°5, Beam No.4$\theta_u^\sigma$ =24°7, Beam No.8$\theta_u^\sigma$ =30°1.

## 4 CONCLUDING REMARKS

a) In thin-webbed I sections an inclined and uniaxial compression field forms as a result of diagonal cracking due to shear: consequently the ensuing assumption, very popular in limit analysis, is fully confirmed;

b) The actual static behavior of the stirrup in tension can be adequately described in crack formation and stabilisation is correctly introduced;

c) As should be expected, stress and strain principal directions may markedly diverge beyond the linear-elastic domain;

d) The concrete strength of the inclined compression field does not increase linearly with the standard compression strength, but is penalized at the higest concrete grades.

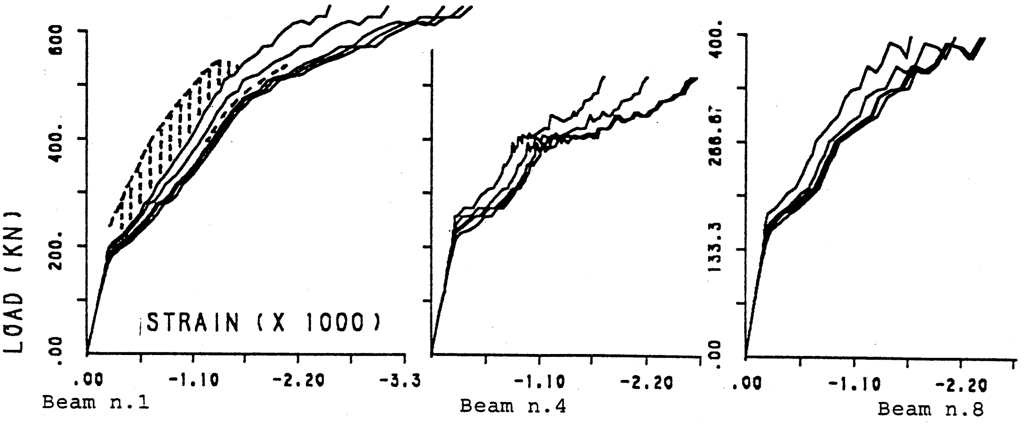


Fig.9-Principal compressive Strains of Web Concrete; (---)test;(——)analysis. Elements23..27.I.p.No.3.Beam No.1$\theta_u^\varepsilon$ =40°1, Beam No.4$\theta_u^\varepsilon$ =41°4, Beam No.8$\theta_u^\varepsilon$ =41°1.

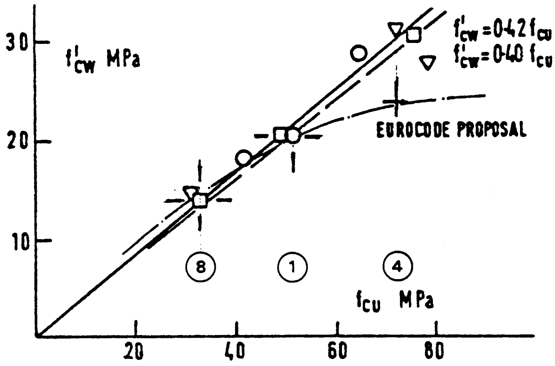


Fig.10-Web crushing strength of concrete as a function of cube strength; test: ○ beams 1-3, △beams 4-6, □beams 7-9; analysis +.

## ACKNOWLEDGEMENTS

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