Numerical and Experimental Studies Concerning New Failure Criteria for Reinforced Concrete Elements

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INTRODUCTION

The reinforced concrete elements behaviour after the elastic limit is made by means of failures criteria as:Mohr-Coulomb,Drucker Prager,energetical criteria,etc.For the materials with a brittle failure we may use the Mohr-Coulomb criterium if the principal stresses before the cracks appearance are known.By the experimental researchers made on the reinforced concrete samples was been determined the cohesion c and the internal friction angle θ.With these values the Mohr-Coulomb criterium gives the errors between 10 % - 30 %.In the paper,authors propose an improved new relation for the Mohr-Coulomb criterium adapted for the reinforced concrete.This relation has the errors between 2%-4%.The errors variation diagrams for the reinforced concrete samples cubes presents a more adapted approach from the reality with this new criterium. The authors have introduced this new criterium as a routine in a computer program with finite elements.The paper presents also some results concerning the behaviour of the reinforced concrete plane elements after the elastic limit.

FAILURE CRITERIA AND CONSTITUTIVE RELATIONS FOR REINFORCED CONCRETE

Structural elements behaviour studies beyond on the elastic limit are made by means of yield or failure criteria.For to choose one criterium it is necessary to know the material behaviour and the stress state in each point of the element,respectively the maximum principal stresses(Mihalache, 1988).
The Mohr-Coulomb criteium is a generalisation of the Coulomb friction failure law defined by(Owen,Hinton, 1980)

$$(\sigma_1 - \sigma_3) + (\sigma_1 + \sigma_3) \sin \theta = 2 c \cos \theta \qquad (1)$$

where σ_1 and σ_3 are the principal stresses,c is the cohesion and θ the angle of internal friction.This criterium is applied to concrete,rock and soil problems.
The Drucker-Prager criterium is one approximation of the Mohr-Coulomb law.The influence of a hydrostatic stress component on yielding was introduced by inclusion of an aditional term in the Von Mises expression to give(Atluri,Kobayashi, 1985)

$$a I_1 + (I_2')^{1/2} = k' \qquad (2)$$

where

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \qquad (3)$$

$$a = \frac{2 \sin \theta}{\sqrt{3}(3 + \sin \theta)}, \quad k' = \frac{6 c \cos \theta}{\sqrt{3}(3 + \sin \theta)}$$

The failure envelope is divided into several regions depending on the ratio of maximum to minimum principal stress α_2 and the stress state.

a) If $0 < \alpha_2 < 1$ when is biaxial compression with(Milford, Schnobrich, 1984)

$$\sigma_{2c} = \frac{1 + 3.65\,\alpha_2}{(1 + \alpha_2)^2} f_c', \quad \sigma_{1c} = \alpha_2 \sigma_{2c} \qquad (4)$$

and failure is assumed to occur due to yielding and crushing of the concrete.

b) If $-0.17 < \alpha_2 < 0$ when is biaxial tension compression with

$$\sigma_{2c} = \frac{1 + 3.28\,\alpha_2}{(1 + \alpha_2)^2} f_c', \quad \sigma_{1t} = |\alpha_2 \sigma_{2c}| \qquad (5)$$

and failure is assumed to occur by yielding and crushing of the concrete in the compression direction.

c) If $-\infty < \alpha_2 < -0.17$ when is biaxial tension compression with $\sigma_{2c} = 0.65 f_c'$, $\sigma_{1t} = f_t$ and failure in this zone is assumed to be due to cracking in the tension direction.

d) If $1 < \alpha_2 < \infty$ when is biaxial tension with $\sigma_{1t} = f_t$ and failure is by cracking or changes in the principal stress direction. Progressive cracking or changes in the crack orientation are accounted for the cracking model by assuming that the crack direction is always normal to the direction of the maximum principal strain(Hinton,Owen, 1984)

NEW RELATION FOR REINFORCED CONCRETE FAILURE

By means of one experimental testing series on the reinforced concrete cubes with differents dimensions the authors have been determined a new relation for the concrete elements failure.The experimental tests was been made on three types concrete prisms with the dimensions:

- Type I- $h_1 = a$; $h_2 = 0.5a$; $h_3 = h_1 = a$;
- Type II-$h_1 = h_2 = h_3 = a$;
- Type III -$h_1 = a$; $h_2 = 1.5$ a ; $h_3 = h_1 = a$.

For the experimental tests are used eight different reinforcements N=1,...,8 and two concrete classes (C_{15} and C_{25}),The load in the instant before cracks appearance (P_c) and the load in the failure instant was been measured,the prisms being actioned in the diagonal direction.The loads values are presented in the Fig.1.

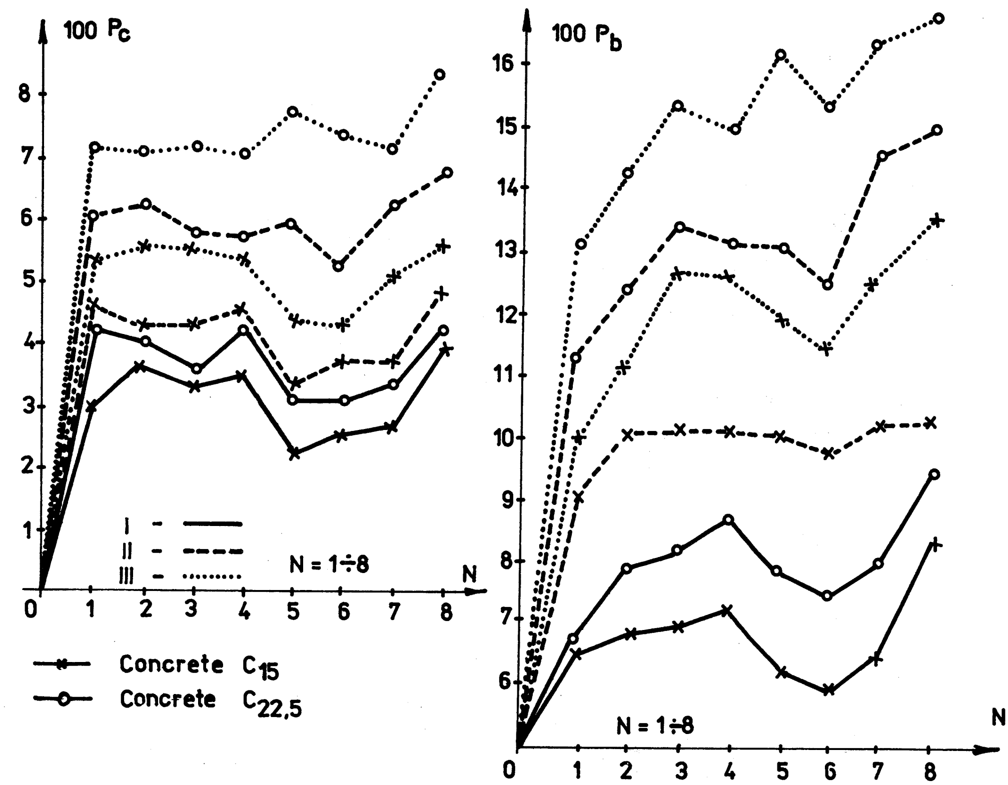

Fig. 1 P_c and P_b loads variation

The mechanical characteristics,the loads and their direction are given in the Fig.2.

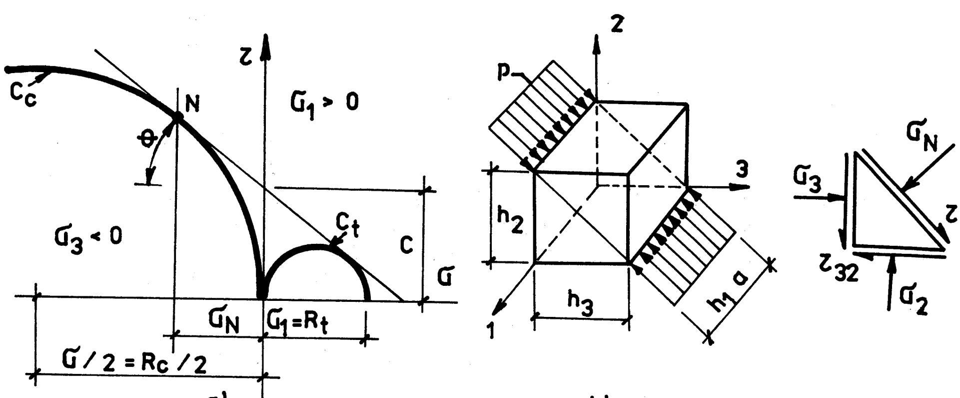

Fig. 2 Mechanical characteristics

Stretching (S_t) and compression (S_c) stresses for two concrete classes,three dimensions types and eight reinforcement variants are illustred in Fig.3a(S_t) and Fig.3b(S_c).With stretching ($S_t = \sigma_1$)stresses,Fig.3a and compression ($S_c = \sigma_3$)stresses was been determined,following Fig.1a,the cohesion c and the internal friction angle θ.

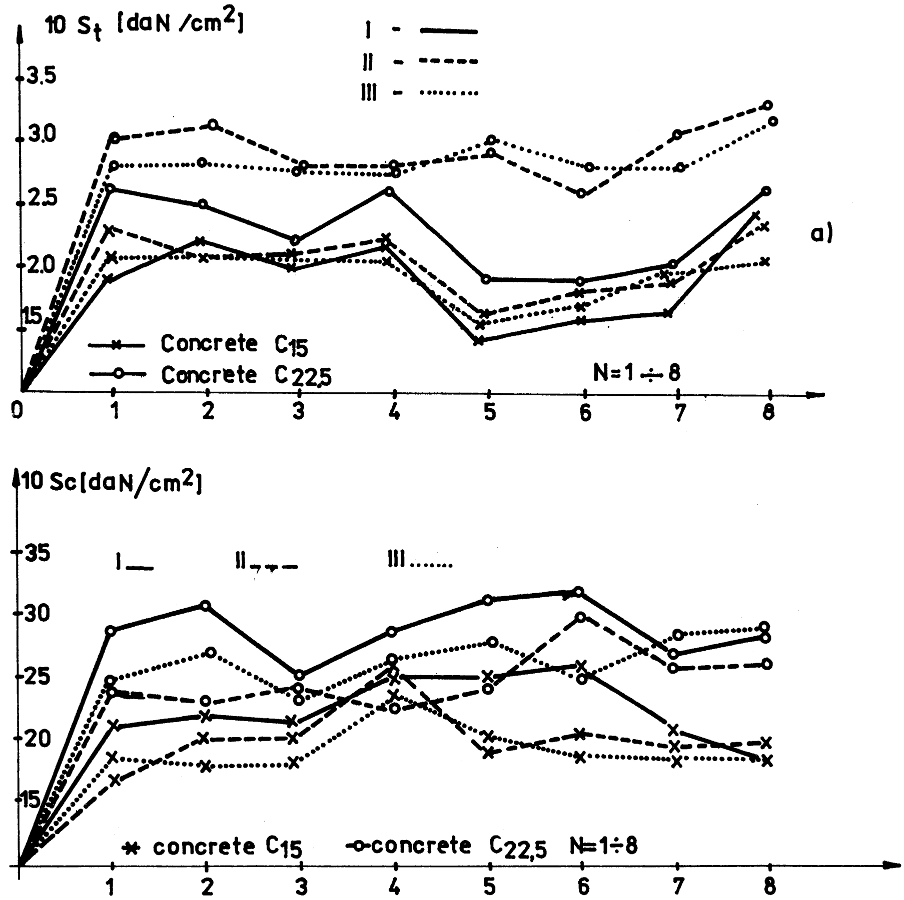

Fig.3 S_t and S_c stresses variation

Stresses values $\sigma_1 = S_t$ and $\sigma_3 = S_c$ was been introduced in the relation (1) resulting the errors between 10.37 % and 26,47 %, graphically presented in the Fig.3.The authors was been determined a new improved relation.On the experimental results,the new failure criterium for reinforced concrete,limited to the cases $\sigma_1 > 0$, $\sigma_3 < 0$, $\alpha_i = \sigma_1 / \sigma_3 < -0.17$ and c)from this paper is

$$\sigma_1 - \sigma_3 + (\sigma_1 + \sigma_3) \sin \theta = c(2 \cos \theta + tg\ \theta) \qquad (6)$$

Introduising the second term from the right hand of the relation (6) for the reinforcement prisms in the variants 1,2,3,4,7,8 it results an error smaller 5%,Fig.4,for two concrete classes considered.From the experimental tests on the 100 concrete samples have been resulted the cohesion $c \cong 32.6$ daN/cm² and $\theta \cong 55°$ for the concrete C_{15} and $c \cong 42.9$ daN/cm², $\theta \cong 54°\ 30'$ for the concrete C_{20}.Supplementary,these constant do not depends on the prism height tested.

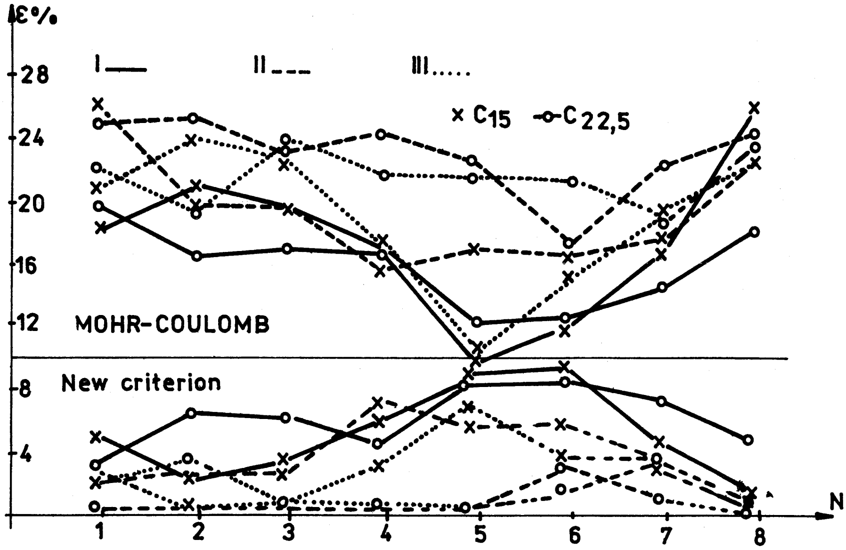

Fig.4 Criteria errors

For the biaxial compression(case a) where $\sigma_3 < 0$ and $\sigma_1 \cong 0.22\sigma_3$ the authors propose the following failure yield relation for reinforced concrete elements

$$\sigma_1 - \sigma_3 + (\sigma_1 + \sigma_3)\sin\theta = 2c(\cos\theta - tg\,\theta) \qquad (7)$$

If it is used the relation (1),the error is greater 50% but with the relation (7) proposed by the authors the error is smaller 6%. The hand member from the relations (6) and (7) represents the yield failure stress value in according with the failure theories from the literature.The relations (6) and (7) may be introduced in the routins of the computer programs based on the finite elements method or the boundary element methods,in order to study the failure of the reinforced concrete structural elements.

CONCLUSIONS

Theoreticaland experimental study made by the authors leads to some original improvings of the failure criterium Mohr-Coulomb. for to using it for the reinforced concrete structural elements with good results.We may make a numerical simulation for the reinforced concrete structures behaviour until the failure,thus obtaining their bearing capacity and the actual safety coefficients.

REFERENCES

Atluri , N. S., Kobayashi, S. A.(1985). Elasto-plastic Fracture (Quasi-static).In Ed. Atluri, S. A.,Comp.Methods in the Mechanics of Fracture,sec.2, North-Holland, pp.55-84.

Gafitanu , M., Poterasu , V.F., Mihalache, N. (1987) FEM and BEM with Machine Elements Applications Design(in romanian). Technical House,Bucuresti.

Mihalache, N.(1988) Behaviour of the Shear Wall Made by Reinforced Concrete to the Shear Forces Action,Doctoral thesis,Polytechnic Institute of Jassy,Civil Engineering Faculty,Romania.

Milford, R.V., Schnobrich, W.C.(1984) Nonlinear Behaviour of Reinforced Concrete Cooling Towers. Civil Engineering Studies. Structural Research Series No.514. University of Illinois, Urbana.

Owen, D.R.J., Hinton, E. (1980) Finite Elements in Plasticity: Theory and Practice. Pineridge Press. Swansea.

Poterasu, V.F., Mihalache, N. (1987) Numerical Simulation for Crack Propagation in a Flexible Dam by BEM Modelling. Simulation & Control, B , AMSE Press, vol. 7, no.3, pp.57-63.

Poterasu, V.F., Mihalache, N.(1985) Elastic Plastic Behaviour of the Seismic Reinforced Concrete Walls by FEM. NUMETA 85, Swansea. Eds. Middleton, J. and Pande, G.N.,Balkema,Rotterdam, pp. 809-814.

Naylor, D.J., Pande, G.N. (1981) Finite Elements in Geotechnical Engineering. Pineridge Press.Swansea.

Hinton, E. , Owen, D.R.J. (1984) Finite Element Software for Plates and Shells. Pineridge Press. Swansea.