An Improved Model for Considering Strain Rate Effects on Reinforced Concrete Elements Behavior under Dynamic Loads

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ABSTRACT

An improved model for predicting the reinforced concrete element behavior under dynamic strain rates was developed using the layer modeling technique. The developed strain rate sensitive model for axial/flexural analysis of reinforced concrete elements was used to predict the test results, performed at different loading rates, and the predictions were reasonable. The developed analysis technique was used to study the loading rate sensitivity of reinforced concrete beams and columns with different geometry and material properties. Two design formulas for computing the loading rate dependent axial and flexural strengths of reinforced concrete sections were also suggested.

INTRODUCTION

Current techniques for analysis and design of reinforced concrete (R/C) structures subjected to dynamic loads are generally based on the quasi-static test results on R/C materials and elements. The rate of straining in such tests is of the order of 10^{-5}/sec (ASTM, 1982) However the typical strain rates in the critical regions of R/C structures under dynamic loads are as high as 10^{-1}/sec or more (Scott, B.D. et al, 1982) To have successful predictions of R/C element behavior under dynamic loads, this discrepancy should be eliminated by developing strain rate sensitive model.

In general, the compressive and tensile strengths and stiffnesses of concrete and the yield strength of steel are observed to increase with increasing strain rate (Sim, J., 1987) As a result of the increases in material strength and stiffness, the axial and flexural strengths and stiffnesses of R/C elements increase at higher loading rates. Hence reliable information of strain rate effects on R/C elements is needed to be modeled in dynamic analysis and design procedures.

MATERIAL CONSTITUTIVE MODELS

Two empirical strain rate-dependent constitutive laws of concrete (Soroushian, P. et al, 1986) and steel (Soroushian, P. et al, 1987) were adopted in this study. The adopted strain rate-dependent constitutive law of concrete is given in Equation (1) :

$$f_c(\epsilon, \dot{\epsilon}) = \begin{cases} K_1 K_2 f'c [[\frac{2\epsilon}{0.002 K_1 K_3} - [\frac{\epsilon}{0.002 K_1 K_3}]^2] & \text{for } \epsilon < 0.002 K_1 K_3 \\ K_1 K_2 f'c [1 - Z(\epsilon - 0.002 K_1 K_3)] \geqq 0.2 K_1 K_2 f'c & \text{for } \epsilon > 0.002 K_1 K_3 \end{cases} \quad (1)$$

Where, f_c = concrete stress (psi), ϵ = strain, $\dot{\epsilon}$ = strain rate (1/sec) $\geqq 10^{-5}$, $K_1 = 1 + \frac{\rho_s f_{yh}}{f'_c}$

ρ_s = valumetric ratio of transverse reinforcement to concrete core

f_{yh} = yield strength of transverse reinforcement (psi)

f'_c = standard (quasi-static) compressive strength of concrete (psi)

$$K_2 = \begin{cases} 1.48 + 0.160 \log_{10} \dot{\epsilon} + 0.0127 (\log_{10} \dot{\epsilon})^2 & \text{for air - dired concrete} \\ 2.54 + 0.580 \log_{10} \dot{\epsilon} + 0.0543 (\log_{10} \dot{\epsilon})^2 & \text{for saturated concrete} \end{cases}$$

$$K_3 = 1.08 + 0.112 \log_{10} \dot{\epsilon} + 0.0193 (\log_{10} \dot{\epsilon})^2$$

$$Z = \frac{0.5}{\frac{3 + 0.002 f'_c}{f'_c - 1000} + \frac{3}{4} \rho_s \sqrt{\frac{h'}{s}} - 0.002 K_1 K_3}$$

h' = width of concrete core measured outside of transverse reinforcement

s = center to center spacing of transverse reinforcement

The empirical strain rate-dependent constitutive law of steel adopted in this study is given in Equation (2):

$$f_s(\epsilon, \dot{\epsilon}) = \begin{cases} E_s \cdot \epsilon & \text{for } \epsilon \leq \frac{f'_y}{E_s} \\ f'_y + E'_h \left(\epsilon - \frac{f'_y}{E_s}\right) & \text{for } \frac{f'_y}{E_s} < \epsilon < \epsilon'_u \\ 0 & \text{for } \epsilon \geq \epsilon'_u \end{cases} \tag{2}$$

where, f_s = steel stress (psi), E_s = modulus of elasticity of steel (psi)

f'_y = dynamic yield strength of steel (psi)

$= f_y [-4.51 \times 10^{-6} f_y + 1.46 + (-9.20 \times 10^{-7} f_y + 0.0927) \log_{10} \dot{\epsilon}]$

f_y = standard (quasi-static) yield strength of steel (psi)

E'_h = dynamic strain hardening modulus of steel (psi), $\dot{\epsilon}$ = strain rate (1/sec) $\geqq 10^{-5}$

ϵ'_u = dynamic ultimate strain of steel

$= \epsilon_u [-8.93 \times 10^{-6} f_y + 0.007 + (4 \times 10^{-6} f_y - 0.1850) \log_{10} \dot{\epsilon}]$

ϵ_u = quasi-static ultimate strain of steel

SECTION MODEL

The axial/flexural tangent stiffness matrix of R/C cross sections was derived using the layer modeling technique (Kaba, S.A. et al, 1984) In this technique, the cross section is divided into a number of concrete and steel layers (Fig. 1). The strains and strain rates at these layers are calculated from the values of strain and strain rate of the section plastic centroid, and curvature and curvature rate of the section (assuming that plain sections remain plain):

$$d\epsilon_i = d\epsilon_p + d\phi \, y_i \tag{3}$$

$$\dot{\epsilon} = \dot{\epsilon}_p + \dot{\phi} \, y_i \tag{4}$$

where, $d\epsilon_i$ = strain increment in the i'th layer, $\dot{\epsilon}_i$ = strain rate in the i'th layer

$d\epsilon_p$ = strain increment at plastic centroid, $\dot{\epsilon}_p$ = strain rate at plastic centroid

$d\phi$ = section curvature increment, $\dot{\phi}$ = section curvature rate

y_i = distance from the centroid of the i'th layer to the plastic centroid

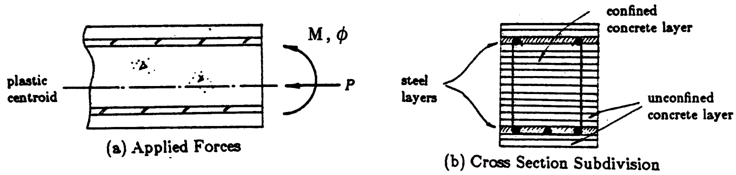


Figure 1. Layer Modeling of R/C Sections.

Once the layer strains and strain rates are obtained, tangent stiffness moduli (E_i) of various layers can be calculated using the strain rate-dependent steel and concrete constitutive models presented earlier:

$$E_i = \frac{df(\epsilon_i, \dot{\epsilon}_i)}{d\epsilon_i} \tag{5}$$

Where, $f(\epsilon_i, \dot{\epsilon}_i)$ = constitutive equation expressing the value of stress at the i'th layer i in terms of strain and strain rate

The section stiffness matrix (K_s) can then be constructed by proper summation of the layer tangent stiffness moduli:

$$\begin{bmatrix} dM \\ dP \end{bmatrix} = K_s \begin{bmatrix} d\phi \\ d\epsilon_p \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \cdot \begin{bmatrix} d\phi \\ d\epsilon_p \end{bmatrix} \tag{6}$$

Where, dM = bending moment increment at the cross section

dP = axial load increment at the cross section

$K_{11} = \sum_i A_i E_i y_i^2$, $K_{12} = K_{21} = \sum_i A_i E_i y_i$, $K_{22} = \sum A_i E_i$, A_i = area of the i'th layer

ELEMENT MODEL

The section stiffness matrix drived above was incorporated into an algorithm for nonlinear axial/flexural analysis of R/C elements. The inflection point under bending is assumed to be in the mid point of beam in the element model for the simplicity purpose. Thus the element model for axial/flexural analysis of R/C cantilever elements was constructed by dividing the element by a number of slices a long its length as shown in Fig. 2. A iterative numerical procedure for inelastic flexural or axial analysis of R/C cantilever elements was used. This method accounts for the loading rate effects as well as the material and geometric nonlinearities.

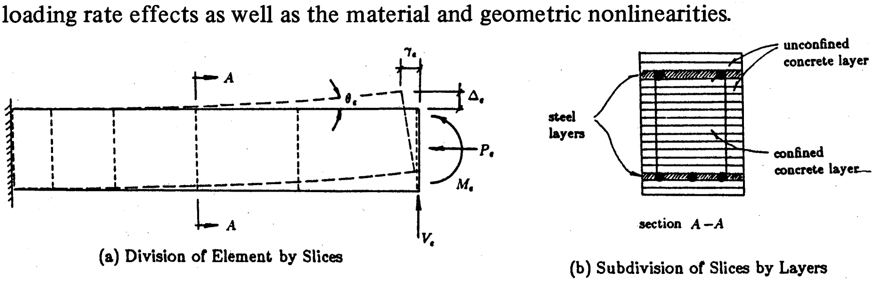
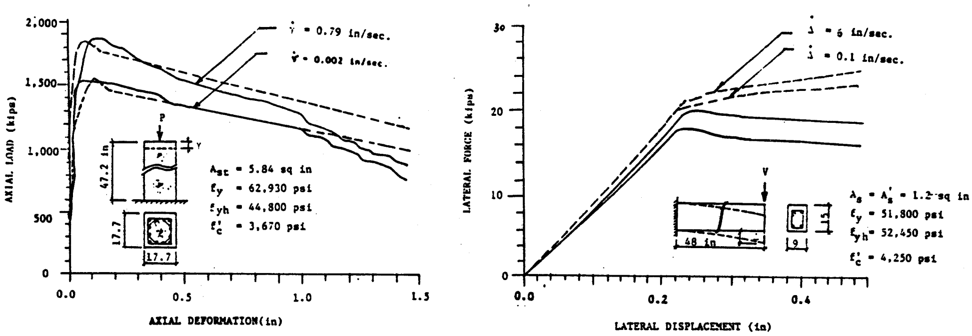


Figure 2. Modeling of R/C Elements.

The developed strain rate sensitive methods for axial/flexural analysis of R/C elements was used to predict the results of some tests on R/C columns (Scott, B.D. et al, 1982) and beams (Celebi, M. et al, 1973) Fig. 3 compares the experimental results with the predictions of the suggested strain rate sensitive analysis technique. The proposed analytical approach is observed to compare satisfactorily with test results.

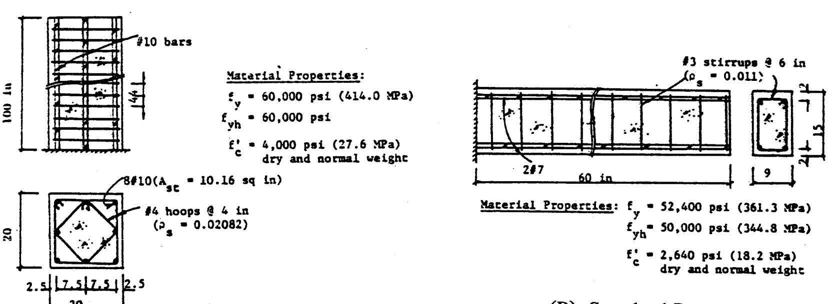


(A) Axial performance (B) Flexural performance.

Figure 3. Experimental and Analytical Performances of R/C Elements at Different Loading Rates.

PARAMETRIC STUDIES

The developed strain rate dependent element model was used to study the effects of the variations in different material and geometric properties of R/C columns and beams on the sensitivity of their axial/flexural behavior. The typical column shown in Fig. 4 (a) and beam shown in Fig. 4 (b) were chosen as standard elements in this study.



(A) Standard Column (B) Standard Beam

Figure 4. Standard Elements.

The effects of loading rate variations on the column axial strength and secant stiffness are summarized, among other information, in Table 1. In the table, Pn and Pn$'$ are the quasi-static and dynamic axial strength, and K and K$'$ are the quasi -static and dynamic secant axial stiffnesses, respectively. Table 1 also summarizes the strain rate effects on columns similar to the standard column, except the change in a single variable which could be confinement, concrete compressive strength, degree of saturation of concrete, or cross sectional shape. From the data presented in Table 1, it may be concluded that the variations in steel ratio, concrete compressive strength, confinement and cross sectional shape of columns do not significantly influence the column loading rate-sensitivity. There is, however, a slight increase in the loading rate-sensitivity of column axial strength with decreasing yield strength of steel. Both the axial strength and stiffness of the saturated concrete column are also observed to be far more loading rate-sensitive than those of dry concrete columns.

Table 2 summarizes the loading rate effects on the lateral strength of the standard R/C beam as well as those similar to the standard one except the change in a single variable which could be steel yield strength, concrete compressive strength, the ratio of compression or tension steel, confinement, degree of concrete saturation, or the beam geometry. In this table, V and V$'$ are the quasi-static and dynamic values of lateral strength, respectively. From the data presented in Table 2, it can be concluded that R/C beams with lower yield strength steels and higher steel ratios are more sensitive to the variations in loading rate. Beams with saturated concrete are also more loading rate-sensitive than those with air-dried concrete. The variations in concrete compressive strength and confinement, element length, and cross sectional shape do not seem to influence the loading rate-sensitivity of the flexural behavior of R/C beams.

Table 1. Summary of the loading rate effects on the axial behavior of R/C columns.

SECTION	$\dot{\gamma}$ (in/sec)	P'_n / P_n refined analysis	P'_n / P_n suggested eqn. (7)	K / K refined analysis
standard	0.5	1.16	1.16	1.22
	5.0	1.25	1.25	1.28
	50.0	1.36	1.35	1.31
low f_y (f_y = 40 ksi)	0.5	1.18	1.17	1.23
	5.0	1.28	1.27	1.28
	50.0	1.40	1.39	1.37
high f_y (f_y = 80 ksi)	0.5	1.12	1.13	1.23
	5.0	1.20	1.21	1.29
	50.0	1.31	1.30	1.32
low A_{st} (A_{st} = 8.0 in^2)	0.5	1.16	1.16	1.23
	5.0	1.26	1.25	1.29
	50.0	1.37	1.37	1.32
high A_{st} (A_{st} = 12.48 in^2)	0.5	1.15	1.15	1.22
	5.0	1.24	1.24	1.27
	50.0	1.35	1.34	1.30
low confinement (ρ_s = 0.0188)	0.5	1.14	1.16	1.23
	5.0	1.23	1.25	1.28
	50.0	1.35	1.35	1.31
high confinement (ρ_s = 0.4164)	0.5	1.17	1.16	1.23
	5.0	1.25	1.24	1.26
	50.0	1.37	1.35	1.31
low f'_c (f_c = 3 ski)	0.5	1.16	1.15	1.21
	5.0	1.25	1.24	1.26
	50.0	1.36	1.34	1.29
high f'_c(f_c = 6 ksi)	0.5	1.15	1.16	1.24
	5.0	1.25	1.20	1.30
	50.0	1.37	1.37	1.33
saturated concrete	0.5	1.39	1.37	1.51
	5.0	1.69	1.65	1.78
	50.0	2.07	2.00	2.05
circular shape (D = 22.56 in)	0.5	1.23	1.16	1.15
	5.0	1.29	1.25	1.24
	50.0	1.31	1.35	1.36

Table 2. Summary of the loading rate on the flexural behavior of R/C beams.

SECTION	$\dot{\Delta}$ (in/sec)	V' refined analysis	V suggested egn. (8)
Standard	0.1	1.08	1.09
	1.0	1.11	1.13
	5.0	1.15	1.18
low f_y	0.1	1.11	1.10
	1.0	1.14	1.15
	5.0	1.17	1.21
high f_y (f_y = 60 ksi)	0.1	1.08	1.08
	1.0	1.10	1.12
	5.0	1.13	1.16
low A_s' (A_s'= 0.61 in^2)	0.1	1.08	1.09
	1.0	1.12	1.13
	5.0	1.17	1.18
high A_s (A_s = 2.0 in^2)	0.1	1.10	1.09
	1.0	1.15	1.14
	5.0	1.18	1.18
high confinement (ρ_s = 0.016)	0.1	1.07	1.09
	1.0	1.10	1.13
	5.0	1.15	1.18
high f'_c (f'_c = 6 ksi)	0.1	1.07	1.09
	1.0	1.12	1.13
	5.0	1.15	1.17
saturated concrete	0.1	1.09	1.09
	1.0	1.12	1.12
	5.0	1.17	1.16
T-beam (flange width = 18 in)	0.1	1.08	1.07
	1.0	1.11	1.12
	5.0	1.15	1.16
short beam (l = 40 in)	0.1	1.09	1.10
	1.0	1.12	1.14
	5.0	1.15	1.19

DESIGN FORMULAS

This study also suggested two practical design formulas for computing the loading rate dependent axial and flexural strengths of R/C sections.

The column axial strength at the high loading rates can be calculated by Eq. (Celebi, M. et al, 1973) which has the same concept of ACI 318-83 code (ACI Committee, 1983) except for the strain rate.

$$P'_n = 0.85 f''_c (A_g - A_{st}) + A_{st} f_y \tag{7}$$

Where, A_g = gross cross sectional area of concrete

A_{st} = total area of longitudinal steel

f''_c = dynamic compressive strength of concrete (psi)

$$= \begin{cases} f'_c \; [1.48 + 0.160 \log_{10} \dot{\epsilon} + 0.0127 \; \log_{10} \dot{\epsilon})^2 \,] & \text{for air-dried concrete.} \\ f'_c \; [2.54 + 0.580 \log_{10} \dot{\epsilon} + 0.0543 \; \log_{10} \dot{\epsilon})^2 \,] & \text{for saturated concrete.} \end{cases}$$

= strain rate (1/sec) $\geqq 10^{-5}$; f_y = standard (quasi - static) yield strength of steel (psi)

Table 1 compares the ratios of dynamic to static compressive strengths obtained from the above equation with those obtained from the refined (layer) analysis procedure described earlier. The suggested simple approach is observed to compare well with the results of the refined analysis.

The beam flexural strength at the high strain rates also can be calculated by Eq. (8) using f_c'' and f_y' instead of f_c' and f_y in the ACI 318-83 code.

$$M_n' = As\,f_y\ (d - d') + 0.85\, f_c''\ a\ b\ (a/2 - d') \qquad (8)$$

The suggested procedure for calculating the loading rate-dependent flexural strength of R/C sections was applied to all the beams used in the numerical study discussed earlier. The results are compared with the predictions of the layer modeling technique in Table 2, and the comparison is observed to be quite satisfactory.

CONCLUSIONS

Empirical strain rate sensitive constitutive laws of concrete and steel were incorporated into an analytical technique for axial/flexural analysis of R/C beam-columns at different loading rates. The suggested analytical predictions compared well with both quasi-static and dynamic test results. This method was also used to perform a parametric study on the loading rate sensitivity of the axial and flexural behavior of R/C elements. Two practical design formulas were suggested for calculating the axial and flexural strengths of R/C columns and beams as functions of the applied loading rates.

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