Channel Walls Interaction Through the Liquid

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I. INTRODUCTION

1.1. Subject. The problem discussed is a part of more large investigation on dynamical behaviour of water-filled cavities with a special geometry and bounded partially by elastic walls. Such cavities are broadly used as structural components in the buildings of the classical (hydraulic) power stations; in nuclear reactor technologies (as tanks and different basins (e.g. for pressure suppression, etc.)); in chemical industry and biotechnologies (as reactors, brawing chambers, etc.); in architecture as usual premises with window plates (or door plates, etc.); in ship structures, etc.

1.2. Physical description. In a thinwalled channel with a rectangular cross section inviscid, compressible and heavy liquid is flowing with a velocity on an infinity $V \ll c$, where c is its sound velocity. Two of the channel walls are infinitely rigid, other pair of them being characterized by a finite given rigidities EI_1 & EI_2 correspondingly. All geometrical dimensions of the cross section are given, Fig. 1. The channel is infinitely long and the whole problem does not depend on the longitudinal co-ordinate. So the problem is planar one.

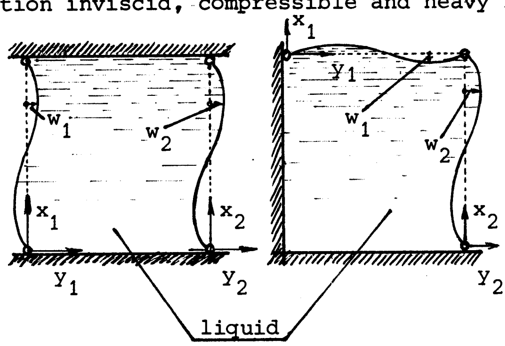

Fig. 1. Two schemes of the channel wall interaction.

The schemes of the channels cross sections with the disposition of the co-ordinate systems and corresponding vibrating walls are shown on Fig. 1. For the first scheme $w_1 = w_1(x,t)$ and $w_1 = w_2(x,t)$ are displacement functions and for the second scheme $w_1 = w_1(y,t)$ and $w_2 = w_2(x,t)$. All demonstration will be provided following No 1 scheme.

1.3. Analytical description of the problem. It is supposed that the "hydroelastic" mechanical system "two plates + liquid" is in state of free vibration with small amplitudes of w_1 and w_2 generating into liquid small acoustical perturbations. The displacement functions $w_1(x,t)$ & $w_2(x,t)$ have to satisfy the boundary problem:

(1) $(EI)_1 w_1^{IV} + \bar{m}_1 \ddot{w}_1 = -p_a - p_g, \quad (EI)_2 w_2^{IV} + \bar{m}_2 \ddot{w}_2 = +p_a + p_g;$

(2) $x = 0,L: \quad w_1 = 0, \quad w_1'' = 0, \quad w_2 = 0, \quad w_2'' = 0,$

where $\bar{m}_1$ & $\bar{m}_2$ - mases per $1m^2$ of the walls vibrated; p_a - acoustical pressure; p_g - gravitational (statical) pressure; $(-)^{IV} = (\partial^4/\partial x^4)$; $(\dot{-}) = (\partial/\partial t)$. Further, the liquid motion is supposed to be potential one with velocity potential function $\varphi = \varphi(x,y;t)$ which has to satisfy the boundary problem:

(3) $\Delta\varphi - (1/c^2).(\partial^2\varphi/\partial t^2) = 0, \quad \Delta = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$;

(4) $$\begin{array}{llll} x = 0 & (\partial\varphi/\partial x) = 0; & y = 0 & (\partial\varphi/\partial y) = (\partial w_1/\partial t), \\ x = h & (\partial\varphi/\partial x) = 0; & y = L & (\partial\varphi/\partial y) = (\partial w_2/\partial t). \end{array}$$

If the velocity potential is known,

(5) $\vec{v} = \text{grad}\varphi \; , \quad p_a = -\rho.(\partial\varphi/\partial t)$.

As the gravitation pressure (including preliminary inner pressure p_o) $p_g = -p_o - \rho.g.(h-x)$ is known, we shall not consider it furter.

II. ANALYTICAL SOLUTION

2.1. Local co-ordinates. To be able to treat the problems of both geometrical schemes (Fig. 1) local co-ordinate systems $(0,x,y_1)$ and $(0,x,y_2)$ are introduced. They are connected by the formulas

(6) $x_1 \equiv x_2 \equiv x; \; y_1 = L + y_2, \; z_1 \equiv z_2 \equiv z$.

2.2. Local potential functions. Let us introduce so called local denotations of a velosity potential function

(7) $\varphi_I = \varphi_1 + \varphi_{2(2\to1)}, \quad \varphi_{II} = \varphi_{1(1\to2)} + \varphi_2$

which have to be read: "The velocity potential of all liquid motion in near vicinity of plate No 1 is equal to the sum of the "own" velocity potential function φ_1 describing the liquid motion generated by No 1 plate vibrations and velocity potential function φ_2, but rewritten down in co-ordinate system No 1, describing the liquid motion generated by No 2 plate vibrations".

2.3. Two parallel boundary problems. Velosity potential function φ_I (or φ_{II}) has to be a solution of differential equation (3) and this leads to:

(9) $\Delta\varphi_1 - (1/c^2).\ddot{\varphi}_1 = 0, \quad \Delta\varphi_2 - (1/c^2).\ddot{\varphi}_2 = 0$.

Boundary conditions (4) transform themselves into:

(10) $$\begin{array}{lll} y_1 = 0 & (\partial\varphi_1/\partial y_1) = \dot{w}_1, & \partial\varphi_{2(2\to1)}/\partial y_1 = 0; \\ y_2 = 0 & \partial\varphi_{1(1\to2)}/\partial y_2 = 0, & (\partial\varphi_2/\partial y_2) = \dot{w}_2. \end{array}$$

2.4. Integrals of Eqs. (9). The integrals of Eqs. (9), satisfying conditions (10) are obtained into forms:

(11) $$\begin{aligned} \varphi_1 &= -\sum_m A_m \cos(m\pi x/h) \cos[k_{2m}(L-y_1)]\, \sigma \sin(\sigma t) \\ \varphi_2 &= -\sum_m B_m \cos(m\ x/h) \cos[k_{2m}(L+y_2)]\, \sigma \sin(\sigma t) \end{aligned}$$

where σ is generally said any circular frequency providing by its special values $\sigma = \omega_1, \sigma = \omega_2, \sigma = \omega_3, \ldots$ the natural frequences spectrum of the hydroelastic system; k_{2m} is wave number which has to be determined from the dispersion equation

(12) $k_{2m}^2 = (\sigma/c)^2 - (m\pi/h)^2$.

Constants A_m & B_m are unknown on the present stage...

2.5. Displacement functions w_1 & w_2. Let the initial separations

$$(13)\quad w_1(x,t) = W_1(x)\cos(\sigma t), \qquad w_2(x,t) = W_2(x)\cos(\sigma t)$$

are introduced. Then, both Eqs. (1) will become

$$(14)\quad \begin{aligned} W_1^{IV} - \Omega_1^2 W_1 &= \sum_m S_1[A_m\cos(k_{2m}L) + B_m]\cos(m\pi x/h), \\ W_2^{IV} - \Omega_2^2 W_2 &= -\sum_m S_2[A_m + B_m\cos(k_{2m}L)]\cos(m\pi x/h), \end{aligned}$$

where

$$(15)\quad \Omega_j^4 = [\bar{m}_j\sigma^2/(EI)_j], \quad S_j = [\rho\sigma^2/(EI)_j], \quad j = 1,2,$$

and

$$(16)\quad W_1 = W_1'' = 0, \quad W_2 = W_2'' = 0 \text{ at } x = 0,h.$$

Following the classical procedure one can obtain the integrals of (14) in the forms:

$$(17)\quad \begin{aligned} W_1 &= W_1^o(\Omega_1 x) + \sum_m (S_1/q_{1m})[A_m\cos(k_{2m}L) + B_m]\cos(m\pi x/h), \\ W_2 &= W_2^o(\Omega_2 x) - \sum_m (S_2/q_{2m})[A_m + B_m\cos(k_{2m}L)]\cos(m\pi x/h), \end{aligned}$$

where

$$(18)\quad \begin{aligned} W_1^o(\Omega_1 x) &= C_1^{(1)}\cos(\Omega_1 x) + C_2^{(1)}\sin(\Omega_1 x) + C_3^{(1)}\mathrm{ch}(\Omega_1 x) + C_4^{(1)}\mathrm{sh}(\Omega_1 x), \\ W_2^o(\Omega_2 x) &= C_1^{(2)}\cos(\Omega_2 x) + C_2^{(2)}\sin(\Omega_2 x) + C_3^{(2)}\mathrm{ch}(\Omega_2 x) + C_4^{(2)}\mathrm{sh}(\Omega_2 x). \end{aligned}$$

Satisfying boundary conditions (10) one can obtain the system

$$(19)\quad \begin{aligned} &\sum_m A_m\left[k_{2m}\sin(k_{2m}L) - \frac{S_1}{q_{1m}}\cos(k_{2m}L)\right] - B_m\frac{S_1}{q_{1m}} \quad \cos\frac{m\pi x}{h} = W_1^o(\Omega_1 x), \\ &\sum_m A_m\frac{S_2}{q_{2m}} - B_m\left[k_{2m}\sin(k_{2m}L) - \frac{S_2}{q_{2m}}\cos(k_{2m}L)\right] \quad \cos\frac{m\pi x}{h} = W_2^o(\Omega_2 x) \end{aligned}$$

from which the constants A_m & B_m can be determined. Putting them into Eqs. (17) the modes sought can be represented into the forms:

$$(20)\quad \begin{aligned} W_1 = {} & C_1^{(1)}\sum_m a_{1m}[1-Q_m^{(11)}]\cos\frac{m\pi n}{h} + C_1^{(2)}\sum_m b_{1m}Q_m^{(21)}\cos\frac{m\pi x}{h} + \\ &+ C_2^{(1)}\sum_m a_{2m}[1-Q_m^{(11)}]\cos\frac{m\pi x}{h} + C_2^{(2)}\sum_m b_{2m}Q_m^{(21)}\cos\frac{m\pi x}{h} + \\ &+ C_3^{(1)}\sum_m a_{3m}[1-Q_m^{(11)}]\cos\frac{m\pi x}{h} + C_3^{(2)}\sum_m b_{3m}Q_m^{(21)}\cos\frac{m\pi x}{h} + \\ &+ C_4^{(1)}\sum_m a_{4m}[1-Q_m^{(11)}]\cos\frac{m\pi x}{h} + C_4^{(2)}\sum_m b_{4m}Q_m^{(21)}\cos\frac{m\pi x}{h}, \end{aligned}$$

where (i) only constants $C_j^{(1)}$ & $C_j^{(2)}$, $j = 1,2,3,4$ are unknown; (ii) a_{jm} & b_{jm} are obtained from

$$\int_o^h W_1(\Omega_1 x)\cos(m\pi x/n)\,dx \;\&\; \int_o^h W_2(\Omega_2 x)\cos(m\pi x/h)\,dx$$

see below; second form will be

$$(21)\quad W_2 = C_1^{(2)} \sum_m b_{1m}[1-Q_m^{(22)}]\cos\frac{m\pi x}{h} + C_1^{(1)} \sum_m a_{1m}Q_m^{(12)}\cos\frac{m\pi x}{h} +$$
$$+ C_2^{(2)} \sum_m b_{2m}[1-Q_m^{(22)}]\cos\frac{m\pi x}{h} + C_2^{(1)} \sum_m a_{2m}Q_m^{(12)}\cos\frac{m\pi x}{h} +$$
$$+ C_3^{(2)} \sum_m b_{3m}[1-Q_m^{(22)}]\cos\frac{m\pi x}{h} + C_3^{(1)} \sum_m a_{3m}Q_m^{(12)}\cos\frac{m\pi x}{h} +$$
$$+ C_4^{(2)} \sum_m b_{4m}[1-Q_m^{(22)}]\cos\frac{m\pi x}{h} + C_4^{(1)} \sum_m a_{4m}Q_m^{(12)}\cos\frac{m\pi x}{h},$$

where:

$$(22)\quad \begin{aligned} a_{1m} &= \int_0^h \cos(\Omega_1 x)\cos\frac{m\pi x}{h}\,dx, & b_{1m} &= \int_0^h \cos(\Omega_2 x)\cos\frac{m\pi x}{h}\,dx,\\ a_{2m} &= \int_0^h \sin(\Omega_1 x)\cos\frac{m\pi x}{h}\,dx, & b_{2m} &= \int_0^h \cos(\Omega_2 x)\cos\frac{m\pi x}{h}\,dx,\\ a_{3m} &= \int_0^h \mathrm{ch}(\Omega_1 x)\cos\frac{m\pi x}{h}\,dx, & b_{3m} &= \int_0^h \mathrm{ch}(\Omega_2 x)\cos\frac{m\pi x}{h}\,dx,\\ a_{4m} &= \int_0^h \mathrm{sh}(\Omega_1 x)\cos\frac{m\pi x}{h}\,dx, & b_{4m} &= \int_0^h \mathrm{sh}(\Omega_2 x)\cos\frac{m\pi x}{h}\,dx, \end{aligned}$$

and additionally:

$$(23)\quad \begin{aligned} Q_m^{(11)} &= d_m^{-1} S_1[k_{2m}q_{2m}\cos(k_{2m}L) + S_2\sin(k_{2m}L)](2/h),\\ Q_m^{(22)} &= d_m^{-1} S_2[k_{2m}q_{1m}\cos(k_{2m}L) + S_1\sin(k_{2m}L)](2/h),\\ Q_m^{(21)} &= d_m^{-1} S_1\, k_{2m}q_{2m}(2/h), \qquad d_m^{-1} = (1/d_m),\\ Q_m^{(12)} &= d_m^{-1} S_2\, k_{2m}q_{1m}(2/h) \end{aligned}$$

$$(24)\quad d_m = (S_1 q_{2m} + S_2 q_{1m}).k_{2m}\cos(k_{2m}L) + S_1 S_2 \sin(k_{2m}L).$$

2.6. Natural frequences spectrum. Together with the integration constants in Eqs. (20) & (21) one parameter only (namely) wave number k_{2m}) is unknown. If one determines it, e.g.

$$(25)\quad (k_{2m}L) = k^*_{2mn},\ n = 1,2,3,\ldots,\infty \Rightarrow k_{2mn} = (k^*_{2mn}/L),$$

then from Eq. (12)

$$(26)\quad \omega_{mn} = +c\sqrt{(m\pi/h)^2 + (k^*_{2mn}/L)^2} \equiv \sigma_{mn},\ m < m^*,$$

where m* is a number of the members remained in (20) & (21).

Well, satisfying boundary conditions (16) and following the habitual procedure is easy to obtain the main determinant with 8×8 elements constructed from the coeficients in front of unknown integration constants $C_j^{(1)},\ldots,C_j^{(2)}$, (j = 1,2,3,4) and to equate it to zero:

$$(27)\quad \Delta_m[\sigma, k_{2m}, \Omega_1, \Omega_2, S_1, S_2, q_{1m}, q_{2m}, (m\pi/h)] = 0.$$

This will be the equation providing unknown wave numbers k_{2mn} and the natural frequenses sought.

2.7. Vibration modes. Following the prescription of modal analysis and eigen values problems, the obtained k_{2mn} & ω_{mn} must be put into (20) & (21). All constants $C_{jn}^{(1)}$, $C_{jn}^{(2)}$ except one only [e.g. $C_{1n}^{(1)}$] will be determined by the ratios

(28) $\beta_{jn}^{(1)} = C_{jn}^{(1)}/C_{1n}^{(1)}$, $\beta_{jn}^{(2)} = C_{jn}^{(2)}/C_{1n}^{(1)}$.

The last constant (namely $C_{1n}^{(1)}$) must be determined from any norming condition taking into account that the forms $W_n^{(1)}$ & $W_n^{(2)}$ are not mutualy orthogonal, [Kito, 1970].

3. ANALYSIS

3.1. Liquid link. The liquid is playing role of additional link between both structural components vibrating. That is why the vibration modes of every plate are linear sum of the vibration modes of both plates.

3.2. Structural components interaction through the liquid can be find in Ref.[5] (Sheinin, 1967) but in pure shape the idea is refined by Dzhupanov (1974).

3.3. Symetry principle. If one want to separate the mutual influence of the plates due to the liquid, he will see that

$$\bar{Q}_m^{(21)} = d_m^{-1} k_{2m} q_{1m} q_{2m} = \bar{Q}_m^{(12)}.$$

This expresses the Symmetry hydroacoustical principle (Shenderov, 1972). The symmetry of interaction matrices is considered in the fundamental work of Chen & Wambsgans (1972) and investigated in details, afterwants, e.g. in the article of Golovkina, Dzhupanov & Yanatchkov (1984).

3.4. Inertia & Damping. It is interesting to be marked that the influence of the "own" interaction of the each plate with the liquid diminishes the amplitudes of the vibration modes having the reciprocal influence of the oposite plate (which increases the same amplitudes).

3.5. On the natural frequences spectrum. Obviously equating to zero the quantity expressed by Eq. (24) one can obtain another spectrum of natural frequenses. If any of the natural frequenses determined by (27) & (26) coincides with any of aforementioned natural frequenses in the liquid will occure a disordered jam of chaotic modes but the vibrational modes of the plates will not become infinite.

4. REFERENCES

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