Study and Analysis for the Flow–Induced Vibration of the Core Barrel of a PWR

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INTRODUCTION

The core barrel of a PWR is one of the key components of the reactor internals. The existence of coolant flow may cause the vibration of the barrel. The effect of protracted vibration may cause the fatigue of the barrel, wear away the connections, and even bring about accident or shutdown. Theoretically, it is very difficult to obtain the flow–induced pulse pressure acting on the barrel structure because of the complexity of fluid flow in the reactor. The objective of this paper lies in demonstrating the effectiveness of the following method to study the flow–induced vibration of the barrel.

In this research, modal analysis tests on a steel pressure vessel and barrel model (1:10) are performed in the static water to obtain the dynamic characteristics of the barrel in the water. The pulse pressure acting on the surface of the barrel, and the acceleration and strain response are measured in a hydraulic test loop. The random vibration equation which considers the effect of the 'additional mass' of the water is solved by the statistic method to verify the hydraulic vibration test. On the basis of correspondence between the analysis and the test, the analysis of hydraulic vibration characteristics for the reactor barrel of 300 MWe Qinshan Nuclear Power Plant is carried out.

THE SIMILITUDE RELATIONS OF THE MODEL TEST

The researches presented here belong to dynamic problems of fluid–structure interaction. The main attention is concentrated on maintaining the dynamic similarity of the Strouhal number while neglecting the static similarity of the Reynolds number. The similitude criteria are listed in Table 1.

Table 1. Similitude criteria of fluid–structure interaction

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Structure</th>
<th>F/S boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>St=V/Lf</td>
<td>$f^*$</td>
<td>$f^*$/f_E</td>
</tr>
<tr>
<td>Eu=p/PeV</td>
<td>$p^*$</td>
<td>$p^*$/E</td>
</tr>
<tr>
<td>Re=VL/ν</td>
<td>V/f</td>
<td>V/f_E</td>
</tr>
</tbody>
</table>

In Table 1, V is the velocity of the fluid, p, the fluid pressure, $p^*$, the pulse pressure acting on fluid surface of the structure, L, the dimension, $f^*$, dynamical viscous coefficient of the fluid, $f$, the amplitude, $f_E$, the frequency, $p^*$, and $f^*$, the densities of the fluid and the solid, and E, the modulus of the solid elasticity.

From Table 1 the proportional relations of various parameters between the
real barrel and the 1:5 model made of the same material can then be acquired
as shown in Table 2.

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
<th>Proportion Model</th>
<th>Real object</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Dimension</td>
<td>cm</td>
<td>L</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Amplitude</td>
<td>cm</td>
<td>S</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>Velocity</td>
<td>m/s</td>
<td>V</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Acceleration</td>
<td>m/s²</td>
<td>a</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>Volumetric flow</td>
<td>m³/hr</td>
<td>Q</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>Time</td>
<td>s</td>
<td>t</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>Frequency</td>
<td>Hz</td>
<td>f</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>Pressure</td>
<td>Pa</td>
<td>p</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>Stress</td>
<td>MPA</td>
<td>s</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>Pressure power</td>
<td>(Pa)/Hz</td>
<td>Gp</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>11</td>
<td>Stress power</td>
<td>(MPa)/Hz</td>
<td>Gs</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>Acceleration power</td>
<td>(m/s³)/Hz</td>
<td>Ga</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

MODAL ANALYSIS TESTS ON A BARREL MODEL

The purpose to undertake modal analysis tests on the model in combination
with the pressure vessel lies in obtaining the natural vibration character-
istics of the barrel both in the air and in the static water, including
mode frequencies, mode shapes, modal mass, modal stiffness and damping ratio
for both the beam and the shell type vibrations of the barrel with very
small gaps on its bottom. The single excitation is performed by the random
white noise and the strike method. 50 points are distributed in both circum-
ferential and axial directions to measure the accelerations. The modal ana-
lysis is undertaken by a linear vibration computer program which considers
non-linear damping. The results, which have been converted to the real ob-
ject according to the proportional relations in Table 2, are listed in
Table 3 and Table 4.

Table 3. Parameters of beam vibration of
the barrel (1st order) *

<table>
<thead>
<tr>
<th>Test state</th>
<th>Natural frequency (Hz)</th>
<th>Damping ratio ([ξ])</th>
<th>ηr</th>
<th>ξr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>9.28</td>
<td>0.012</td>
<td>0.598</td>
<td>2.01</td>
</tr>
<tr>
<td>Static water</td>
<td>5.58</td>
<td>0.040</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Table 3 and Table 4 ηr = r²/r²; and ξr = (Mr - Mr)/Mr is
the additional mass of water for rth mode; r is the mode
order; w and a indicate the air
and the water respectively; m
and n are respectively the
axial and the circumferential
mode shape numbers for the
barrel cylinder.

*: The tests are performed with very
small gaps on the barrel bottom.
Table 4. Parameters of shell vibration of the barrel

<table>
<thead>
<tr>
<th>Order</th>
<th>In the air</th>
<th>In static water</th>
<th>Mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Natural frequency (Hz)</td>
<td>Damping ratio ($\xi_r$)</td>
<td>Natural frequency (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>57.6</td>
<td>0.0071</td>
<td>24.1</td>
</tr>
<tr>
<td>2</td>
<td>68.7</td>
<td>0.0045</td>
<td>35.5</td>
</tr>
<tr>
<td>3</td>
<td>114.6</td>
<td>0.0013</td>
<td>66.4</td>
</tr>
<tr>
<td>4</td>
<td>118.5</td>
<td>0.0038</td>
<td>62.8</td>
</tr>
<tr>
<td>5</td>
<td>140.9</td>
<td>0.0019</td>
<td>83.8</td>
</tr>
<tr>
<td>6</td>
<td>151.6</td>
<td>0.0135</td>
<td>78.8</td>
</tr>
<tr>
<td>7</td>
<td>175.0</td>
<td>0.0017</td>
<td>110.6</td>
</tr>
<tr>
<td>8</td>
<td>198.6</td>
<td>0.0012</td>
<td>127.9</td>
</tr>
</tbody>
</table>

HYDRAULIC VIBRATION TEST FOR THE BARREL MODEL

The model of test loop is 1:5 (Fig. 1) and the flow velocity of the loop is adjustable. 11 miniature pressure sensors are put on the wall of the barrel to measure the pulse pressure and 14 points on the barrel and 6 on the pressure vessel are selected to measure the accelerations. In addition, 4 points on the piping and the outer surface of the pump are arranged to do the synchronism monitoring. The measuring instrument, signal analysis and record system is shown in Fig. 2. The signals are processed by amplitude, spectrum and correlation in the analysis program.

![Figure 1. Hydraulic vibration test loop of the barrel](image)

1. reactor model  2. inlet pipe  3. outlet pipe
4. vent  5. gas storage tank  6. pressurizer  7. pump
8. backwater collector  9-10. flowmeters  11. cooler
12. filter  13. water tank  14. pottery filter
15. make-up water pump  16. water source
17. gas source
Figure 2. Hydraulic vibration measuring system
1. reactor hydraulic model 2. barrel
3. miniature pressure sensor 4. strain gauge
5. accelerometer 6. accelerocalibrator
7. electric charge amplifier 8. standard pressure calibrator 9. DC strain meter
10. tape recorder 11. signal analysis processor
12. signal analysis programs 13. printer and plotter 14. data memory

The design pulse pressure spectrum of the real reactor, correlating to the flow velocity, the position and the frequencies, is obtained by data processing the power spectrum density of the pulse pressure signals gained from the test. It can be expressed as follows:

$$G_p(f, v, \zeta) = \frac{G_p(0, v) \times 16 \cdot \eta(\zeta)}{(4 + f^2) \times (400 + f^2)} \text{ (MPa)}^2 / \text{Hz}$$

Here $v = V/V_p$; $G_p(0, v) = (1.509 \times 10^{-1} + 1.913 \times 10^{-3} v)$; $f$ is the frequency (Hz); $\zeta = \frac{z}{L}$ (on the upper end of the barrel; $z = 0$; the height is $L$);

$$\eta(\zeta) = (-2.07 + 25.30 \zeta - 64.04 \zeta^2 + 46.54 \zeta^3)$$; $V$ is the flow velocity in the channel between the barrel and the pressure vessel; and $V_p$, the flow velocity under the rated power.

From the amplitude analysis of the strain information the accumulative fatigue factor of the barrel in its 30 years lifetime is then acquired as $D = 6.7 \times 10^{-6}$ $\ll 1$, indicating the safety of the barrel vibration induced by hydraulic flow.

The tests are performed under different flow velocities ($V = 0.8V_p$ to 1.2 $V_p$) with single pump running, the relations between the acceleration response and the variation of flow velocity are then obtained. The results of signal analysis indicate that the hydraulic vibration of the barrel is a combination of a Gaussian ergodic stationary random process and acoustic pulse with a single frequency (197 Hz).

The flow-induced vibration is of the shell type, due to the narrow clearance between the radial support keyway of the pressure vessel and the support key on the lower part of the barrel (0.2 mm). The potential damage caused by beam type vibration can, therefore, be avoided.
THEORETICAL ANALYSIS

Fundamental equations and the relevant computer code VAC-F

Given the hydraulic pulse pressure loading, the vibration equation can be expressed as follows:

\[ (M_a + M_w) \ddot{Y} + C \dot{Y} + K Y = X(t) \]  \hspace{1cm} (1)

Here, \( X(t) \) is the pulse pressure loading; \( Y = (Y_1, Y_2) \), \( Y_1, Y_2 \), the amplitudes of the barrel and the pressure vessel; \( M_a = \begin{pmatrix} m_{11}^{a1} & 0 \\ 0 & m_{11}^{a2} \end{pmatrix} \), the mass matrix of the barrel and the pressure vessel in the air; \( M_w = \begin{pmatrix} m_{11}^{w1} & m_{11}^{w2} \\ m_{21}^{w1} & m_{22}^{w2} \end{pmatrix} \), the same mass matrix in the water; \( C \) and \( K \), the damping and stiffness matrix respectively; \( m_{11}^{w1} \) and \( m_{11}^{w2} \), the additional mass added to the barrel and the reactor vessel respectively, and \( m_{21}^{w1} \) and \( m_{22}^{w2} \), the additional mass added from the barrel to the vessel and from the vessel to the barrel respectively. The wave equation of the fluid can be expressed as:

\[ \nabla^2 \phi = C_d \frac{\partial^2 \phi}{\partial t^2} \]  \hspace{1cm} (2)

Here, \( \phi \) is the power function of the flow velocity; \( C_d \), the velocity of sound under the water; \( \nu^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \).

From the homogeneous form of Eq. (1) and Eq. (2), the additional mass of the barrel under the water \( M_w \) and its natural frequencies in static water can be solved. And then the response \( Y(t) \) can be obtained from Eq. (1) by substituting the pulse pressure of the fluid \( X(t) \) acquired from the tests.

On the basis of Eq. (1) and Eq. (2), a computer code, VAC-F, is accordingly developed to specially solve the hydraulic vibration problems of the barrel, the fluid and pressure vessel coupled together.

Comparisons between the test and the calculated results

Table 5 is the calculated natural frequencies fmn of the barrel in the static water.

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>39.7</td>
<td>26.8</td>
<td>36.8</td>
<td>68.9</td>
<td>116.9</td>
<td>179.2</td>
<td>255.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(24.1)</td>
<td>(35.5)</td>
<td>(66.3)(110.7)(164.1)(228.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>75.2</td>
<td>71.9</td>
<td>64.6</td>
<td>83.8</td>
<td>127.4</td>
<td>188.5</td>
<td>264.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(78.8)(62.8)(83.8)(127.9)(186.5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* : The data in the ( ) are the test values.

Using the design pulse pressure and the power spectrum density from the test and the Eq. (1), the barrel hydraulic vibration can be calculated as given
in Table 6.

<table>
<thead>
<tr>
<th>Flow quantity (m/hr)</th>
<th>v=V/Vp</th>
<th>Amplitude (mm)</th>
<th>Acceleration (G)</th>
<th>Circumferential stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30625</td>
<td>0.901</td>
<td>0.00872</td>
<td>0.374</td>
<td>2.48</td>
</tr>
<tr>
<td>33675</td>
<td>0.991</td>
<td>0.00957</td>
<td>0.420</td>
<td>2.82</td>
</tr>
<tr>
<td>35800</td>
<td>1.053</td>
<td>0.01010</td>
<td>0.450</td>
<td>2.99</td>
</tr>
<tr>
<td>single pump</td>
<td>0.520</td>
<td>0.01370</td>
<td>0.597</td>
<td>3.97</td>
</tr>
</tbody>
</table>

*: The data in the () are the test values.

CONCLUSIONS

(1) The flow-induced vibration of the barrel is mainly of the shell type, due to the narrow clearance between the radial support keyway of the pressure vessel and the support key on the lower part of the barrel (0.2 mm). The potential damage caused by beam type vibration can, therefore, be avoided.

(2) The accumulative fatigue factor of the barrel in its 30 years lifetime stays well within the safety limit.

(3) The mathematical formula of the design pulse pressure power density derived by enveloping the pulse pressure power density from the tests is of significance for designing the similar PWR.

(4) This research is a step forward to the understanding of the mechanism of flow-induced vibrations. However it is still a difficult problem in reactor structural mechanics entirely on a theoretical base to solve pulse pressure loading acting on the barrel by the fluid.

REFERENCES


