Heat Transfer Analysis of Thermal Stratification in Pressurizer Surge Line

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ABSTRACT
This paper presents a numerical method for analyzing thermally stratified flow in the PWR pressurizer surge lines. The finite volume method presented in this paper employs a body-fitted, non-orthogonal grid system to accommodate the pipe wall of circular geometry and the variable interface of the two fluids at different temperatures. This study investigates in detail the effects of surge flow and interface level of the two thermally stratified fluids on the determination of the transient temperature distributions in the pipe wall. As the result, the circumferential temperature distributions in the pipe wall obtained by changing the interface level of the stratified level were found to be reasonable. In addition, it was shown that the predictions without taking account of the effect of surge flow yielded less conservative results of the temperature gradients and thermal stresses in the pipe wall. Therefore, it is recommended to take into account the surge flow effect in the analysis for determining the temperature distributions in the pipe wall subjected to internally stratified flow.

INTRODUCTION
The integrity of pressurizer surge pipeline at operating pressurized water reactor (PWR) systems is susceptible to be threatened by the thermal stratification causing unacceptable stresses in the pipe wall composing the primary pressure boundary. This led the USNRC to issue its Bulletin 88-11 [1], requesting licensees to take proper actions for the resolution of the issue. Hence, the potential thermal stratification in the pressurizer surge line became one of the significant safety concerns of all the country holding PWRs. For addressing this matter, an assessment of the potential for piping damage due to the thermal stratification is needed. Thus, it is very important to determine, as realistic as possible, the transient temperature distribution in the wall of the piping in which thermally stratified flow occurs, as a prerequisite for the assessment.

Several investigators [2-6] have made efforts to determine the temperature distributions in the pipe wall by means of laboratory testing of the particular geometry, measurement of the temperatures on the outer surface of the pipe in the field, or theoretical predictions. There are much difficulties and limitations in applying the first two approaches for operating plants. Only a few literatures addressing the theoretical analyses are available. Smith et al. [2] presented an approximate analytical solution for the steady-state temperature distributions in a pipe wall. Yu et al. [6] obtained temperature distributions for the steady-state heat transfer model of PWR pressurizer surge line which was simplified by using the computer code
ANSYS based on the assumptions that the inside of the pipe is exposed to two distinct ambient fluids of which the temperatures are constant. Jung et al. [5] proposed an unsteady two-dimensional natural convection model for the same problem as that considered by Yu et al. [6]. However the grid system used in the numerical calculations was not satisfactory to simulate the initial geometrical condition of the fluid interface. Recently Jo et al. [7] obtained finite volume solutions of the same problem, employing a body-fitted, non-orthogonal grid system to accommodate the pipe wall of circular geometry and the interface of the two fluids at different temperatures, of which the level is variable. However, in those analyses [5-7] the effect of surge flow was neglected.

This paper presents a numerical method for the prediction of thermally stratified flow in the PWR pressurizer surge lines. The governing equations are discretized using the finite volume method and the convection term is approximated by a higher-order bounded scheme named COPLA [8], which is known as a high-resolution and bounded discretization scheme. The method presented in this paper employs a body-fitted, non-orthogonal grid system to accommodate the pipe wall of circular geometry and the interface of the two fluids at different temperatures, of which the level is variable. The cell-centered, non-staggered grid arrangement is adopted and the resulting checkerboard pressure oscillation is prevented by the application of a modified momentum interpolation scheme [7]. The present method employs the SIMPLE algorithm [9] for the pressure and velocity coupling. This study investigates the effects of surge flow in the line and interface level of the two stratified fluids on the determination of the transient temperature distributions in the pipe.

MATHEMATICAL MODEL

Governing Equations
Consider a situation that hot fluid and cold fluid flow in the PWR pressurizer surge line with a constant bulk velocity. The hot fluid occupies upper portion of the pipe (see Fig. 1). Consequently this leads to the formation of two distinct fluid layers in the pipe. For simplicity, it is assumed in this study that the flow is fully developed and the axial gradients of velocities and temperature are negligible. Thus the dimensionless governing equation of this thermally stratified flow model can be expressed in a generalized coordinate system $x'$ as,

$$ \frac{\partial}{\partial x'} U_1 + \frac{\partial}{\partial x'^2} U_2 = 0 $$

(1)

$$ \frac{\partial}{\partial t} (J u_1) + \frac{\partial}{\partial x'} \left[ U_1 u_1 - \frac{1}{Re} J \left( \frac{\partial u_1}{\partial x'} D_1 + \frac{\partial u_1}{\partial x'^2} D_2 + b_1 w_1 + b_2 w_2^2 \right) + P b_1 \right]$$

$$ + \frac{\partial}{\partial x'^2} \left[ U_2 u_1 - \frac{1}{Re} J \left( \frac{\partial u_1}{\partial x'} D_1 + \frac{\partial u_1}{\partial x'^2} D_2 + b_1 w_1 + b_2 w_2^2 \right) + P b_2 \right] = \frac{Gr}{Re^2 TJ} $$

(2)

$$ \frac{\partial}{\partial t} (J u_2) + \frac{\partial}{\partial x'} \left[ U_1 u_2 - \frac{1}{Re} J \left( \frac{\partial u_2}{\partial x'} D_1 + \frac{\partial u_2}{\partial x'^2} D_2 + b_1 w_1 + b_2 w_2^2 \right) + P b_1 \right]$$

$$ + \frac{\partial}{\partial x'^2} \left[ U_2 u_2 - \frac{1}{Re} J \left( \frac{\partial u_2}{\partial x'} D_1 + \frac{\partial u_2}{\partial x'^2} D_2 + b_1 w_1 + b_2 w_2^2 \right) + P b_2 \right] = 0 $$

(3)
\[
\frac{\partial}{\partial t} (J T) + \frac{\partial}{\partial x^i} \left[ U_i T - \frac{1}{\text{Re Pr} J} \left( \frac{\partial T}{\partial x^i} D^2 + \frac{\partial T}{\partial x^2} D^2 \right) \right] + \frac{\partial}{\partial x^2} \left[ U_2 T - \frac{1}{\text{Re Pr} J} \left( \frac{\partial T}{\partial x^2} D^2 + \frac{\partial T}{\partial x^2} D^2 \right) \right] = 0
\]

where

\[U_1 = \left( u_1 b_1^2 + u_2 b_2^2 \right), \quad U_2 = \left( u_1 b_1^2 + u_2 b_2^2 \right), \quad D_{ij} = b_i^j b^m k \quad \text{and} \quad w_i^j = \frac{\partial u_i}{\partial x^j} \]

and the geometric coefficients \( b_{ij} \) represent the cofactors of \( \partial y^i / \partial x^j \) in the Jacobian matrix of the coordinate transformation \( y^i = y^i(x^j) \), and \( J \) is the determinant of the Jacobian matrix. In the above equations (1) - (4), \( \rho, \mu, p, k, c_p, \beta, \) and \( g \) denote respectively density, dynamic viscosity, pressure, thermal conductivity, the specific heat, volumetric coefficient of thermal expansion, and the gravitational acceleration. In addition, \( u_i \) are the Cartesian velocity components in the \( y^i \) direction, \( W_0 \) is a specified constant bulk velocity of stratified fluid in the \( x^3 \) direction, and \( r_i \) is the inner radius of the pipe.

Initial and Boundary Conditions

As mentioned previously, the pipe wall is initially at the temperature of cold fluid \( T_c \), and is suddenly exposed to hot fluid at \( T_h \). The initial conditions for this are given as

\[u_i = 0 \quad (i = 1, 2) \quad \text{in the whole solution domain,} \quad t = 0 \quad \text{(6a)}\]
\[T = 0 \quad \text{in the pipe wall and the cold fluid layer,} \quad t = 0 \quad \text{(6b)}\]
\[T = 1 \quad \text{in the hot fluid layer,} \quad t = 0 \quad \text{(6c)}\]

Because the solution domain is symmetrical thermally and geometrically, only half of the region is needed to analyze. Thus along the symmetry line, the symmetry boundary conditions is applied for both velocity and temperature. On the solid wall, the velocity of the fluids vanishes. For this situation the boundary conditions are given by

\[u_i = 0 \quad (i = 1, 2) \quad \text{at the inner surface of the pipe,} \quad t \geq 0 \quad \text{(7a)}\]
\[\frac{\partial T}{\partial n} = -B_i(T - T_w) \quad \text{at the outer surface of the pipe,} \quad t \geq 0 \quad \text{(7b)}\]

where \( a = (r_o - r_i) / r_i \) and \( B_i = h(r_o - r_i) / k_s \)

\[u_2 = 0, \quad \frac{\partial u_i}{\partial x^j} = 0, \quad \frac{\partial T}{\partial x^j} = 0 \quad \text{at the symmetry plane,} \quad t \geq 0 \quad \text{(7c)}\]

where \( n \) is the outward normal to the surface of the wall, \( T_w \) is the temperature of environment outside the pipe, \( h \) is heat transfer coefficient, \( k_s \) is the thermal conductivity of the pipe material, and \( r_o \) is the outer radius of the pipe.

**NUMERICAL METHOD OF SOLUTION**

Solution Domain Discretization

The governing equations (1) - (3) are solved numerically by a finite volume approach, requiring the discretization of the solution domain into a finite number of quadrilateral control volume cell whose faces are coincided with the non-orthogonal curvilinear coordinate.
lines. A typical discretized domain is presented in Fig. 2, and also a typical control volume cell is shown in Fig. 3. The values of all computed variables are stored at the geometric center of each control volume cell. The interface between the hot and cold fluids is arranged here to align with a boundary between two rows of cells, i.e. a gridline.

To obtain the curvilinear non-orthogonal mesh shown in Fig. 2, it is assumed that the solution domain is the cross-section of a pair of eccentric cylinders as shown in Fig. 4. The center of the inner solid cylinder is coincided with the intersecting point of the fluid interface and the vertical symmetry line passing the center of the pipe. The outer cylinder is the pipe subjected to internally stratified flow, and the inner cylinder has such a small size of diameter that the effect of its presence on the calculations can be negligible. Thus, the following boundary conditions are applied to the outer surface of the inner solid cylinder with such an infinitesimal diameter.

\[
\frac{\partial u_i}{\partial x} = 0, \quad \frac{\partial T}{\partial x} = 0 \quad (i = 1, 2) \text{ at the outer surface of the infinitesimal inner cylinder, } t \geq 0 \quad (8)
\]

Dislocating the inner solid cylinder either downward or upward can easily control the level of the fluid interface with a horizontal straight-line configuration. The grid is generated by using an algebraic method. In this study, the calculations are performed with a grid of $52 \times 42$, forming 51 divisions in the circumferential direction and 41 divisions in the radial direction.

Discretization of Governing Equation

The discretization of the governing equations is performed following the finite volume approach, and the convection terms are approximated by the COPLA scheme developed by Choi et al. [8] and the unsteady term is treated by the backward differencing scheme. The resulting algebraic equation for a variable \( \varphi \) can be written in the following general form.

\[
A_p \varphi_p = A_E \varphi_E + A_W \varphi_W + A_N \varphi_N + A_S \varphi_S + b_\varphi \quad (9)
\]

where \( A_j (j = P, E, W, N \text{ or } S) \) are coefficients and \( b_\varphi \) is a source term for variable \( \varphi \).

Momentum Interpolation Method

For a better resolution of flow field in complex geometries, recently several investigators have developed various calculation methods of momentum equations employing the non-orthogonal, body-fitted coordinates. Among these methods, the non-staggered, momentum interpolation method originally developed by Rhie and Chow [10] is known to be one of the efficient methods and has been widely used because of its simplicity feature of algorithm. In this method, the momentum equations are solved at the cell centered locations using the Cartesian velocity components as dependent variables and the cell face velocities are obtained through the interpolation of the momentum equations for the neighboring cell centered Cartesian velocity components. In the present analysis, the modified version of the Rhie and Chow's scheme [7] is used to obtain a converged solution of unsteady flows which is independent of the size of time step.

RESULTS AND DISCUSSION

The geometry of the surge line and most of the computational parameters used here are the same as those in references [5, 7]. In operating reactors, the outer surface of the surge line is
insulated with a little heat loss. The overall heat transfer coefficient of \( h = 0.79 \text{ W/m}^2\text{K} \) was used in the present analysis. The surge flow rate of hot fluid in the line coming from the pressurizer was considered to be \( 1.26 \times 10^{-2} \text{ m}^3/\text{sec} \). To investigate the effect of fluid interface level on the temperature distributions in the pipe wall, three different cases were examined. The interface levels for the three cases, respectively, are at heights of \( 0.25d_i \), \( 0.5d_i \), and \( 0.75d_i \) from the horizontal reference line passing through the bottom point of inner wall surface, where \( d_i \) is the inner diameter of the surge line. For simplifying the calculations, the fluids are assumed to be Newtonian with constant properties and the Boussinesq approximation is assumed to be valid. And on the basis of these assumptions the variables of length, time, velocity and temperature are nondimensionalized, respectively, using the reference scales of \( r_i \), \( r_i/W_0 \), \( W_0 \), and \( \Delta T = (T_b - T_c) \). The dimensionless time step used in the computations is 0.1. The iterative computation for each time step ceases when the maximum of the absolute sum of dimensionless residuals of momentum equations or energy equation, or pressure correction equation is less than \( 10^{-6} \). Relaxation factors of 0.7 and 1.0 were used for momentum equations and energy equation, respectively.

The typical calculation results for both cases of \( u_3 = 0 \) and \( u_3 = W_0 \) are presented in Figs. 5-7 to discuss the effects of surge flow and interface level. Fig. 5 presents the variation of the local Nusselt number as a function of the angle at two different elapsed times for the two different interface levels of \( 0.5d_i \) and \( 0.75d_i \). The average Nusselt number decreases to zero with elapsing of time, and increases with increasing the amount of hot fluid flowing into the pipe section. These are plausible from the fact that the greater the difference in temperature between the fluid and the wall surface, the higher the Nusselt number. It is known that the major effect of thermal stratification in the pressurizer surge line of operating nuclear power plants are displacements, bending, and associated stresses resulting from a significant circumferential temperature difference, which was not considered in the original design. In addition, axial stratification profile affects both the local stresses and the global bending effect at a given pipe cross-section. Therefore, the circumferential temperature difference and distribution as well as their variation along the axial direction of pipe are the most important factors to be examined in the assessment of the piping integrity. Fig. 6 shows the transient maximum circumferential temperature differences both at the inner and outer wall surfaces for the three different cases of fluid interface levels. Fig. 7 displays the transient temperature variations with the change in the fluid interface level at the bottom and mid-level positions on the inner and outer wall surfaces of the subjected piping at the specified non-dimensional times of 800 and 1500. As can be seen in Figs. 6 and 7, the maximum value of the temperature difference is higher at the inner wall surface than that at the outer wall surface. It increases to their maximum values at the elapsed dimensionless time zone ranging from 500 to 1800 and then decreases as time elapses further. Figs. 6 and 7 also show that the maximum temperature difference is affected by the axial stratification profile that is characterized by the variation of fluid interface level. The piping section at which the maximum temperature difference is produced is expected to be the case where the fluid interface level is slightly under the height of \( 0.5d_i \). As shown in the figures, the temperatures decrease as the interface level increases and the gradients of temperature distributions at the mid-level positions are steeper than at the bottom-level positions. It is seen from Figs. 6 and 7 that the calculation results of temperature variations (gradients) in the pipe wall both in the circumferential and longitudinal directions for the case of \( u_3 = 0 \) are less significant than for the case of \( u_3 = W_0 \) where the surge flow is taken into account. Thus the stress analysis using the temperature distributions predicted by neglecting the effect of surge flow may yield under-conservative results. In addition, it can be re-confirmed from those figures that the temperatures on the inner wall surface are much higher than on the outer wall surface during
the early transient time period. As can be expected, such effects may cause the excessive longitudinal and circumferential stresses in the pipe wall, which can eventually result in unacceptable mechanical damages to the pipe such as global bending, dislocation, failure, etc.

CONCLUSIONS

The transient behaviors of fluid flow and temperature distribution in a PWR pressurizer surge line subjected to internally stratified flow were simulated using the finite volume approach. For the numerical simulation, a body-fitted non-orthogonal grid system was employed to accommodate the pipe wall of circular geometry and the interface of two fluids at different temperatures of which the level is variable. This study investigated in detail the effects of surge flow in the line and interface level of the two stratified fluids on the determination of the transient temperature distributions in the pipe wall.

It was shown that the predictions without taking account of the effect of surge flow yielded less conservative results of the temperature gradients and thermal stresses in the pipe wall of pressurizer surge line. Therefore, it is recommended to take into account the surge flow effect in the analysis for determining the temperature distributions in the pipe wall subjected to internally stratified flow. In addition, the circumferential temperature distributions in the pipe wall obtained by changing the interface level of the stratified level were considered to be reasonable. Although in this study only the thermal stratification problem for a PWR pressurizer surge line was addressed, it is emphasized that the present method can be extended for applications to various cases of thermally stratified flows in pipes and tanks with complex geometry and different flow conditions.

REFERENCES

Fig. 1 Thermally stratified flow in a circular pipe.

Fig. 2 The curvilinear non-orthogonal mesh.

Fig. 3 A typical control volume cell in the computing mesh.

Fig. 4 The imaginary eccentric cylinder for mesh generation.

Fig. 5 The variation of the local Nusselt number (Nu).

(a) Non-dimensional time = 800

(b) Non-dimensional time = 1500

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Fig. 6 Transient maximum wall temperature differences both on the inner and outer wall surfaces.

Fig. 7 Non-dimensional temperatures on the inner and outer wall surfaces at the bottom and mid-level positions.