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# Finite Element Crash Analysis of Framed Structures by the Adaptively Shifted Integration Technique

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## ABSTRACT

In the present study, each member of a framed structure is subdivided with two linear Timoshenko beam elements at both ends and a cubic beam element based on Bernoulli-Euler hypothesis at the center. The adaptively shifted integration(ASI) technique is used only in the linear Timoshenko beam elements. The proposed model is applied to the explicit finite element analysis of crashing behaviors of impulsively loaded framed structures considering the effect of large deformation by the updated Largrangian formulation. Serveral numerical studies have been carried out in order to show the validity of the proposed numerical technique.

## 1. INTRODUCTION

In the field of nonlinear structural dynamics based on finite element formulation, the commercial codes such as PAM-CRASH and DYNA-3D are now available to simulate the crashing behavior of structures under impact. However, these codes require large computer resources and involve time-consuming preparation of the modelling data, which make them rather unsuitable as a design tool during the initial design stage. In many impact situations, the individual structural members are overloaded principally in bending giving rise to plastic deformations in highly localized regions, called plastic hinges. Therefore, the structural model idealized as a frame by the use of linear or cubic beam elements is commonly used in the finite element analysis at the initial design stage.

The new computational algorithm identified as the adaptively shifted integration technique[1]-[3](abbreviated to the ASI technique) has been developed by one of the authors as a smart algorithm for the nonlinear finite element analysis of framed structures and has been applied to the various static and dynamic collapse problems of steel as well as concrete framed structures. In the nonlinear finite element analysis by the ASI technique, the highest computational efficiency for large-scale frame analysis has been achieved by adaptively shifting the numerical integration points for the evaluation of element stiffness matrix (i.e, strain energy), depending upon the occurence and the location of plastic hinges in the element. The ASI technique is applicable to the linear Timoshenko beam element and the cubic beam element based on Bernoulli-Euler hypothesis. It is noted that the ASI technique can be easily implemented in the existing codes.

In this paper the ASI technique is applied to the crash analysis of steel frames.Each member of a framed structure is subdivided with three elements, which are two linear Timoshenko beam elements at both sides and a cubic beam element based on Bernoulli-Euler hypothesis at the center. The ASI technique is employed in the two linear elements in order to consider the occurence of plastic hinges

and fracture at the exact positions (usually member joints), while the numerical integration points are fixed at the Gaussian points in the cubic element to maintain the computational stability. Large deformation is considered by the updated Largrangean formulation with convected element coordinates. Material nonlinearity is taken into account by the non-layered formulation expressed in terms of resultant forces.

## 2. DYNAMIC CRASH ANALYSIS OF FRAMED STRUCTURES

### *2.1 Finite element model and ASI technique*

As mentioned above, one structural member such as a beam, a column and a bracing is subdivided with three elements as shown in Fig. 1. Here, this model is identified as the LCL model in which 'L' and 'C' mean linear elements and cubic elements respectively. The length of linear elements is expressed by the parameter $\beta$ which is the fraction between the length of a linear element and the length of a member.

For the linear element, the relationship between the locations of the numerical integration point and a plastic hinge is written as[1]

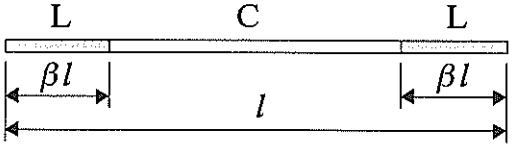
$$s_1^L = -r_1^L \text{ or } r_1^L = -s_1^L \tag{1}$$

The integration point is located at the center of the element when it is elastic or unloaded. Immediately after the occurence of a plastic hinge, the integration point is shifted according to the relation given in Eq. (1). By the adaptive shifting of a numerical integration point, the location of a plastic hinge can be exactly considered and higher computational efficiency can be achieved.

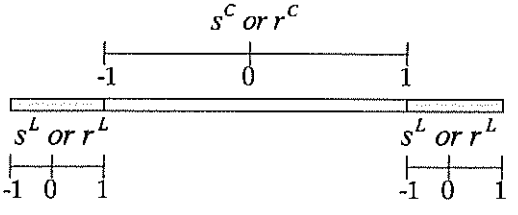
For the cubic element, the location of numerical integration points is fixed on Gaussian points. In this case, the relationship between numerical integration points and plastic hinge positions is written as[1]

$$s_1^C = -r_1^C = -1/\sqrt{3} \text{ and } s_2^C = -r_2^C = 1/\sqrt{3} \tag{2}$$

The cubic element which is located at the center of a member is not always subjected to large bending moment. Computational accuracy is not necessarily improved by the shifting of numerical integration points because this element experience complicated loading and unloading accompanied by the elasto-plastic wave propagation[3]. Therefore, ASI technique is not applied to the cubic element in order to maintain the computational stability and accuracy.




(a) LCL model (L : Linear Timoshenko beam element, C : Cubic beam element)




(b) Nondimensional coordinate system
($s$ : coordinate of numerical integration point, $r$ : coordinate of plastic hinge point)

Figure 1. LCL model and nondimensional coordinate system

*2.2 Incremental finite element formulation*

Updated Largrangian formulation in incremental form[4] using the LCL model which is proposed in the present study is adopted for the derivation of nonlinear stiffness equation. The relation between the incremental generalized strain vector and the incremental nodal displacement vector at the time step $n$ is written as

$$\{\Delta_n \bar{\varepsilon}^L\} = [\bar{B}^L]\{\Delta_n \bar{u}\} \tag{3,a}$$

$$\{\Delta_n \bar{\varepsilon}^C\} = [\bar{B}^C]\{\Delta_n \bar{u}\} \tag{3,b}$$

where, the subscript '$L$' and '$C$' denote the linear Timoshenko beam element and the cubic beam element based on Bernoulli-Euler hypothesis.

The incremental nodal displacement vector is calculated as

$$\{\Delta_n \bar{u}\} = [{}^{u}T][{}^{0}T]\{\Delta_n u\} \tag{4}$$

where $[{}^{0}T]$ is the transformation matrix from global coordinate system to initial element coordinate system, $[{}^{u}T]$ is the transformation matrix from initial element coordinate system to element coordinate system at time step $n$ which is obtained by multiple calculation method[4].

The relation between the incemental generalized stress (resultant force) vector and the incemental generalized strain vector is expressed as

$$\{\Delta_n \bar{R}^L\} = [D^L]\{\Delta_n \bar{\varepsilon}^L\} \tag{4,a}$$

$$\{\Delta_n \bar{R}^C\} = [D^C]\{\Delta_n \bar{\varepsilon}^C\} \tag{4,b}$$

The stress-strain matrix $[D]$ for elastic deformation consist of Young's modulus and Poisson's ratio, while for plastic deformation it can be determined by the plastic flow theory using the following form of yield function as a plastic potential :

$$[D^p] = [D^e] - \frac{[D^e]\{\partial f / \partial \bar{R}\}\{\partial f / \partial \bar{R}\}[D^e]}{H' / A + \{\partial f / \partial \bar{R}\}[D^e]\{\partial f / \partial \bar{R}\}} \tag{5}$$

The yield function assumed in the frame analysis is expressed by the following equation :

$$f = \left(\frac{\bar{R}_1}{M_{x0}}\right)^2 + \left(\frac{\bar{R}_2}{M_{y0}}\right)^2 + \left(\frac{\bar{R}_3}{N_0}\right)^2 + \left(\frac{\bar{R}_4}{M_{t0}}\right)^2 - 1 \tag{6}$$

where $M_x$, $M_y$, $N$ and $M_t$ are two components of bending moment, an axial force and a torsional moment, respectively. The subscript '0' indicates a fully plastic value under the condition that each component of resultant forces acts independently on a cross-section of a member. The effect of shear forces on the yield condition has been neglected in the analysis using the linear Timoshenko beam element as well as in that employing the cubic element.

*2.3 Solution of equilibrium equations in dynamic analysis*

The equation of equilibrium governing the dynamic response of a system of finite elements at the time step $t = t_n$ is written in the following form :

$$[M]\{{}^{n}\ddot{u}\} = \{{}^{n}F_{out}\} - \{{}^{n}F_{int}\} \tag{7}$$

where $[M]$ is the lumped mass matrix. $\{{}^{n}\ddot{u}\}$, $\{{}^{n}F_{out}\}$ and $\{{}^{n}F_{int}\}$ are the acceleration vector, the equivalent external nodal force vecor and the equivalent internal nodal force vector, respectively.

Using the central difference method, the acceleration vector at time step $t = t_n$ is approximated as

$$\{{}^n\ddot{u}\} = \left(\{{}^{n-1}u\} - 2\{{}^n u\} + \{{}^{n+1}u\}\right)/(\Delta t)^2 \tag{8}$$

Substitution of Eq. (8) into Eq. (7) leads to the following equation :

$$[M]\{{}^{n+1}u\} = (\Delta t)^2\left(\{{}^nF_{ext}\} - \{{}^nF_{int}\}\right) + [M]\left(2\{{}^n u\} - \{{}^{n-1}u\}\right) \tag{9}$$

The equivalent internal nodal vector $\{{}^nF_{int}\}$ is expressed as

$$\{{}^nF_{int}\} = \int [{}^0T][{}^uT]^T [{}^n\overline{B}(s_i)]\{{}^n\overline{R}(r_i)\}dl_n \tag{10}$$

where $[{}^n\overline{B}(s_i)]$ is the strain-displacement matrix based on finite strain theory and $\{{}^n\overline{R}(r_i)\}$ is the resultant force vector. It should be noted that $[{}^n\overline{B}(s_i)]$ and $\{{}^n\overline{R}(r_i)\}$ are the fucntion of the location of numereical integration points and plastic hinges, respectively.

In Eq. (9), the time increment $\Delta t$ should satisfy Courant condition which is given as

$$\Delta t = l^e_{min} / v_L \tag{11}$$

where $l^e_{min}$ and $v_L$ are the minimum length of elements and the longitudinal wave speed, respectively.

## 3. COMPUTATIONAL RESULTS

The LCL model with the ASI technique has been applied to the dynamic crash analysis of a rotating beam, simple plane frames as well as an automobile body structure to verify the validity and computational efficiency of the proposed numerical technique.

### *3.1 Dynamic collapse analysis of a rotating beam*

Physical model of a rotating beam[5] is shown in Fig. 2(a). The member is an E36 steel box-beam tube, 1 *m* in length, with a 50x25 *mm* hollow rectangular section and 2.65 *mm* in thickness. One of the beam's ends is clamped allowing the rotation of the member. A ballasting mass of 5.95 kg is attached to the other end of the member. The member is accelerated and the angular velocity is 11.85 *rad/sec* when the middle point of the member is impacted. Fig. 2(b) illustrates the finite element model which is represented by two LCL models.

Figure 3 shows the time history of deflection($\delta_{beam}$) after the middle point of the beam is impacted.

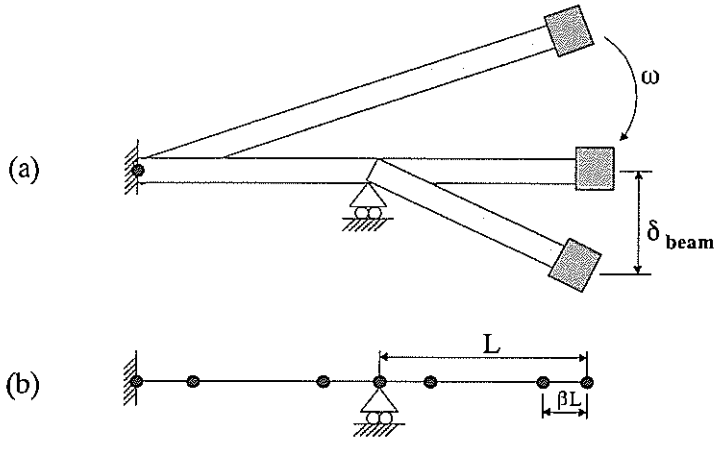



Figure 2. Rotating beam ; (a) physical model; (b) finite element representation with LCL model

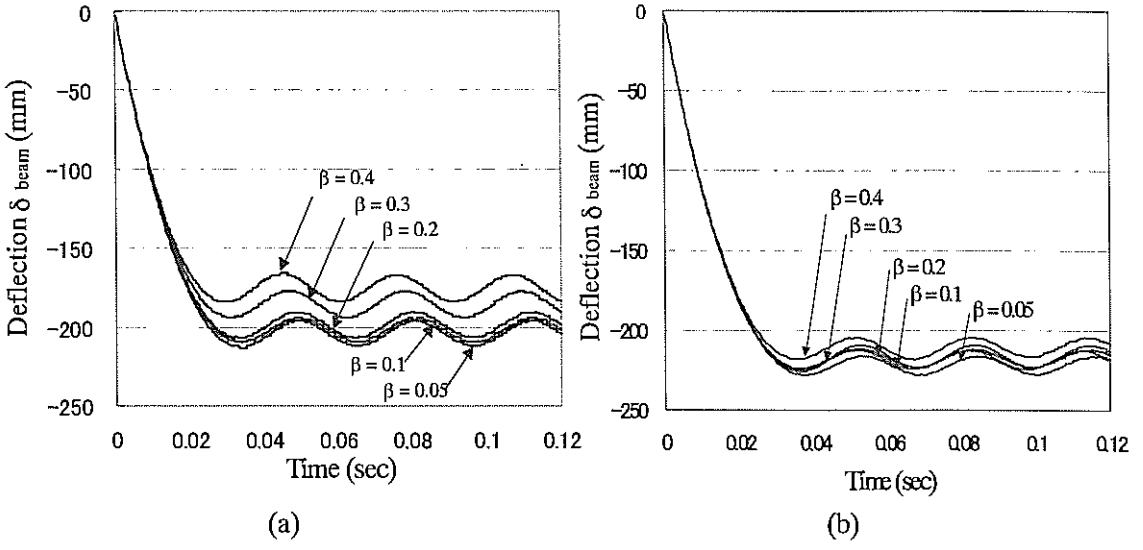



Figure 3. Time history of deflection of a rotating beam with LCL model; (a) Without ASI technique ; (b) With ASI technique

As shown in Fig. 3(a), the solutions given by the LCL model without ASI technique are highly dependent on the length of linear elements. This is due to the assumption that the plastic hinges are formed only at the central point of linear elements in this method. On the other hand, the LCL model with ASI technique has shown fast convergence of solutions not so much influenced by the length of linear elements, as shown in Fig. 3(b). This is because the numerical integraion points have been adaptively shifted according to plastic yielding and unloading.

Based on the results of Fig. (3), we have chosen the model without ASI technique for $\beta$=0.05(LCL-PL-0.05 model) and the model with ASI technique for $\beta$ = 0.3(LCL-ASI-0.3 model) to make the following comparisons. These two models have given similiar solutions but time incerment of the latter can be assumed six times longer than the former.

*3.2 Impulsive collapse analysis of a rectangular frame*

The impacting behavior of a rectangular frame[6] subjected to a uniformly distributed impulsive loading which is shown schematically in Fig. 4(a) has been studied as an example to show the validity of the LCL model with the ASI technique. The model which is uniformly subdivided only by a number of linear elements(standard model) and the above-mentioned two models(LCL-PL-0.05 model and LCL-ASI-0.3 model) have been used in the analysis. The assumed material data is as follows :

Young's modulus $E = 69\ Gpa$
Tangent modulus $E_t = 5.7\ Gpa$
Yield stress $\sigma_o = 82.8\ Mpa$
Density $\rho = 2.68\ sec/m^3$
Breadth $B = 12.7\ mm$
Height $H = 0.9144\ mm$

Figures 4(b) and 4(c) show deformed shapes($t$ =2.88$msec$) and the time history of deflection($\delta_{rect}$) given by the three models and the experiment. In each figure, the result of LCL-PL-0.05 model is not similar to those of the other models because the location of plastic hinges is not exact. On the other hand, the behavior of LCL-ASI-0.3 model is in good correspondence with those of the standard model and the experiment[5]. It is seen that LCL-ASI-0.3 model can give practically sufficient solutions in the present problem even by using not only a minimum number of elements but also nearly uniform mesh.

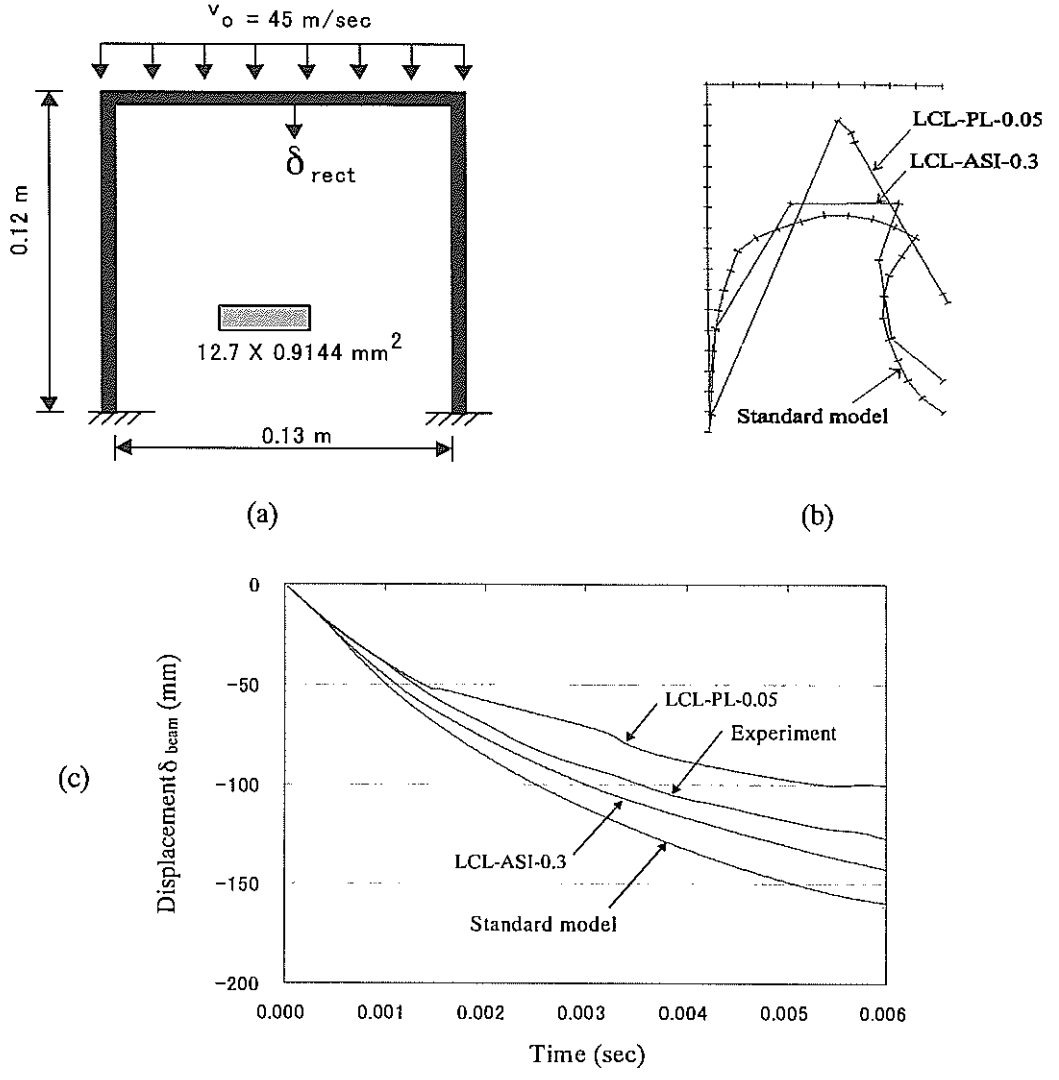



Figure 4 Rectangular frame under uniformly distributed impulsive loading; (a) schematic diagram; (b) deformed shape($t$ =2.88 *msec*); (c) time history of deflection

*3.3 Impulsive collapse analysis of a triangular frame*

Similar numerical studies have been conducted for the impact analysis of a triangular frame[6] subjected to a concentrated impulsive loading which is shown schematically in Fig. 5(a).

The above-mentioned three models are used in the analysis. The assumed material data is the same as of section 3.2.

Figures 5(b) and 5(c) show deformed shapes($t$ =1.28*msec*) and the time history of deflection($\delta_{tri}$) for each model. As in the preceding section, the deformed shape of the LCL-ASI-0.3 model is very similar to that of the standard model as shown in Fig. 5(b) and the calculated time history shown in Fig. 5(c) well corresponds with those of the standard model and experiment[6].

*3.4 Crashing collapse analysis of an automobile body structure*

Automobile body structure is subjected to a frontal collision. Each member is idealized by the LCL model. The frontal end is fixed and all nodes are subjected to the initial velocity of 50 km/h. The LCL-PL-0.05 model and the LCL-ASI-0.3 model have been used in the analysis.

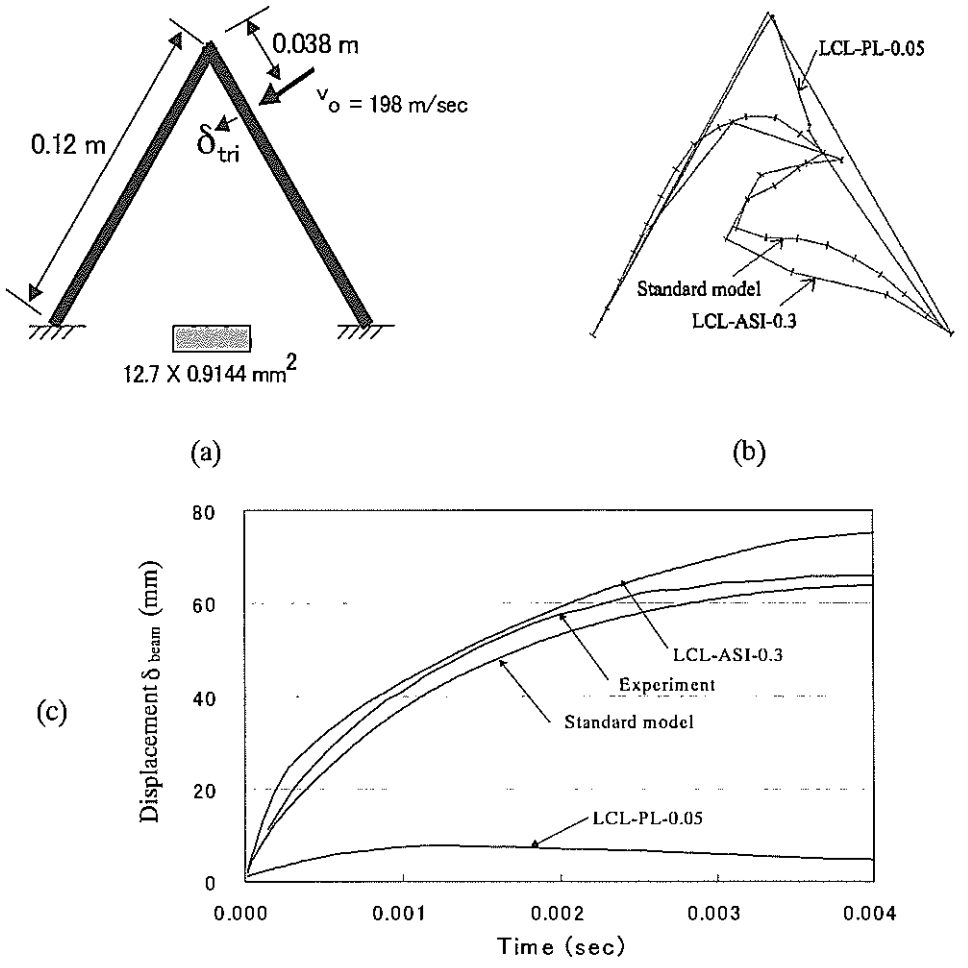



Figure 5 Triangular frame under concentrated impulsive loading; (a) schematic diagram; (b) deformed shape($t$ =1.28 *msec*); (c) time history of deflection

Figures 6 and 7 show the deformed shapes($t$ =0.05*sec*) and the time history of displacement of front-pillar($\delta_{front}$) for each model, respectively. In the LCL-ASI-0.3 solution, the front side member is vertically deflected (Fig. 6(a)) as shown in the actual frontal collision of automobiles. However, in the LCL-PL-0.05 solution, the maximum deflection is overestimated because of the transverse deflection as observed in Fig. 6(b). From these results it is seen that the LCL-ASI-0.3 model has given better solution in comparison with the actual crashing deformation of automobile body structures.

## 4. CONCLUDING REMARKS

The LCL model with the ASI technique for linear elements has been applied to the explicit finite element analysis of crashing behaviors of impulsively loaded framed structures. The proposed technique has given very fast convergence of solutions in the dynamic analysis of framed structures even by using a small number of elements and nearly uniform mesh. Therefore, the present method can be used as an accurate, computationally stable and cost-effective simplified numerical technique at the initial design stage against impact and collision. The present technique can be easily extended to consider the fracture of member joints and numerical studies are now under way.

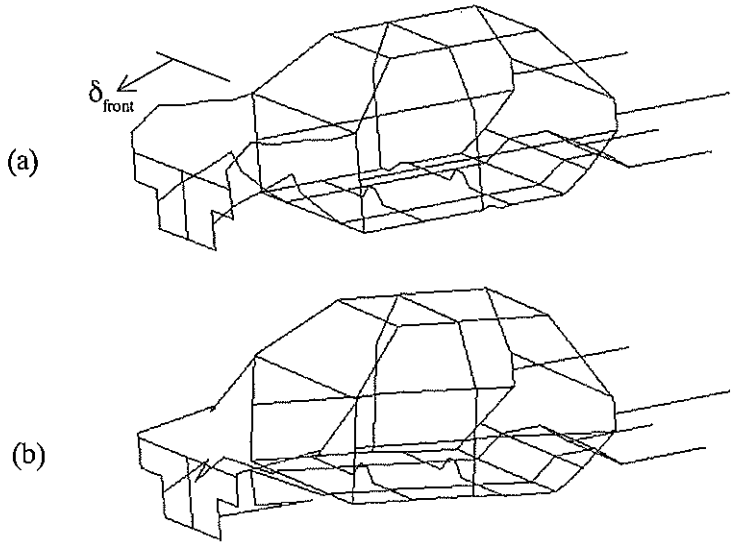



Figure 6 Deformed shape of a automobile body structure ($t$ =0.05*sec*); (a) LCL-ASI-0.3 model; (b) LCL-PL-0.05 model

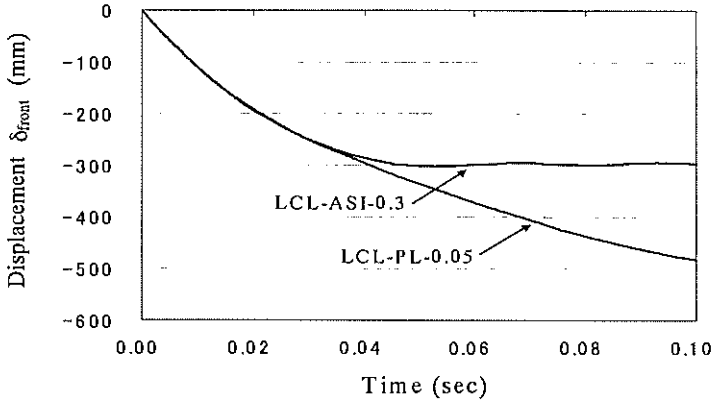



Figure 7 Time history of front pillar displacement of a automobile body structure

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