Extension of an Elastic Stiffness Formula on the Leaf Type Holddown Spring Assembly of the PWR Fuel Assembly

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ABSTRACT: Based on the Euler beam theory and the elastic strain energy method, the elastic stiffness formula on the holddown spring assembly consisting of several leaves is derived, where the friction forces acting on the interfaces between the leaves are considered. The elastic stiffness from the formula is compared with the characteristic test results on several kinds of leaf spring specimens. As a result of comparisons, the formula is found to be able to estimate the elastic stiffness of the holddown spring assembly within the maximum error range of 12%. And also, the elastic stiffness from the finite element model using contact elements between the leaf springs agrees well with that from the elastic stiffness formula.

1. INTRODUCTION

A HoldDown Spring (HDS) assembly, which is attached to the uppermost part of the fuel assembly in pressurized water reactors, has two main functions [1]. The first is to keep the fuel assembly firmly seated on the lower core plate during normal plant operation with enough holddown force to resist buoyancy forces and upward hydraulic flow forces that act on fuel assemblies due to normal reactor coolant flow. The second is to allow changes to occur in the length of the fuel assembly relative to the space between the upper and lower core plate, while still providing an acceptable holddown force. These changes in relative length can occur due to differential thermal expansion between the fuel assembly structure made of Zircaloy-4 and the core support structure made of stainless steel, and due to the neutron irradiation-induced growth of the fuel assembly. Maintaining these two functions during the entire residence time of the fuel assembly in the reactor core should be evaluated through the analysis of the hold down force [2].

Currently, two kinds of leaf type HDS assemblies are attached to the fuel assembly: the tapered-thickness leaf type HDS assembly named TT-HDS and the tapered-width leaf type HDS assembly named TW-HDS. The leaf type HDS assembly consists of a number of leaves which are bent into design shapes and machined to have a uniformly tapered thickness or width along the leaf length. Reliably estimating the elastic stiffness of the leaf type HDS assembly is known to be difficult because of the complicated geometric shape of each leaf, even though the elastic stiffness of the spring assembly is considered to be one of the fundamental parameters in the analysis of the holddown force. Therefore, some foreign nuclear fuel vendors have developed their own methodologies to estimate the elastic stiffness of the TT-HDS, and have used them only for the initial estimates of the holddown force [1,2]. However, the methodologies have some limitations in case of design changes in
the leaf springs because they are based on either too simplified assumptions or empirical formul constructed from the characteristic test data of the leaf springs.

In this paper we propose two general methodologies to estimate the elastic stiffness of the leaf type HDS assembly. One is the derivation and verification of the extended elastic stiffness formula of the leaf type HDS assemblies, based on the Euler beam theory and the strain energy method considering normal reaction forces and friction forces acting on interfaces between leaves. Another is a finite element model using contact elements between the leaf springs.

2. DERIVATION OF THE EXTENDED ELASTIC STIFFNESS FORMULA

In order to analytically derive the extended elastic stiffness formula of the leaf type HDS assembly shown in Fig. 1, each leaf spring is divided into regions, as designated in Figs. 2 and 3, for the TT-HDS and the TW-HDS, respectively. When a leaf spring is deformed, normal reaction forces and friction forces are acting on interfaces between the leaves. The bending moments, axial and shear forces are obtained from the equilibrium conditions of the free-body diagram in each region of the leaf spring. The procedure to derive the extended elastic stiffness formula is summarized in the following three stages. First, the bending moments, axial and shear forces in each region are put to use to calculate the total strain energy in each leaf. Second, in-line deflections at the loading and reaction points are obtained by applying Castigliano's theorem. Third, the extended elastic stiffness formula is obtained by imposing constraint conditions on the in-line deflections of each leaf. In the following subsections, these procedures are described in detail.

2.1 Total Strain Energy in Each Leaf

When a leaf is deformed, the total strain energy \( U_n \) in each leaf is expressed as [3]:

\[
U_n = \sum_{i=1}^{V} \left( \int \frac{M_i^2}{2E_iI_i} \, dS + \int \frac{P_i^2}{2A_iE_i} \, dS + \int \frac{\tau_i^2}{2G_i} \, dV \right)
\]  

(1)

where,

- \( dV \): Element of volume (or Differential volume)
- \( dS \): Differential length
- \( U_n \): Total strain energy in \( n \)-th leaf
- \( M_i \): Bending moment
- \( E_i \): Elastic modulus
- \( A_i \): Cross-sectional area
- \( P_i \): Axial force
- \( G_i \): Shear modulus
- \( I_i \): Second moment of the beam cross-sectional area
- \( \tau \): Shear stress
- \( i=I, II, III, IV \): Region number of the leaf

2.2 In-line Deflections at Loading (F) and Reaction (F_n) Points

In-line deflections \( \delta \) at the loading and reaction points are obtained by differentiating the total strain energy with respect to the load at that point (Castigliano's theorem [3]).

2.2.1 For the Top Leaf

II-398
\[ \delta_{1n} = \frac{\partial U_1}{\partial F} = AA_1F - AB_1F_{R1} \]  
\[ \delta_{2n} = \frac{\partial U_1}{\partial F_{R1}} = -AB_1F + BB_1F_{R1} \]  

2.2.2 For the Lower (n ≥ 2) Leaf

\[ \delta_{2F_{R1}} = \frac{\partial U_2}{\partial F_{R1}} = BB_2(F_{R1} - F_{R2}) , \text{ for the 2nd leaf} \]  
\[ \delta_{3F_{R2}} = \frac{\partial U_2}{\partial F_{R2}} = BB_2F_{R2} , \text{ for the 3rd leaf} \]  

\[ AA_1, AB_1, BB_1, BB_2, \text{ and } BB, \text{ are coefficients expressed as a function of the design variables and the coefficient of friction } (\mu) \text{ between leaves [4]. } F_{R1} \text{ and } F_{R2} \text{ are the reactions at the reaction points of each leaf, as shown in Figs. 2 and 3.} \]

2.3 Constraint Conditions on the In-line Deflections at the Reaction Points

Assuming that the in-line deflections at the reaction points between the leaves are equal, the constraint conditions are as follows:

\[ \delta_{1F_{R1}} = -\delta_{2F_{R1}} , \text{ for the top and 2nd leaf} \]  
\[ \delta_{2F_{R1}} = \delta_{3F_{R1}} , \text{ for the 2nd and 3rd leaf} \]
a) For the uppermost leaf

![Diagram of uppermost leaf]

b) For the lower ($n \geq 2$) leaf

![Diagram of lower leaf]

Fig. 2. Design variables for each leaf of TT-HDS

2.4 Extended Elastic Stiffness Formula

From the in-line deflections of Eq. (2), Eq. (3), and Eq. (4-a, b) and constraint conditions of Eq. (5-a, b), we can obtain the extended elastic stiffness formula of the leaf type HDS assembly as Eq. (6). Because of different dimensions and shape, the coefficients in Eq. (6) for the TT-HDS are differently expressed from those for the TW-HDS.
\[ K_{\text{anx}} = \frac{F}{\delta_{1F}} = \frac{1}{AA_1 + \frac{AB_1^2}{BB_1} + \sum_{i=2}^{3} \frac{1}{BB_i}} \]  

(6)

a) For the uppermost leaf

b) For the lower leaf

Fig. 3. Design variables for each leaf of TW-HDS
3. VERIFICATION OF THE EXTENDED ELASTIC STIFFNESS FORMULA

In order to check the validity of the extended elastic stiffness formula, characteristic tests on three kinds of specimens that are composed of one leaf, two leaves, and three leaves, were carried out. For each kind of the HDS specimens, five sets of test specimens were prepared. The characteristic test results were compared with the elastic stiffness from the extended formula in the case of both only considering the reaction force ($\mu = 0.0$) and additionally considering the friction force ($\mu = 0.2$). The comparisons for the HDS specimen are shown in Figs. 4 and 5.

![Graphs showing elastic stiffness comparison](image)

a) TT-HDS  
b) TW-HDS  
Fig. 4. Comparison of elastic stiffness from the formulas and test results

![Graphs showing ratio comparison](image)

a) TT-HDS  
b) TW-HDS  
Fig. 5. Comparison of the ratio of elastic stiffness from the formulas and test results
For both the TT-HDS and the TW-HDS specimens composed of only a top leaf, Figs. 4 and 5 show that the ratio of the elastic stiffness from both the previously derived formula [5] and the extended elastic stiffness formula, designated as "present" in Figs. 4 and 5, to the characteristic test result is $1.087 \sim 1.098$. Such an over-estimation of the elastic stiffness from the formulas was assumed to the imposition of different boundary conditions at the root part of the leaf springs. I.e., actually the test specimen was fixed at the root part of the leaf springs by the screw, which allowed the leaf springs to rotate and lead to more deflections. While in the analytical method, all displacements at the root part of the leaf were constrained as clamped conditions. In addition, Figs. 4 and 5 show that as the number of leaves was increased, the ratio of elastic stiffness from the previously derived formula to the test result deviated greatly from the test results up to 1.145 while the ratio for the extended elastic stiffness formula maintained around 1.085 $\sim$ 1.101 ($\mu = 0.0$) and 1.110 $\sim$ 1.121 ($\mu = 0.2$). This fact denotes that the extended elastic stiffness formula was properly considering the reaction forces and friction forces acting on interfaces between leaves, which were not properly considered in the previously derived formula. The reason that the extended elastic stiffness formula gave closer results to the test result was that the friction forces were more realistically considered for the extended elastic stiffness formula ($\mu \leq 0.2$) than that for the previously derived formula ($\mu = \tan \alpha_1 \approx 0.515$).

4. FINITE ELEMENT MODEL

Based on the finite element code ANSYS, finite element models were developed for the leaf type HDS assemblies. Fig. 6 shows a finite element model for the TW-HDS. The model was assembled from 8 node brick elements. Three element layers were used for each leaf and contact elements were used on the interfaces between the leaf springs where the friction force and the normal force are acting. Table 1 represents comparisons of the elastic stiffness of the TW-HDS from between the finite element model and the extended elastic stiffness formula. Table 1 shows that the elastic stiffness from the finite element model agrees well with that from the extended elastic stiffness formula.

![Fig. 6. Finite Element Model for the TW-HDS](image-url)
Table 1. Comparisons of the elastic stiffness of the TW-HDS from the F.E model and
the extended elastic stiffness formula

<table>
<thead>
<tr>
<th>Number of leaves</th>
<th>Elastic stiffness (N/mm)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F.E. Model (A)</td>
<td>Extended elastic stiffness formula (B)</td>
</tr>
<tr>
<td>1</td>
<td>32.17</td>
<td>31.92</td>
</tr>
<tr>
<td>2</td>
<td>63.55</td>
<td>63.93</td>
</tr>
<tr>
<td>3</td>
<td>93.81</td>
<td>96.36</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

The elastic stiffness formula of the leaf type HDS assembly, which was previously
derived, has been extended to additionally consider the normal reaction forces and friction
forces acting on interfaces between leaves. The extended elastic stiffness formula was
verified by comparing the values of the elastic stiffness from the extended formula with the
characteristic test results. Also, finite element models for the leaf type HDS assemblies were
developed using contact elements between the leaf springs. The results from this study are as
follows:

1. For the HDS specimen composed of only the top leaf, the ratios of the elastic
stiffness from the extended elastic stiffness formula to the characteristic test result was
around 1.087 ~ 1.098. And this deviation from the test results was attributed to the
presumptions in derivation of the extended elastic stiffness formula.

2. As the number of leaves was increased, the ratio of the elastic stiffness from the
extended formula to the test result maintains around 1.085 ~ 1.121, while the ratio for the
previously derived formula deviated much. This fact denotes that the extended elastic
stiffness formula was properly considering the reaction forces and friction forces acting on
interfaces between leaves, which were not properly considered in the previously derived
formula.

3. The elastic stiffness from the finite element models of the leaf type HDS assembly
using contact elements between the leaf springs agrees well with that from the elastic
stiffness formula.

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II-404