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Piping Systems Instability-Margin on Elastic Analysis with Local Instability Criterion

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ABSTRACT

Limit load calculation by Elastic Compensation Method is extended to "beam" modelled pipings, using a very conventional equivalent stress. This calculation can be performed with the CASTEM2000 code.

Piping line instability (buckling) calculation is proposed with this method. The aim is to evaluate margins when using the RCC-MR rule (RB3600 chapter) to provide against instability. In fact, this rule is somewhat conservative as it prescribes the use of a local instability criterion after an elastic analysis of loadings of primary and secondary origins. Therefore, it does not integrate the redistribution effects due to plasticity, hence it can be deemed too restrictive.

After a few simple examples which relate to cases whose solution is analytically known, realistic lines are calculated. The significance of line supporting is clearly shown : in some cases, margins in excess of 2 are obtained.

This study illustrates the interest of using such a simple method, requiring little CPU time, for solving the problem of thin piping instability. It could be applied to all piping problems which can be solved by calculation of limit loading, for example, the study of real primarity of loadings of primary origin (weight or seismic loading).

1. INTRODUCTION

The RCC-MR rules (reference 1) as preventive steps against instability (or buckling) of pipings are based on local criteria: from an elastic calculation, one must check, with a given margin based on the analysis level, whether the moments (and forces) exceed or not certain permissible values corresponding to the considered item's failure (straight pipe or elbow). These values were determined based on failure tests or calculations (see reference 2) on a straight pipe or an elbow, hence they correspond to very simple cases where the piping is statically determined.

In the case of actual lines which are often highly statically undetermined particularly due to the presence of many support points, it seems obvious that a local instability can be supported by the entire line which will be really unstable only when several plastic hinges appear. A significant improvement can be expected in terms of failure.

In the scope of a thesis on behaviour of branch pipe connections and (references 3 and 4), a limit load calculation method called "Elastic Compensation Method" (ECM) was integrated into CASTEM 2000 as a procedure. ECM was initially developed for "brick" elements, then extended to "thin shell" elements and tested on numerous loading cases and structures. Therefore, we envisioned the option to extend the ECM method to "piping" elements in order to solve the instability problem of statically undetermined line.

2. PREVENTION RULE AGAINST RCC-MR PIPING INSTABILITY

2.1. Rule Description

The RCC-MR RB3600 chapter rule is expressed as:

$$D_1 \frac{PD}{2e} + \frac{1}{Z} \cdot \sqrt{D_{21}^2 (M_t + g.m_t)^2 + D_{22}^2 (M_f + g.m_f)^2} \quad \leq S* \qquad (1)$$

where S* is a permissible limit linked to the material properties and the criterion level.

P	pressure	
M	primary origin moments	Calculated elastically
m	secondary origin moments	
indices t and f characterise torsion and bending		
g	secondary load relaxation coefficient	
e	thickness	
D	outer diameter	
Z	section modulus	

D coefficients were determined experimentally (reference 2):

D_1 = 0.5

D_{21} = 1.1

D_{22} = $0.35\ (D/e)^{1/3}$ in straight pipes

D_{22} = $1.2 / \lambda^{2/3}$ in a 90° elbow, if the beneficial effect from pressure is neglected.

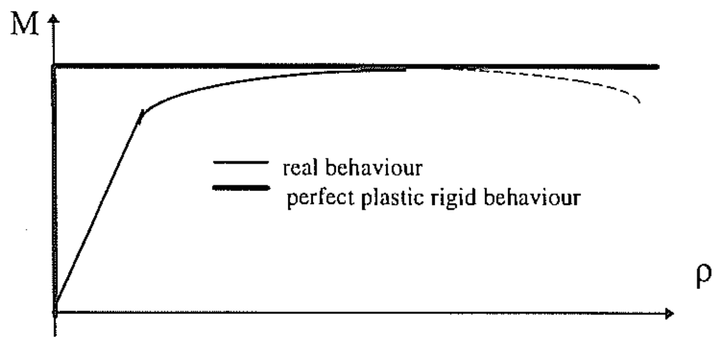
where:
- r_m mean radius
- R elbow curvature radius
- λ eR/ r_m^2 characteristic parameter of elbows

The tests which permitted these coefficients to be defined were carried out on elbows or straight pipes fixed at one end and submitted to a force (or a moment) at the other end, that is to say on statically determined pipings.

The resulting criterion is a local criterion applicable to forces and moments calculated elastically. Therefore, it will be a conservative criterion which can in no case take line static indetermination into account.

2.2. How to Reduce Margins through the Calculation of Line Limit Load

Assuming that the failure behaviour of a component (straight pipe or elbow) can be simulated by a perfect plastic rigid behaviour as in the following diagram in the moment-curvature space,



the available margin can be determined by calculating the limit load of the entire piping line, and by comparing it to the local limit load values.

We could have performed an incremental perfect plastic calculation and therefore approximately determine the limit load at calculation non-convergence. However, we favoured a much more efficient method, which directly provides the limit load.

3. PRESENTATION OF CASTEM 2000 @ANA_LIM PROCEDURE FOR LIMIT LOAD CALCULATION IN A STRUCTURE

3.1. Limit Load Calculation in a Structure through the Elastic Compensation Method (ECM)

The theory of this method already used by various authors were carried out by A.R.S. Ponter (reference 5).

It provides the limit load of a structure (perfect rigid plastic material) through a sequence of uncompressible elastic calculations, with variable Young's modulus values. A.R.S. Ponter prouves that this is equivalent to the perfect plastic calculation in the case of a von Mises type criterion.

A complete description of this method is given in references 3, 4 and 7.

The result obtained is theoretically a upper bound limit load; however, it tends towards the true limit if the meshing is very fine. Simultaneously, a lower bound limit is proposed, derived from the upper bound.

3.2. @ ANA_LIM Procedure in Castem 2000

The ECM method as described in reference 3 was integrated as a "user" procedure (called @ANA_LIM) into CASTEM 2000 (reference 6). This was initially carried out for brick elements, then extended to thin shell elements which use a degenerated von Mises criterion of Illyouchine type.

This method formed the subject of many tests on various structures:

- comparison with incremental plastic calculations
- shell-brick comparison
- comparison with results obtained through simplified limit analysis methods
- comparison with tests

These studies are grouped in references 3 and 4. They illustrate the valuable qualities of this method which provides the same results as an incremental calculation without the load step and convergence problems.

A complete description of @ANA_LIM and its operation is given in reference 7.

4. ECM METHOD EXTENSION TO PIPING INSTABILITY CALCULATION

4.1. Equivalent Stress Loading Used in Castem 2000

The equivalent stress calculated in CASTEM for piping components (and which thus influences the "pipe" plasticity criterion) can be considered as a degenerated Von Mises stress, as in the case of shells. Therefore, the use of ECM with such a total stress should provide a result close to a total incremental calculation, using the same equivalent stress in its plasticity criterion.

In CASTEM, the equivalent stress used in the plasticity criterion is:

$$\sigma^* = \sqrt{\left(\alpha_n \frac{N}{S}\right)^2 + \left(\alpha_p \frac{PD}{2e}\right)^2 + \left(\alpha_t \frac{M_t}{Z_m}\right)^2 + \left(\alpha_f \frac{M_f}{Z_m}\right)^2} \qquad (2)$$

$$\text{with} \begin{cases} \alpha_p = \sqrt{2/2} \\ \alpha_n = 1 \\ \alpha_t = \sqrt{3/2} \\ \alpha_f = \dfrac{\pi}{4}\gamma \quad \begin{cases} \gamma = 1 & \text{in straight pipes} \\ \gamma = 0.89 \dfrac{1}{\lambda^{2/3}} & \text{in elbows} \end{cases} \end{cases}$$

N	normal load
M_t	torsional moment
M_f	resulting bending moment
P	pressure
r_m	mean radius
e	thickness
D	outer diameter
R	elbow radius of curvature
S	area of cross section
λ	eR/r_m^2 elbow characteristic parameter
Z_m	section modulus calculated with the mean radius.

4.2. Piping Formulation of @ANA_LIM

Owing to the form of (2), only very small modifications were required to extend @ANA_LIM to pipings.

4.3. Instability Calculation According Chapter RB 3600

In order to correctly address our problem, i.e. to apply the local instability criteria, the coefficients in Eq. (2) must be those of Eq. (1) or, at least, must be proportional to them, as the perfect plastic material's elastic limit value can be adjusted.

So far, we cannot change CASTEM's α_i coefficients. Until we have access to these coefficients, the calculations presented in this paper are such that:

– P and N are nul

– The elastic limit values are different between the elbows and straight pipes, so that the ratio between bending failure loads of these two components is verified.

Therefore, the ratio between bending failure load and torsion failure load in (1) is not verified and the calculations presented hereafter, concerning the tridimensional lines, are affected by a slight error. However, as the coefficients related to torsion are much lower than those related to bending, our conclusions on the interest of developing such a method will not be amended by more conform calculations.

As we deal with the ratio between entire line failure load and local failure load of the weakest component, the results will be presented as a coefficient defined by:

$$h = \frac{C_L}{C_l} \quad (3)$$

C_L	limit load of the line
C_l	local limit load

5. A FEW RESULTS

5.1. Straight Pipe Fixed at the Ends, under a Distributed Loading

This very conventional case is illustrated in figure 1. In the elastic envelope, the bending moment is twice lower at the centre than at the ends. For a perfect plastic material, at failure, the moments are equal at these three points, which make up three plastic hinges. In this case, @ANA_LIM gives h = 1.3, which is close to the theoretical result (4/3).

5.2. Bidimensional Line Submitted to an Imposed Force

This case is illustrated in figure 2. Three out of four elbows plastify at failure (the plastic hinges are shown in figure 2). @ANA_LIM gives h = 1.07, which is indicative of a rather low margin with respect to plasticity beginning.

5.3. Tridimensional Line under Imposed Displacements

This example is illustrated in figure 3. It is strongly representative of a real line: many elbows and many support points, imposed displacements at the ends, simulating restrained thermal expansion. The straight pipe sections, which are thinner, plastify before the elbows. @ANA_LIM gives h = 2.5, which is indicative of a significant margin with respect to plasticity beginning.

6. SUPPORTS INFLUENCE

The highly interesting result obtained in the latter case led us to study the influence of supports. In fact, this influence seems significant for line static indetermination. A parameter study of bidimensional lines was conducted: h calculations for various line lengths, with supports as defined in the RCC-MR (reference 1), and either weight, or imposed displacement loading. When the line length is increased, h tends towards an asymptotic value. In the case of weight, this value is close to that of the pipe fixed at its ends, under its weight. In the case of imposed displacement at one end, the values are higher and can exceed 2, as was the case with the tridimensional line example. The asymptotic value seems to be dependent on line geometry, and in particular on the position of elbows next to the anchorage points.

9. CONCLUSIONS

By limit load loading calculation in a piping line, we obtained significant instability margins with respect to the local instability criteria. The example of a tridimensional line representative of RNR reactor systems is extremely striking, as a margin of 2.5 is obtained against the local rule of the RCC-MR RB 3600.

For this study, the Elastic Compensation Method (ECM) programmed in CASTEM 2000's @ANA_LIM procedure was extended to pipings and found satisfactory: convergence is easily achieved and the deviations between upper bound and lower bound limit loads are very low. This particular study demonstrates the interest of using such a simple method requiring little CPU time, for the study of instability in thin pipings.

Then, we proceeded with the study of the influence of supports and the type of loading. A fine analysis of the results obtained, and the continuation of the parameter calculations, should allow us to achieve a complete understanding of the involved phenomena. As regards line instability, our final purpose is to propose an alternative method, when the local instability rule is not verified.

The application of this study to piping line instability is only one of the possible application cases for this method. It could also be applied in determining the real primarity of loadings of primary origin in highly statically undetermined systems.

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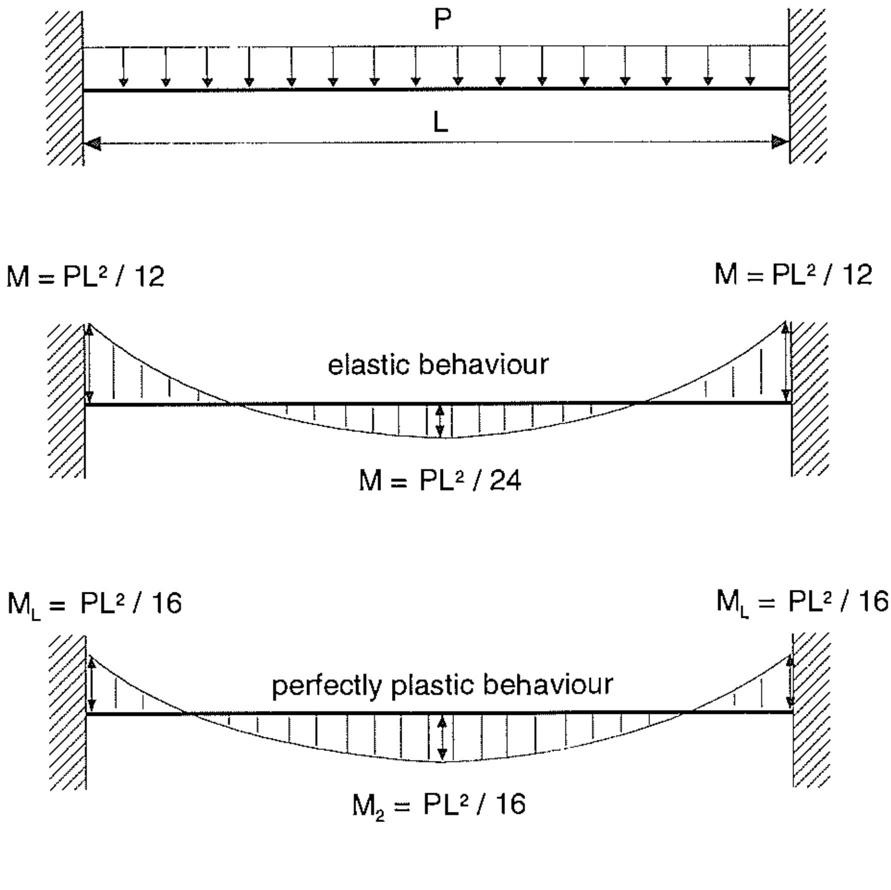

Figure 1 : Pipe embedded at the ends submitted to a distributed load

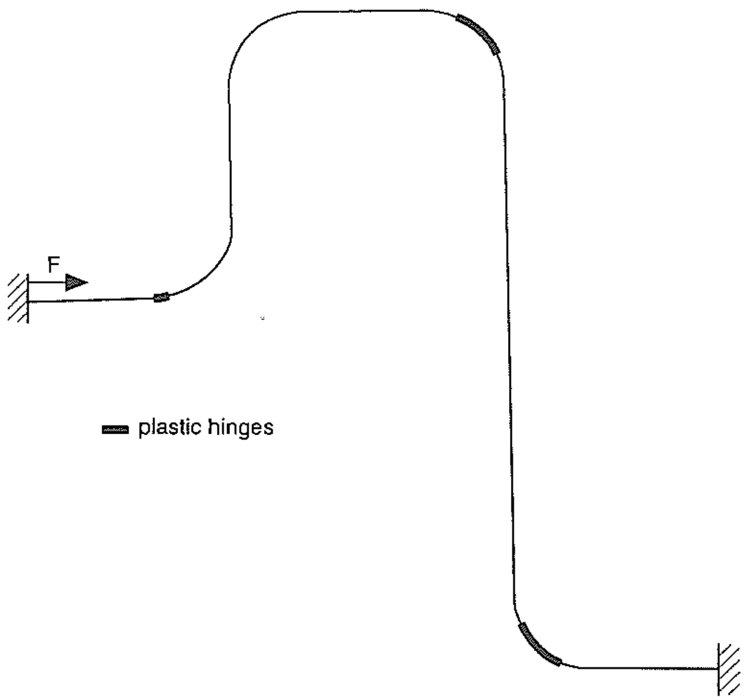

Figure 2 : Bidimensional line

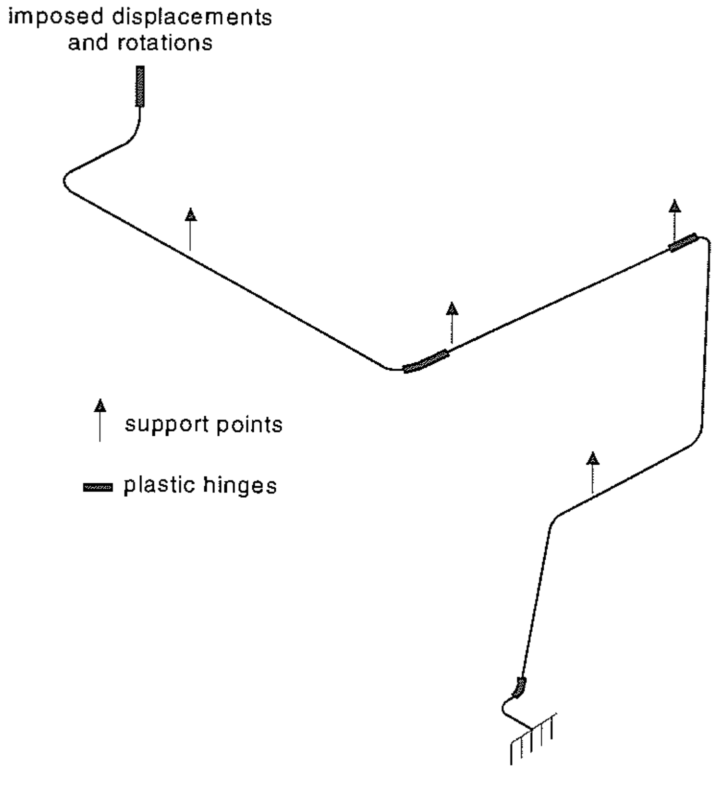

Figure 3 : Tridimensional line